Physics 70006 **Exam 2** **April 12, 2006**

This examination must be completed *independently* by each student. Consulting books and notes is permitted; consulting other students is not! If questions about the meaning or wording arise, contact me [Office: 334-335 NSH, phone: 1-6651, email: johnson@nd.edu]

Return the exam in class on April 19.

1. The electronic charge density of a hydrogen-like ion in a $2p$ state with magnetic quantum number $m = 0,\ \pm 1$ is

$$\rho_m(\boldsymbol{r}) = \frac{e}{r^2}|P_{2p}(r)|^2|Y_{1m}(\theta,\phi)|^2,$$

where e is the charge on the electron and where the radial wave function is

$$P_{2p}(r) = \frac{1}{2\sqrt{6}}\left(\frac{Z}{a_0}\right)^{5/2} r^2 e^{-Zr/(2a_0)}$$

   (a) For each value of m, determine *all* multipole moments of the charge distribution and give the complete multipole expansion of the potential.

   (b) For each value of m, determine the leading terms (through order r^2/a_0^2) in a series expansion of the potential for *small* values of r.

2. A charge q is located on the z axis at distance $d < a$ from the center of a dielectric spherical shell of inner radius a, outer radius b, and dielectric constant ϵ_r. (Caution: Avoid the method of images.)

   (a) Show that the first two terms in the asymptotic expansion of the potential in the region $r > b$ are

$$\Phi(r,\theta) = \frac{1}{4\pi\epsilon_0}\left[\frac{1}{r} + \frac{9\,d\,\epsilon_r}{(\epsilon_r+2)(2\epsilon_r+1) - 2(a/b)^3(\epsilon_r-1)^2}\,\frac{\cos\theta}{r^2}\right]$$

   (b) Find the *cartesian* coordinates of the electric field at the origin.

   (c) Verify the correctness of your answers in the limit $\epsilon_r \to 1$.

   (d) Verify the correctness of your answers in the special case $b = a$.

3. A long cylindrical tube (inner radius a, outer radius b, permeability μ) with axis along z is placed in a uniform field of magnitude B_0 directed along the x axis.

   (a) Determine $\boldsymbol{B}$ everywhere. (Hint: Introduce a magnetic scalar potential Φ_m and solve the resulting boundary value problem.)

   (b) Show that $\boldsymbol{B}_{\text{in}}$, the flux density inside the tube, is uniform and directed along the x axis. Find the ratio B_{in}/B_0 assuming that $\mu/\mu_0 = 5000$ (99.8% pure iron) and $b/a = 5/4$.

(c) Sketch B-field for the case $b/a = 5/4$ and $\mu >> \mu_0$.

4. A surface current of density $\boldsymbol{K} = (I/2a)\sin\phi\,\hat{z}$ flows in the axial direction on the surface of a hollow cylinder of radius a.

   (a) Determine the $\boldsymbol{B}$ field inside and outside the cylinder.

   (b) Determine the energy per unit length stored inside and outside the cylinder. How is the energy divided between the two regions?

   (c) Determine the self-inductance/length of this system.