The special theory of relativity and its paradoxes

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1 Introduction to the special theory of relativity

1.1 The principle of relativity, the speed of light, and Galilean relativity

One way into the the theory of relativity is by seeing it as the resolution of a kind of paradox: the incompatibility of the following three plausible claims about the physical world:

The principle of relativity. If you are moving at a constant speed with respect to me, the laws of nature are the same in your frame of reference as in mine.

Galilean relativity. For any two objects moving at any speeds, their relative speed if they're moving in the same direction is the difference between their speeds, and if moving in opposite directions is the sum of their speeds.

> Of course we must refer the process of the propagation of light (and indeed every other process) to a rigid reference-body (co-ordinate system). As such a system let us again choose our embankment. We shall imagine the air above it to have been removed. If a ray of light be sent along the embankment, we see from the above that the tip of the ray will be transmitted with the velocity c relative to the embankment. Now let us suppose that our railway carriage is again travelling along the railway lines with the velocity v, and that its direction is the same as that of the ray of light, but its velocity of course much less. Let us inquire about the velocity of propagation of the ray of light relative to the carriage. It is obvious that we can here apply the consideration of the previous section, since the ray of light plays the part of the man walking along relatively to the carriage. The velocity W of the man relative to the embankment is here replaced by the velocity of light relative to the embankment. w is the required velocity of light with respect to the carriage, and we have
>
> $$w = c - v.$$
>
> The velocity of propagation of a ray of light relative to the carriage thus comes out smaller than c.

Einstein's discussion of the conflict between Galilean relativity and the constancy of the speed of light.

It is a law of nature that the speed of light is constant. (We'll follow convention by using 'c' to stand for the speed of light.)

If these three theses are inconsistent, the question arises which one we should give up. The principle of relativity seems untouchable, and Galilean relativity seems obviously correct. This makes it plausible that we should give up the idea that the speed of a beam of light is the same in all frames of reference. However, experiments designed to detect differences in the speed of light in different intertial frames failed to detect any differences. Einstein's response was to hold on to the idea that the speed of light is the same in all inertial frames, and give up Galilean relativity.

1.2 The relativity of simultaneity

Suppose that we hold to the idea that the speed of light is the same in all frames of reference, and give up Galilean relativity. It seems to follow that simultaneity is relative: that two events can be simultaneous viewed from one frame of reference, but not from another.

Consider the example (from Sartori, *Understanding Relativity*): a light is turned on at the midpoint of a moving train. Suppose first that you are on the train; in this case, the light travels the same distance to the front and rear of the train, so that the events of the light arriving at the front and rear of the train are simultaneous.

Suppose now that you are outside the train, watching it go by. In the time between the light being turned on and it reaching the front and rear of the train, the train moves some distance. So, it seems that the light must travel further to reach the front of the train than the rear of the train. Since the speed of light is (contra Galilean relativity) the same in both directions from the perspective of an observer outside the train as well as the observer on the train, the light arrives at the rear of the train earlier than at the front.

So the two events are simultaneous in one frame of reference, but not the other. Can you think of a way to change the set-up so that the time order of the events is reversed (so that one is earlier in one frame of reference, and the other earlier in the other)?

1.3 Time dilation

Now ask a slightly different question about the example of the moving train. Suppose as before that the light is turned on in the center of a carriage at the midpoint of the train, and that it bounces off of a mirror placed on a window at the midpoint of the train, back to the center of the carriage. Consider the time it takes from the light being turned on until it is reflected back to its origin, as measured by someone in the train, and then by someone on the embankment, outside the train. How would the two measurements compare? Would each observe the light traveling the same distance?

This kind of case is enough to show that the time interval between two events can differ depending one one's frame of reference. In particular, the time interval between any two events at one place in one frame of reference will be less than the time interval between those two events in some other frame of reference (with respect to which they will not occur at the same place). You could say that if A is in motion relative to B, time 'slows down' for A (relative to time for B). How *much* time slows down will depend on A's speed, as reflection on the example of the train shows.

1.4 Length contraction

Suppose you to measure the length of a train which is at rest. The natural procedure is to mark where the front of the train is, mark where the rear of the train is, and see what the distance is between the two marks.

Now suppose you want to measure the length of a train which is passing by as you stand on the platform. One procedure you could use, if you know the speed of the train, would be to measure the time interval between the front of the train passing you and the rear of the train passing you. Using the speed, you could then calculate the length of the train.

Consider now the event of the front of the train passing you, and the event of the rear of the train passing you, as you stand on the platform outside of the train. From the discussion of time dilation above, it follows that the interval between these events is shorter from your perspective than from the perspective of someone on the train.

But now suppose that we use these measurements to determine the length of the train. Since you and the people on the train agree about the relative speed of the train, it seems that you will arrive at different views about the length of the train. In particular, it seems that you will arrive at the view that the length of the train is less than it is relative to the frame of reference in which the train is located.

2 Paradoxes of special relativity

2.1 The twin paradox

Think about the argument above that if A is in motion relative to B, time 'slows down' for A Isn't B also in motion relative to A? Does that mean that time also slows down for B, relative to A? But how can time be relatively slower for each of them? What if they meet up, and compare watches?

Imagine, in particular, two twins, each of whom is moving relative to the other. We seem to have arguments that time moves more slowly — and hence that aging happens more slowly — for each, since each is in motion relative to the other. But if they met up, it surely couldn't be the case that each is younger than the other.

What would it take for them to meet up?

2.2 The pole and barn paradox

Suppose that a pole 10 meters in length is moving very fast in the direction of a barn whose front and back doors are located on the axis on which the pole is moving. The doors are 10 meters apart.

Since the pole is moving very fast, it is contracted in length quite a bit from the perspective of the frame of reference of the barn. So, it fits easily into the barn.

Since the barn is also moving quite fast from the frame of reference of the pole, it is contracted relative to the pole. So the distance between the barn doors is less than 10 m, and the pole can at no point be wholly located within the barn.

Which is it?

2.3 The relativity of the present

One immediate consequence of the theory of special relativity seems to be a challenge to views like the following:

> Past and future things do not exist; only the present exists.

The problem is with understanding what 'the present' could mean. Clearly, it is functioning as the name of a time. But which events are present ones? The natural thought is that present events are ones simultaneous with what I am doing right *now*. But which events these are will depend upon my frame of reference. This seems to imply that which events are present is relative to a frame of reference; which, with the above, implies that which things exist is relative to a frame of reference. But it is hard to understand how existence could be relative to a frame of reference in this way.

One response is to give up views such as the one stated above, and say that the present has no special status as compared to the past and future.