Zeno's paradoxes

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Zeno presented four main paradoxes, each of which was designed to show the impossibility of motion. They can be thought of as breaking down into two sub-arguments: one assumes that space and time are continuous — in the sense that between any two moments of time, or locations in space, there is another moment in time, or location, in space — or one assumes that they are not. Either way, Zeno will argue, motion is impossible.

(An example of something which is continuous is the set of real numbers; the series of natural numbers, on the other hand, is not continuous.)

1 If space and time are continuous, motion is impossible

First, suppose that space and time are continuous. Zeno presents two paradoxes to show that, on this supposition, motion is impossible.

<u>The Racetrack</u>

Imagine that you are trying to move from point A to point B. Suppose C is the midpoint of the distance from A to B. It seems that you have to first get from A to C, before you can get from A to B. Now suppose that D is the midpoint between A and C; just as above, it seems that you have to first get from A to D before you can get from A to C. Since space is infinitely divisible, this process can be continued indefinitely. So it seems that you need to complete an infinite series of journeys before you can travel any distance — even a very short one!

The argument can be laid out like this:

1. Any distance is divisible into infinitely many smaller distances.
2. To move from a point x to a point y, one has to move through all the distances into which the distance from x to y is divisible.
3. To move from one point to another in a finite time, one has to traverse infinitely many distances in a finite time. (1,2)
4. It is impossible to traverse infinitely many distances in a finite time.

C. It is impossible to move from one point to another in a finite time. (3,4)

<u>The Achilles</u>

Suppose that the Tortoise and Achilles are racing to some point z, and that Achilles begins at point x, and the Tortoise begins at point y, where y is between x and z. Then we argue as follows that no matter what distances are involved, no matter how slow the Tortoise is, and no matter how fast Achilles is, Achilles can never catch the Tortoise, so long as the Tortoise never stops moving:

1. To traverse the distance between x and y, Achilles requires some interval of time.
2. During every interval of time, the Tortoise moves some distance.

C. By the time Achilles reaches y, the Tortoise is some distance beyond y. (1,2)

Since we made no particular assumptions about the distance between x and y or the length of the interval of time, it appears that this argument can be repeated infinitely many times, so show that even after an infinite number of movements and intervals of time, the Tortoise is still ahead of Achilles.

This seems to allow us to conclude:

> For every interval of time, at the end of that interval the Tortoise is still ahead of Achilles.

which means that Achilles never catches the Tortoise.

Does this argument require any assumptions about the infinite divisibility of time or space?

2 Infinite tasks

Clearly, both of these arguments turn on the impossibility of completing tasks that are divisible into infinitely many sub-tasks. But why is this supposed to be impossible?

One thought is that it is supposed to be impossible for finite beings because completing infinitely many sub-tasks would take an infinitely long time. Suppose that each of the sub-tasks takes a finite (nonzero) amount of time. Isn't the product of an infinite number and a finite number an infinite number? Does this show that completing infinitely many sub-tasks each of which takes some finite amount of time would take an infinite length of time?

Here's a different kind of argument to show that it is impossible to perform infinitely many tasks in a finite time:

> Suppose that a lamp is turned on and off infinitely many times during some finite interval — say, between 3 and 4 pm — that is, the button on the lamp is pressed infinitely many times during that hour. Before 3:00 and after 4:00, the button is never pushed. Now, at 4:15, is the light on or off? Every time the light is turned on, this is followed by it being turned off; so it cannot be on. But every time the light is turned off, this is followed by it being turned on; so it cannot be off. So the light must be neither on nor off. But that is impossible.

3 If space and time are discrete, motion is impossible

Suppose that we avoid these arguments by holding that space and time are discrete rather than continuous. Zeno presents two further paradoxes to show that, even if this is true, motion is still impossible.

<u>The Arrow</u>

1. At any one instant, an arrow does not move.
2. Nothing happens between one instant and the next.
3. The arrow does not move between instants. (2)

C. The arrow does not move. (1,3)

What assumption about time is required to make sense of talk about 'the next instant'?

Is it valid to infer that something does not move from the assumption that there is no moment at which it moves? Compare the following inference: I did not walk to school at any of the moments between my waking and now; therefore, I did not walk to school today.

The Stadium

Remember that we are supposing that space and time are not continuous, so that we can speak of the point in space next to another (i.e., with no space in between), and the moment in time after another (i.e., with no intervening moment).

Now consider the following arrangement of particles, each occupying a spot in a grid of adjacent points in space:

Moment 1

	a	a'	a''	
	b	b'	b''	
	c	c'	c''	

Now suppose that in the next moment, each of the a-particles moves one spot to the right, and each of the c-particles moves one spot to the left. Then we're left with the following configuration of particles in space:

Moment 2

		a	a'	a''
	b	b'	b''	
c	c'	c''		

Now consider, for example, particle a and particle c'. In Moment 1, a is to the left of c'; in Moment 2, a is to the right of c'. So it seems that at some point, a must have passed c'. But this is impossible, since it did not happen at Moment 1, and did not happen at Moment 2, and by hypothesis there is no moment between the two.