Decentralized Event-triggered Broadcasts over Networked Control Systems

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Abstract. This paper examines event-triggered broadcasting of state information in networked control systems. Event-triggering has the agent broadcast its state information when its local "error" signal exceeds a given threshold. We present a decentralized approach for determining event-triggering thresholds for both linear and nonlinear subsystems with the assumption that each agent only has access to its local state. The main results of this paper show that our decentralized event triggering scheme guarantees the asymptotic stability of the entire networked control system. For nonlinear systems these conditions are characterized as Hamilton-Jacobi-Isaacs (HJI) inequalities. For linear systems the conditions simplify to a linear matrix inequality (LMI) feasibility problem. Simulation results show that event-triggered systems outperform comparable periodically triggered systems when the number of subsystems is relatively small. Contention within the communication network, however, eventually erodes the performance benefits of the event-triggered scheme so that in highly congested networks periodically triggered broadcasts have the performance edge.

1 Introduction

A networked control system (NCS) is a collection of control systems where individual controllers exchange information over some communication network. Networking refers to not only the communication infrastructure supporting feedback control, it also refers to the fact that individual subsystems may be interconnected in a way that can be modelled as a network. Specific examples of NCS include electrical power grids and transportation networks. The networking of control effort can be advantageous in terms of lower system costs due to streamlined installation and maintenance costs. Such distributed systems may have higher reliability since the failure of no single subcomponent will bring down the entire system.

The introduction of communication network infrastructure, however, raises new challenges regarding the impact that communication reliability has on the control system's performance. Communication channels are customarily accessed

* The authors gratefully acknowledge the partial financial support of the National Science Foundation (NSF-ECS0400479)

in a mutually exclusive manner. In other words, only one agent can broadcast its state information at a time. So one important issue in the implementation of such NCSs is to identify the broadcast decision logics that can provide guarantees on overall system performance. In addition, the broadcast decision should be made locally since there is no central computer to make broadcast scheduling decisions.

This paper addresses this issue through the use of an event-triggered broadcast scheme. Event-triggering has the agent broadcast its state information when its local "error" signal exceeds a given threshold. We present an approach for selecting event-triggering thresholds that assure the asymptotic stability of the group. Our analysis applies to both linear and nonlinear subsystems. The event-triggering scheme is "decentralized" in that a controller's broadcast decisions are made using its local state and the last received state information from its neighbors. It is also "decentralized" in that the designer's selection of the threshold also only requires information about an individual subsystem and its immediate neighbors. For nonlinear subsystems, these thresholds are characterized by feasible solutions to Hamilton-Jacobi-Isaacs (HJI) inequalities. For linear systems, these conditions simplify to an LMI (linear matrix inequality) feasibility problem. Preliminary simulation results show that when a controller has a limited number of neighbors, the event-triggered systems perform better than systems using periodically triggered broadcasts. As the number of neighbors increases, our results show that periodically triggered broadcasts eventually have better performance than comparable event-triggered schemes.

This paper is organized as follows. Section 2 discusses the prior work. The problem is formulated in section 3. Our event-triggering scheme for a general class of nonlinear subsystems is presented in section 4. Section 5 specializes these results to linear subsystems. Simulation results examining the scalability of event-triggered and periodically-triggered broadcasts are presented in section 6. Final conclusions are found in section 7.

2 Prior Work

Early works analyzing scheduling of real-time network traffic were presented in [2] and [3]. However, the impact of communication constraints on system performance was not been addressed in these works. [4], [5], [6] noticed the harmful effect of the communication delay on the system stability and considered the one packet transmission problem, where all of the system outputs were packaged into a single packet. As a result, agents in the network do not have to compete for channel access. One packet transmission strategies, however, use a supervisor to summarize all subsystem data into this single packet. As a result such schemes may be impractical for large-scale systems.

Asynchronous broadcasts were considered in [7]. This work derived bounds on the maximum admissible time interval (MATI) that a message can be delayed while still maintaining closed loop system stability. It led to scheduling methods [8] that were able to assure the MATI was not violated. This work, however, estimated MATI bounds in a "centralized" way since information from all systems was needed to estimate the bound. Furthermore, all of the previous work confined its attention to control area network (CAN) buses where centralized computers can be used to schedule communication.

In recent years there has been considerable interest in developing distributed controllers over ad hoc wireless networks [9]. The problem faced in using wireless networks is that their throughput capacity is limited [10]. As network density increases, the throughput seen by an individual agent asymptotically approach zero. There is, therefore, great interest in being able to develop networked control systems which are extremely frugal in their use of network bandwidth.

One approach for reducing the bandwidth requirements within a networked control system is to reduce the frequency with which agents communicate. Unlike the aforementioned CAN buses, NCS that use multi-hop wireless networks as their communication infrastructure must schedule message transmission in a decentralized manner. Each controller in such systems is a potential router for a message. The problem, therefore, turns to finding a localized strategy for deciding when to broadcast state information.

This paper addresses this problem through a decentralized event-triggering scheme. In particular, we want to adaptively adjust agent broadcasts in a manner that is sensitive to what is currently happening within the system. One approach for doing this is to use event-triggered broadcasts. Event-triggering has a subsystem broadcast its state information only when "needed". In this case, "needed" means that some measure of the agent's state error is above a specified threshold. There is a great deal of recent research [1], [11], [12], [13] dealing with event-triggered feedback. All of this prior work, however, has focused on using event-triggered feedback in single processor real-time systems. The novelty of our paper is its consideration of event-triggering in networked systems following an approach we laid out earlier in [14].

3 Problem Formulation

In this section, the system dynamics and control objective are defined. Consider an N-agent nonlinear distributed system. Let $\mathcal{N} = \{1, 2, \cdots, N\}$.

Notation: $Z_i \subset \mathcal{N}$ denotes the set of agents whose state information is accessible by agent i (so-called "information set of agent i"). $D_i \subset \mathcal{N}$ denotes the set of agents that directly drive agent i's dynamics. $U_i \subset \mathcal{N}$ denotes the set of agents that can receive agent i's broadcasted information. $S_i \subset \mathcal{N}$ denotes the set of agents who are directly driven by agent i's dynamics. $x_i : \mathbb{R} \to \mathbb{R}^{n_i}$ is the ith agent's state trajectory, $u_i : \mathbb{R} \to \mathbb{R}^{m_i}$ is the ith agent's control variable, and $x_{i0} \in \mathbb{R}^{n_i}$ is the initial state of agent i. $x = (x_1^T, \cdots, x_N^T)^T$ is the overall system state, $x_0 = (x_{10}^T, \cdots, x_{N0}^T)^T$ is the overall initial state, and $u = (u_1^T, \cdots, u_N^T)^T$ is the overall input. Let $T_i = D_i \cup Z_i$, $\bar{n} = \Sigma_{j\in\mathcal{N}} n_j$, and $\bar{m} = \Sigma_{j\in\mathcal{N}} m_j$; for a given set $S \subseteq \mathcal{N}$, we let $n_S = \sum_{j\in S} n_j$ and $x_S = \{x_j\}_{j\in S}$.

The system dynamics of agent $i \in \mathcal{N}$ are defined by the following equations

$$\begin{aligned} \dot{x}_i(t) &= f_i(x_{D_i}, u_i) \\ u_i(t) &= \gamma_i(x_{Z_i}) \\ x_i(0) &= x_{i0} \end{aligned} \tag{1}$$

where $\gamma_i : \mathbb{R}^{n_{Z_i}} \to \mathbb{R}^{m_i}$ is the given feedback strategy of agent i satisfying $\gamma_i(0) = 0$, and $f_i : \mathbb{R}^{n_{D_i}} \times \mathbb{R}^{m_i} \to \mathbb{R}^{n_i}$ is a given function satisfying $f_i(0,0) = 0$.

In particular we assume the closed-loop system 1 is asymptotically stable. So there exists a smooth, proper, positive-definite function $V : \mathbb{R}^{\bar{n}} \to \mathbb{R}$, such that

$$\sum_{i \in \mathcal{N}} \frac{\partial V}{\partial x_i} f_i(x, \gamma_i(x)) \leq 0 \tag{2}$$

and the equality holds if and only if $x = 0$.

This paper focuses on distributed control systems, where each agent broadcasts its state information to its neighbors if its local "error" signal exceeds a given threshold. In this system, each agent can only detect its own state and broadcast it to other agents in an aperiodic fashion. We assume there is no delay, namely the time spent in sampling and receiving the signal is negligible (The delay case can be easily extended by using the techniques in [1] and [15]). Agent i's broadcasting task is characterized by a monotone increasing sequence of time instants, $\{b_k^i\}_{k=1}^{\infty}$, where b_k^i denotes the time instant when agent i broadcasts its state for the kth time (so-called "broadcast times"). b_k^i can also be viewed as agent i's sampling time since we assume there is no delay between sampling and broadcasts.

Agent i's control, u_i, at time t is computed based on its neighbors' latest broadcast states (also called "measured states") at time t, denoted as $\hat{x}_{Z_i}(t)$. Notice that, in our discussion, i's neighbors include agent i itself. The control signal used by agent i is held constant by a zero-order hold (ZOH) until one of its neighbors makes another broadcast. This means that the distributed system satisfies the following state equations,

$$\begin{aligned} \dot{x}_i(t) &= f_i(x_{D_i}, u_i) \\ u_i(t) &= \gamma_i(\hat{x}_{Z_i}(t_k)) \end{aligned} \tag{3}$$

for $t \in [t_k, t_{k+1})$, $k = 1, \ldots, \infty$. Here t_k represents the kth time instant when any agent broadcasts. In fact, the sequence of $\{t_k\}_{k=1}^{\infty}$ is the sorted sequence of the elements in the set $\{b_k^j \mid k \in \mathbb{N}, j \in \mathcal{N}\}$. Notice that $\hat{x}_{Z_i}(t) = \hat{x}_{Z_i}(t_k)$ for all $t \in [t_k, t_{k+1})$.

4 Decentralized Broadcast-triggering Events Design

This section derives a threshold condition for event-triggering. The triggered event causes the agent to broadcast its state information to its neighbors. We're interested in determining condition under which such event-triggering preserves the system's asymptotical stability. In the following discussion, we use $|S| \in \mathbb{N}$ to denote the number of the elements in a given set S, $\|\cdot\|_2$ to denote 2-norm of a vector, and $\|\cdot\|$ to denote the matrix norm.

Theorem 1. *For system 3, assume that there exists a smooth, proper, positive-definite function $V : \mathbb{R}^{\bar{n}} \to \mathbb{R}$, such that the following inequality*

$$\sum_{i\in\mathcal{N}} \frac{\partial V}{\partial x_i} f_i(x_{D_i}, \gamma_i(y_{Z_i})) \leq \sum_{i\in\mathcal{N}} \phi_i(x_i, y_i) \tag{4}$$

holds, where $\phi_i : \mathbb{R}^{n_i} \times \mathbb{R}^{n_i} \to \mathbb{R}$ is a continuous function and $\phi_i(x_i, x_i)$ is negative definite for all $i \in \mathcal{N}$. If for any $i \in \mathcal{N}$, the broadcast sequence $\{b^i_k\}_{k=1}^{\infty}$ satisfies

$$\phi_i(x_i(t), x_i(b^i_k)) \leq 0 \tag{5}$$

for $t \in [b^i_k, b^i_{k+1})$, then system 3 is asymptotically stable.

Proof. Note that, if agent i broadcasts its state, equation 5 will be trivially satisfied because $\phi_i(x_i, x_i)$ is negative definite. Therefore, $b^i_k \leq b^i_{k+1}$ will always hold.

Consider $\dot{V}$ over the time interval $[t_k, t_{k+1})$. Assume that the current measured state at time t is $\left(x_1(b^1_{k_1})^T, \cdots, x_N(b^N_{k_N})^T\right)^T$. Therefore, according to equation 4, the inequality

$$\dot{V} \leq \sum_{i\in\mathcal{N}} \phi_i \left(x_i(t), x_i(b^i_{k_i})\right) \tag{6}$$

holds for $t \in [t_k, t_{k+1})$.

Based on the definition of $\{t_k\}_{k=1}^{\infty}$ in equation 3, we have $[t_k, t_{k+1}) \subseteq [b^i_{k_i}, b^i_{k_i+1})$ for any $i \in \mathcal{N}$. Therefore, by equation 5 and 6, we have

$$\dot{V} \leq 0 \tag{7}$$

for any $t \in [t_k, t_{k+1})$. Since k is arbitrarily selected, $\dot{V} \leq 0$ holds for all $t > 0$. By the assumption that $\phi_i(x_i, x_i)$ is negative definite, we know $\dot{V}$ will stay at 0 if and only if $x_i = x_i(b^i_{k_i}) = 0$ for all $i \in \mathcal{N}$, which implies that system 3 is asymptotically stable. □

Remark 1. Equation 4 implies that the growth rate of the total system's "energy" can be partitioned in N pieces, $\phi_i(x_i, x_i(b^i_{k_i}))$. Each piece is related to only one agent so that the agent just needs to take care of its own piece.

Theorem 1 shows the structure of the broadcast event trigger. Essentially, the theorem says that under the structural conditions in equations 4, the threshold function implicit in equation 5 can be used to assure the overall system's asymptotic stability. We now address the question of how to locally construct such threshold functions.

Before addressing this question in theorem 2, we must define $\delta \in \mathbb{R}^+$ and continuous functions $\beta_i, \psi_i : \mathbb{R}^{n_i} \to \mathbb{R}^{n_i}$, $i = 1, \cdots, N$, which are known to each agent. In other words, δ and the collection of $\{\beta_j\}_{j\in\mathcal{N}}$, $\{\psi_j\}_{j\in\mathcal{N}}$ are selected at the very beginning of the entire design procedure.

The following theorem presents a decentralized design scheme by which each agent constructs its threshold function.

Theorem 2. *For system 3, assume that there exist continuous functions $\psi_i : \mathbb{R}^{n_i} \to \mathbb{R}^{n_i}$ and $L_i : \mathbb{R}^{n_{T_i}} \to \mathbb{R}^+$, $i = 1, \cdots, N$, such that for any $x_{Z_i}, y_{Z_i} \in \mathbb{R}^{n_{Z_i}}$*

$$\|f_i(x_{D_i}, \gamma_i(y_{Z_i})) - f_i(x_{D_i}, \gamma_i(x_{Z_i}))\|_2 \leq L_i(x_{T_i})\|\psi_{Z_i}(y_{Z_i}) - \psi_{Z_i}(x_{Z_i})\|_2, \quad (8)$$

where $\psi_{Z_i}(x_{Z_i}) = \{\psi_j(x_j)\}_{j\in Z_i} \in \mathbb{R}^{n_{Z_i}}$. Given a constant $\delta \in \mathbb{R}^+$ and continuous functions $\beta_i(x_i)$, $i = 1, \cdots, N$, if there exist smooth positive-definite functions $V_i : \mathbb{R}^{n_i} \to \mathbb{R}$ and continuous functions $\alpha_i : \mathbb{R}^{n_i} \to \mathbb{R}$, $i = 1, \cdots, N$, such that

$$-\alpha_i(x_i) + (|S_i \cup U_i| - 1)\beta_i(x_i) \text{ is negative definite} \quad (9)$$

$$\frac{\partial V_i}{\partial x_i} f_i(x_{D_i}, \gamma_i(x_{Z_i})) + \frac{1}{2\delta} L_i^2(x_{T_i}) \frac{\partial V_i}{\partial x_i}\left(\frac{\partial V_i}{\partial x_i}\right)^T \leq -\alpha_i(x_i) + \Sigma_{j\neq i, j\in T_i}\beta_j(x_j). \quad (10)$$

then the threshold functions $\phi_i : \mathbb{R}^{n_i} \times \mathbb{R}^{n_i} \to \mathbb{R}$ defined by

$$\phi_i(x_i, y_i) = -\alpha_i(x_i) + (|S_i \cup U_i| - 1)\beta_i(x_i) + \frac{|U_i|\delta}{2}\|\psi_i(y_i) - \psi_i(x_i)\|_2^2 \quad (11)$$

satisfy equation 4.

Proof. It is obvious that $\phi_i(x_i, x_i)$ is negative definite because of assumption 9. We now consider ϕ_i's satisfaction of equation 4 with $V(x) = \sum_{i\in\mathcal{N}} V_i(x_i)$. In particular using equation 8 we see that

$$\begin{aligned}
&\sum_{i\in\mathcal{N}} \frac{\partial V_i}{\partial x_i} f_i(x_{D_i}, \gamma_i(y_{Z_i})) \\
&\leq \sum_{i\in\mathcal{N}} \frac{\partial V_i}{\partial x_i} f_i(x_{D_i}, \gamma_i(x_{Z_i})) + \sum_{i\in\mathcal{N}} \left\|\frac{\partial V_i}{\partial x_i}\right\|_2 \|f_i(x_{D_i}, \gamma_i(y_{Z_i})) - f_i(x_{D_i}, \gamma_i(x_{Z_i}))\|_2 \\
&\leq \sum_{i\in\mathcal{N}} \frac{\partial V_i}{\partial x_i} f_i(x_{D_i}, \gamma_i(x_{Z_i})) + \sum_{i\in\mathcal{N}} \left\|\frac{\partial V_i}{\partial x_i}\right\|_2 L_i(x_{T_i})\|\psi_{Z_i}(y_{Z_i}) - \psi_{Z_i}(x_{Z_i})\|_2 \\
&\leq \sum_{i\in\mathcal{N}} \frac{\partial V_i}{\partial x_i} f_i(x_{D_i}, \gamma_i(x_{Z_i})) + \sum_{i\in\mathcal{N}} \left[\frac{L_i^2(x_{T_i})}{2\delta}\left\|\frac{\partial V_i}{\partial x_i}\right\|_2^2 + \frac{\delta}{2}\|\psi_{Z_i}(y_{Z_i}) - \psi_{Z_i}(x_{Z_i})\|_2^2\right] \\
&= \sum_{i\in\mathcal{N}} \left[\frac{\partial V_i}{\partial x_i} f_i(x_{D_i}, \gamma_i(x_{Z_i})) + \frac{L_i^2(x_{T_i})}{2\delta}\frac{\partial V_i}{\partial x_i}\left(\frac{\partial V_i}{\partial x_i}\right)^T + \frac{|U_i|\delta}{2}\|\psi_i(y_i) - \psi_i(x_i)\|_2^2\right]
\end{aligned}$$

From equation 10 this can be reduced to

$$\begin{aligned}
&\sum_{i\in\mathcal{N}} \frac{\partial V_i}{\partial x_i} f_i(x_{D_i}, \gamma_i(y_{Z_i})) \\
\le &\sum_{i\in\mathcal{N}} \left[(|S_i \cup U_i| - 1)\beta_i(x_i) - \alpha_i(x_i) + \frac{|U_i|\delta}{2} \|\psi_i(y_i) - \psi_i(x_i)\|_2^2 \right] \\
= &\sum_{i\in\mathcal{N}} \phi_i(x_i, y_i)
\end{aligned}$$

which implies the satisfaction of equation 4. □

Remark 2. In theorem 2, the only things agent i can determine are V_i and α_i. Therefore, agent i's local problem is to construct V_i and α_i such that equation 9 and 10 hold. Once such V_i and α_i are found, we can use equation 5 to construct the event-triggering threshold logic. In this case the ith agent's $k+1$st broadcast time b^i_{k+1} is triggered by the violation of the following inequality

$$-\alpha_i(x_i) + (|S_i \cup U_i| - 1)\beta_i(x_i) + \frac{|U_i|\delta}{2} \|\psi_i(x_i(b^i_k)) - \psi_i(x_i)\|_2^2 < 0 \quad (12)$$

Remark 3. Equation 10 is in the form of Hamilton-Jacobi-Isaacs (HJI) inequality. If f_i is linear, the existence of V_i and α_i can be guaranteed. If f_i is nonlinear, f_i has to satisfy some additional requirements to ensure the existence of V_i and α_i. These necessary conditions for the solution to the HJI inequality are not presented here due to space limitations.

5 Linear System

Consider the distributed system 3 in linear form:

$$\begin{aligned}
\dot{x}_i &= A_i x_{D_i} + B_i u_i \\
u_i &= K_i \hat{x}_{Z_i}(t_k)
\end{aligned} \quad (13)$$

for $t \in [t_k, t_{k+1}), k = 1, \cdots, \infty$, where $A_i \in \mathbb{R}^{n_i \times n_{D_i}}$, $B_i \in \mathbb{R}^{n_i \times m_i}$, and $K_i \in \mathbb{R}^{m_i \times n_{Z_i}}$.

We know that there always exist matrices $C_i \in \mathbb{R}^{n_{D_i} \times \bar{n}}$ and $R_i \in \mathbb{R}^{n_{Z_i} \times \bar{n}}$ such that $x_{D_i} = C_i x$ and $x_{Z_i} = R_i x$ hold. Let $A = \big((A_1 C_1^1)^T, \cdots, (A_N C_N^1)^T\big)^T$, $B = ((B_1 K_1 R_1)^T, \cdots, (B_N K_N R_N)^T)^T$. Therefore, equation 2 is equivalent to the inequality:

$$P(A + B) + (A + B)^T P \le -Q \quad (14)$$

where $V(x) = x^T P x$ and $P, Q \in \mathbb{R}^{\bar{n} \times \bar{n}}$ are positive definite matrices.

We first show the general structure of the threshold functions ϕ_i in linear systems satisfying equation 4.

Theorem 3. *For system 13, if the matrices $P, Q \in \mathbb{R}^{\bar{n}\times\bar{n}}$ and $W_i, M_i \in \mathbb{R}^{n_i\times n_i}$, $i = 1, 2, \cdots, N$ satisfy:*

$$P(A+B) + (A+B)^T P \leq -Q \tag{15}$$

$$Q - PBM^{-1}B^T P \geq W \tag{16}$$

$$P, Q, M_i, W_i > 0 \tag{17}$$

where $M = diag\{M_j\}_{j\in\mathcal{N}}$ and $W = diag\{W_j\}_{j\in\mathcal{N}}$, then the threshold functions $\phi_i : \mathbb{R}^{n_i} \times \mathbb{R}^{n_i} \to \mathbb{R}$ defined by

$$\phi_i(x_i, y_i) = -x_i^T W_i x_i + (y_i - x_i)^T M_i (y_i - x_i) \tag{18}$$

satisfy equation 4 with $V(x) = x^T P x$.

Proof. It is obvious that $\phi_i(x_i, x_i) = -x_i^T W_i x_i$ is negative definite.

We now show that $\phi_i(x_i, y_i)$ defined in equation 18 satisfies equation 4 in theorem 1. For any $x, y \in \mathbb{R}^{\bar{n}}$, let $x_{D_i} = C_i x$ and $y_{Z_i} = R_i y$. Then

$$\sum_{i\in\mathcal{N}} \frac{\partial V}{\partial x_i}(A_i x_{D_i} + B_i K_i y_{Z_i}) = x^T(PA + A^T P + PB + B^T P)x + 2x^T PB(y - x)$$

Since equation 15 holds and using equation 16, the inequality reduces to

$$\begin{aligned}
&\sum_{i\in\mathcal{N}} \frac{\partial V}{\partial x_i}(A_i x_{D_i} + B_i K_i y_{Z_i}) \leq -x^T Q x + 2x^T PB(y - x) \\
&\leq -x^T(Q - PBM^{-1}B^T P)x + (y - x)^T M (y - x) \\
&\leq -\sum_{i\in\mathcal{N}} x_i^T W_i x_i + \sum_{i\in\mathcal{N}} (y_i - x_i)^T M_i (y_i - x_i) = \sum_{i\in\mathcal{N}} \phi_i(x_i, y_i)
\end{aligned} \tag{19}$$

which means equation 4 is satisfied. □

It can be shown that the matrices $\{W_j\}_{j\in\mathcal{N}}$ and $\{M_j\}_{j\in\mathcal{N}}$ required in theorem 3 always exist, provided equation 15 holds (for example, let $W_i = \varepsilon I_{n_i\times n_i}$ and $M_i = \frac{\|PB\|^2}{\lambda_{\min}(Q)-\varepsilon} I_{n_i\times n_i}$, where $\varepsilon \in (0, \lambda_{\min}(Q))$).

Remark 4. Notice that equation 16 can be rewritten as the following

$$\begin{bmatrix} Q - W & PB \\ B^T P & M \end{bmatrix} \geq 0 \tag{20}$$

Therefore, equation 15, 17, 20 form a linear matrix inequality (LMI), which characterizes the desired matrices.

Theorem 3 presents the general structure of valid threshold functions. As mentioned in remark 4, the assumptions in theorem 3 can be posed as an LMI. However, directly solving this LMI for an admissible solution is a centralized approach. Since decentralization is desired, we need to find a way to transform the centralized LMI into several local LMI problems. In the following discussion, we introduce a decentralized event-design scheme, where each agent solves its local LMI and constructs its threshold function based on the local information.

The first step in the design is to select $N+1$ constants $\delta, \beta_1, \cdots, \beta_N \in \mathbb{R}$. To outline the main idea of this approach, we study the case when $\beta_1, \cdots, \beta_N$ are the same, say $\beta_i = \beta \in \mathbb{R}$ for all $i \in \mathcal{N}$. The general case can be easily extended.

Let $\underline{s}_i = \Sigma_{j \in T_i,\, j<i} n_j$ and $\bar{s}_i = \Sigma_{j \in T_i,\, j>i} n_j$ for $i \in \mathcal{N}$. Assuming matrices $C_i' \in \mathbb{R}^{n_{D_i} \times n_{T_i}}$ and $R_i' \in \mathbb{R}^{n_{Z_i} \times n_{T_i}}$ satisfy $x_{D_i} = C_i' x_{T_i}$ and $x_{Z_i} = R_i' x_{T_i}$, we define functions $F_i : \mathbb{R}^{n_i \times n_i} \to \mathbb{R}^{n_{T_i} \times n_{T_i}}$ as

$$F_i(P_i) = \begin{pmatrix} 0_{\underline{s}_i \times n_{T_i}} \\ P_i(A_i C_i' + B_i K_i R_i') \\ 0_{\bar{s}_i \times n_{T_i}} \end{pmatrix} \tag{21}$$

and $G_i : \mathbb{R} \to \mathbb{R}^{n_{T_i} \times n_{T_i}}$ as

$$G_i(q_i) = \text{diag}\left\{ \beta I_{\underline{s}_i \times \underline{s}_i}, -q_i I_{n_i \times n_i}, \beta I_{\bar{s}_i \times \bar{s}_i} \right\}. \tag{22}$$

We now introduce the local LMI problem for agent i:

Problem 1. For two given constants $\delta > 0$ and β, find w_i^* such that

$$\begin{aligned} & w_i^* = \max_{w_i,\, q_i \in \mathbb{R},\, P_i \in \mathbb{R}^{n_i \times n_i}} w_i \\ s.t.\quad & \\ & F_i(P_i) + F_i^T(P_i) \leq G_i(q_i) && (23) \\ & P_i > 0 && (24) \\ & \begin{bmatrix} (q_i - (|S_i \cup U_i| - 1)\beta - w_i)\, I & P_i B_i K_i \\ K_i^T B_i^T P_i & \delta I \end{bmatrix} \geq 0 && (25) \end{aligned}$$

The following theorem shows that if the optimal solution to problem 1 is greater than zero, then the required threshold functions can be constructed in a decentralized manner.

Theorem 4. *For system 13, if for any $i \in \mathcal{N}$, the solution of problem 1, w_i^*, satisfies $w_i^* > 0$, then the threshold functions $\phi_i : \mathbb{R}^{n_i} \times \mathbb{R}^{n_i} \to \mathbb{R}$ defined by*

$$\phi_i(x_i, y_i) = -w_i^* x_i^T x_i + \delta |U_i| \|y_i - x_i\|_2^2 \tag{26}$$

satisfy equation 4 with $V(x) = \sum_{i \in \mathcal{N}} x_i^T P_i x_i$.

Proof. Assume w_i^*, P_i, and q_i are the solution of LMI problem 1. If $w_i^* > 0$ for all $i \in \mathcal{N}$, then it is easy to show that the matrices

$$\begin{aligned} P &= \text{diag}\{P_i\}_{i=1}^N \\ Q &= \text{diag}\{[-q_i + (|S_i \cup U_i| - 1)\beta] I_{n_i \times n_i}\}_{i=1}^N \\ W_i &= w_i^* I_{n_i \times n_i} \\ M &= \text{diag}\{\delta |U_i| I_{n_i \times n_i}\}_{i=1}^N \end{aligned}$$

satisfy the assumptions in theorem 3. According to theorem 3, the conclusion of this theorem is drawn. □

Remark 5. As shown in the proof of theorem 4, if $w_i^* > 0$ holds for all $i \in \mathcal{N}$, then a solution to the centralized LMI defined by equation 15, 17, 20 can be constructed by the solutions of local LMIs.

Remark 6. The LMI defined in problem 1 actually is equivalent to the linear form of equation 10

$$\begin{aligned} &x_i^T P_i Y_i x_{T_i} + x_{T_i}^T Y_i^T P_i x_i + \frac{1}{\delta} x_i^T P_i B_i K_i K_i^T B_i^T P_i x_i \\ &\leq -\left(w_i^* + (|S_i \cup U_i| - 1)\beta\right) x_i^T x_i + \sum_{j \neq i, j \in T_i} \beta x_j^T x_j \end{aligned} \tag{27}$$

where $Y_i \triangleq A_i C_i' + B_i K_i R_i'$. In linear case, $\alpha_i(x_i) = (w_i^* + (|S_i \cup U_i| - 1)\beta)\, x_i^T x_i$ and $\beta_i(x_i) = \beta x_i^T x_i$. The assumption $w_i^* > 0$ is equivalent to the requirement in equation 9. For linear systems, it is natural to set $\psi_i(x_i) \triangleq x_i$.

6 Simulation

The simulation results demonstrate the value of decentralized event-triggered broadcasts over a networked system. A collection of N inverted pendulums is considered, where every pendulum arm is connected to every other pendulum arm by springs as shown in Fig. 1. The plant's state equation is

$$\dot{x}_i = \begin{pmatrix} \dot{\theta}_i \\ \left(\frac{g}{\ell} - \frac{(N-1)ka^2}{m\ell^2}\right)\sin(\theta_i) + \sum_{j \neq i} \frac{ka^2}{m\ell^2}\sin(\theta_j) + \frac{1}{m\ell^2} u_i \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} v_i \tag{28}$$

where m is the mass of the pendulum bob, ℓ is the length of the pendulum arm, g is the gravitational acceleration, k is the spring constant, and $v_i : \mathbb{R} \to \mathbb{R}$ is the external disturbance in agent i. The system state is the vector $x_i = \begin{bmatrix} \theta_i \ \dot{\theta}_i \end{bmatrix}^T$, where θ_i is the ith pendulum bob's angle with respect to the vertical. In these simulations, we set $g = 10$, $l = 2$, $m = 1$, $k = 5$, and $a = 1$.

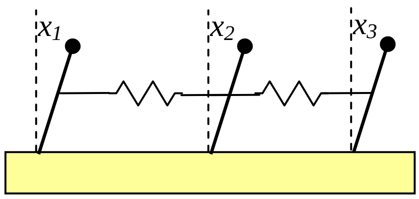

Fig. 1. Network of three inverted pendulums

We assume in all simulations, every broadcast takes 0.001 seconds. If there is a broadcast conflict, we assume agents compete for access to the channel using carrier sense media access (CSMA) protocols. In this case, the probability of an agent accessing the medium is 1/3.

6.1 Event Design for Nonlinear Systems

This simulation considered the broadcast-triggering event design in the inverted pendulums system 28. We set $N = 3$ and $v_i(t) = 0$. The controllers designed for the continuous closed-loop system were:

$$u_i = m\ell^2 \left[-\theta_i - 2\dot{\theta}_i - \left(\frac{g}{\ell} - \frac{2ka^2}{m\ell^2} \right) \sin(\theta_i) \sum_{j \neq i} \frac{ka^2}{m\ell^2} \sin(\theta_j) \right], \text{ for } i = 1, 2, 3$$

Let $\delta = 1000$, $\beta_i(x_i) = x_i^T x_i$, $\psi_i(x_i) = x_i$ and the system dynamics in equation 28 satisfied equation 8 for $L_1 = L_2 = L_3 = \sqrt{5} + 5$. Solving equation 9 and 10, we found

$$V_1(x_1) = x_1^T P x_1 \quad V_2(x_2) = x_2^T P x_2 \quad V_3(x_3) = x_3^T P x_3$$
$$\alpha_1(x_1) = 11.13 x_1^T x_1 \; \alpha_2(x_2) = 11.13 x_2^T x_2 \; \alpha_3(x_3) = 11.13 x_3^T x_3$$

where $P = \begin{pmatrix} 58.27 & 56.11 \\ 56.11 & 81.58 \end{pmatrix}$.

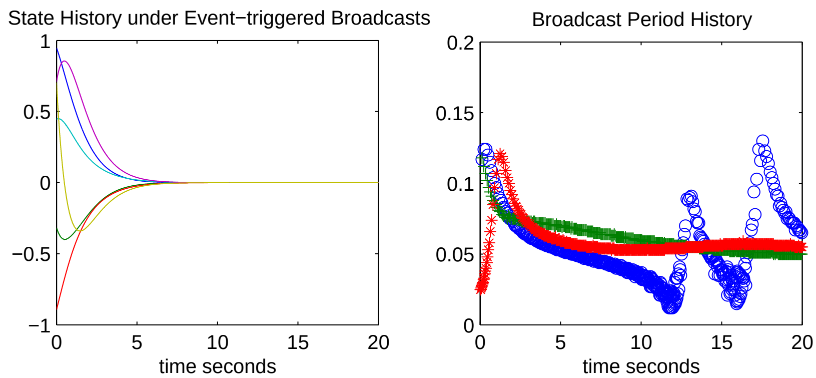

Fig. 2. Event-triggered broadcast simulation results in the distributed nonlinear system

Agent i used the violation of equation 12 to trigger its broadcasts. The left plot in Fig. 2 shows the state time history for all three inverted pendulums. Obviously the system is asymptotically stable. The right plot of Fig. 2 is the history of broadcast periods generated by the violation of equation 12, where the periods of agents 1, 2, 3 are represented as circles, crosses, and stars in the plot, respectively. As shown in the plot, the broadcast periods vary considerably. The minimum, mean, maximum of periods are $[0.012, 0.048, 0.130]$ for agent 1, $[0.050, 0.061, 0.118]$ for agent 2, $[0.025, 0.058, 0.121]$ for agent 3, respectively.

6.2 Event-triggered Model versus Periodic Model

This simulation compared the performance of the event-triggered model and the "comparable" periodic model. By "comparable", we mean the total numbers of broadcasts in these two models are the same over the same system running time. In this way, these two models use the same amount of communication resource. The plant's state equation used in this simulation is the linear version of equation 28:

$$\dot{x}_i = \begin{pmatrix} 0 & 1 \\ \frac{g}{\ell} - \frac{(N-1)ka^2}{m\ell^2} & 0 \end{pmatrix} x_i + \sum_{j \neq i} \begin{pmatrix} 0 & 0 \\ \frac{ka^2}{m\ell^2} & 0 \end{pmatrix} x_j + \begin{pmatrix} 0 \\ \frac{1}{m\ell^2} \end{pmatrix} u_i \tag{29}$$

and the controllers designed for the continuous system were

$$u_i = m\ell^2 \left[-\sum_{j \neq i} \left(\frac{ka^2}{m\ell^2}, 0 \right) x_j - \left(2 + \frac{g}{\ell} - \frac{(N-1)ka^2}{m\ell^2}, \ 3 \right) x_i \right]. \tag{30}$$

We also introduced the external disturbance, $v_i : \mathbb{R} \to \mathbb{R}$, to agent i, where

$$v_i(t) = \begin{cases} 300/N \ , & t - \frac{2(i-1)}{N} \in (k, k+0.02], \ \text{for } k = 0, 2, 4, \cdots, \infty \\ 0 & , \ \text{otherwise} \end{cases} . \tag{31}$$

The case when $N = 3$ was considered. With $\delta = 1000$ and $\beta = 1$, each agent solved problem 1 and resulted in $w_1^* = w_2^* = w_3^* = 59.7$. Therefore, the broadcast-triggering events were determined using LMI techniques.:

$$-0.0597 \, \|x_i\|_2^2 + 3\|x_i - x_i(b_k^i)\|_2^2 < 0 \tag{32}$$

Using the broadcast-triggering events in equation 32 and the disturbance defined in equation 31, we first ran the event-triggered model. The period in the periodic model was the average period in the event-triggered model. In this way, the periodic model used the same average transmission rate as the event-triggered model did.

We varied the system running time from 6 to 50 seconds. Let $x_e(t)$ and $x_f(t)$ denote the state trajectories in the event-triggered model and the periodic model, respectively. The system performance over $[0, t]$ is defined as

$$E[x|t] = \int_0^t x^T x \, dt$$

Fig. 3 shows the improvement of the performance in the event-triggered model compared with the periodic model. The horizonal axis denotes the system running time t and the vertical axis $p(t)$ is defined as

$$p(t) = \frac{E[x_f|t] - E[x_e|t]}{E[x_f|t]} \tag{33}$$

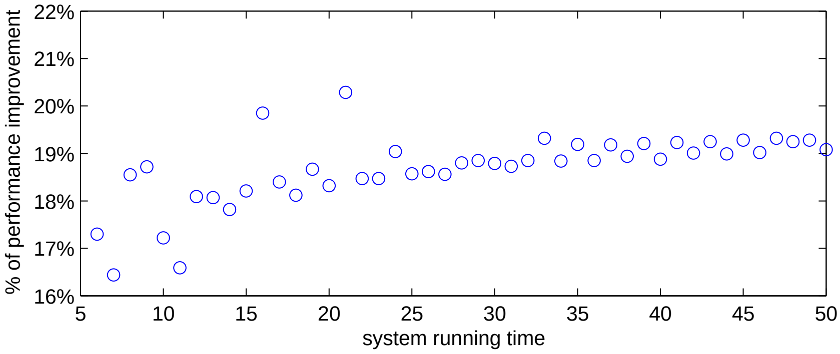

Fig. 3. The percentage of performance improvement by the event-triggered model compared with the periodic model

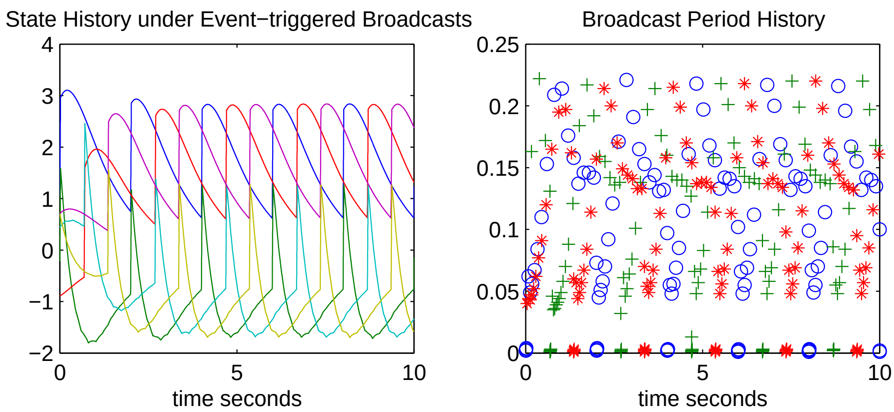

Fig. 4. Event-triggered broadcasts simulation results in linear systems with disturbance

As shown in Fig. 3, the percentage tends to increase as the system running time increases. It eventually settles down around 19%. It shows that, with the same amount of channel access and the same disturbance, the event-triggered broadcast model achieves a better performance than the periodic model.

For the case when the system running time $t = 10$, the state trajectory of the overall system is shown in the left plot of Fig. 4, where the state shows a periodic pattern after $t = 3$. The reason is that after $t = 3$, the state is dominated by the disturbance. Since the disturbance is periodic, the state also changes periodically. The right plot of Fig. 4 shows the broadcast periods. Notice that, every time when the disturbance in agent i is non-zero, the broadcast periods becomes shorter. It is because the event-triggered broadcasts have the ability of adjusting periods according to the current state.

6.3 Scalability in Event-triggered Model and Periodic Model

In the previous simulations, because we limited the number of pendulums to 3, there was no broadcast conflict in the network. However, as the number of pendulums increases, broadcast conflicts will occur, which introduces broadcast delays in the network. By "delay", we mean the time length from the violation of agent i's event to its first broadcast after that violation. Two simulations were done by varying N from 3 to 50. The first one compared the broadcast delay in the event-triggered models and the periodic model; the second one compared the system performance of these two models. In these two simulation, the external disturbance is still the one defined in equation 31 and the system running time is set to be 10 seconds.

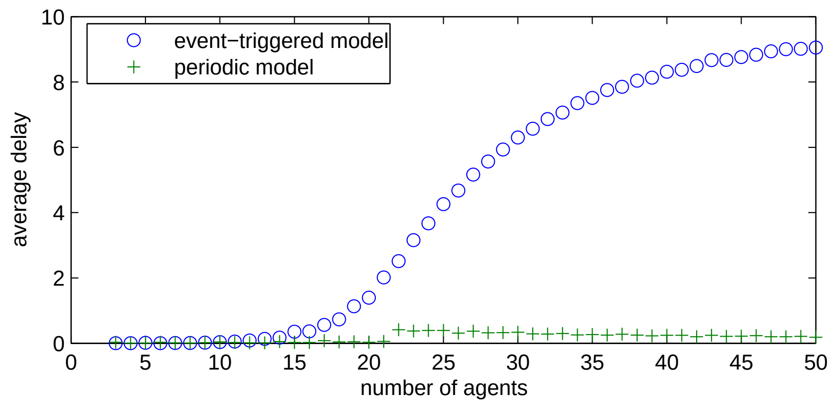

Fig. 5. The relationship between broadcast delay and the scale of the event-triggered model

The first result is shown in Fig. 5. Circles and crosses represent the average delay in the event-triggered model and the periodic model, respectively. By "average", we mean its total delay over N. From the plot, the periodic model introduced less delay than the event-triggered model, especially when N gets larger. In the event-triggered model, after $N > 15$, the delay increases quickly as the number of agents increases. We notice that when $N = 15$, the channel usage is 90% during the entire running time and when $N \geq 23$, the channel occupation is 100%, which means agents always compete for the channel access. This is the indication that the bandwidth of the network is not high enough.

Two reasons lead to the fast increase of delay. The first one is that as the number of agents increases, more agents compete for the channel access. The second reason is that, because every pendulum arm is connected to every other pendulum arm, the coefficient of the local error in the event is $N\delta$. As N increases, the error becomes a larger weight, which leads to a short broadcast period.

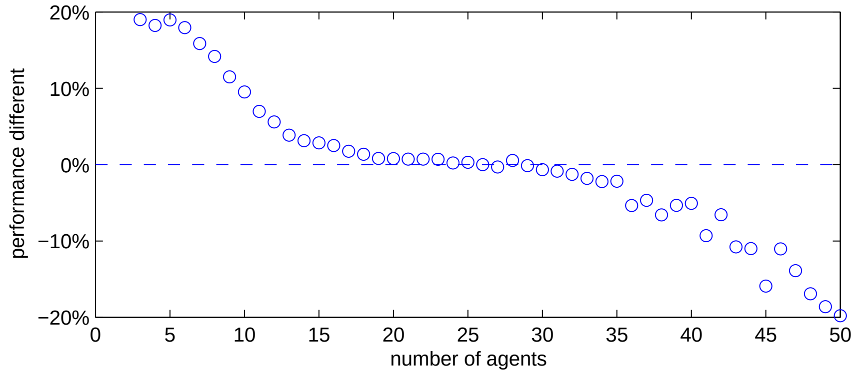

Fig. 6. The performance comparison between the event-triggered model and the periodic model

The comparison of system performance in the event-triggered model and the periodic model is shown in Fig. 6. The vertical axis is $p(t)$ defined in 33. When $N = 3$, the event-triggered model improves the performance by 20% compared with the periodic model. However, the improvement keeps decreasing as the number of agents increases until $N = 30$ when the performance of the event-triggered model becomes worse than the periodic model. Remember that when $N \geq 23$, agents always compete for the channel access. In that case, event-triggering becomes meaningless since the agent's decision is always broadcasting, not waiting. Therefore, when N is extremely large, the periodic model seems better since the periodic model introduces less delay into the system. This suggests that event-triggered feedback, by itself, does not scale well with communication network density.

7 Conclusion

This paper examines event-triggered broadcasting of state information in distributed networked control systems. We showed that it is possible to design event-triggering thresholds that preserve asymptotic stability while only requiring "local" subsystem information to make their decisions. We showed that it is possible to design such thresholds using only local information about the dynamics of a subsystem. The approach applies to both nonlinear and linear systems, though determining the threshold for nonlinear systems requires the solution of a HJI inequality. For linear systems, this condition reduces to a much simpler LMI feasibility problem. We investigated the scalability of event-triggering through simulation studies. These results indicate that event-triggered systems outperform periodically triggered systems when there is little communication network congestion. As network density increases, thereby resulting in increased congestion, the performance of the event-triggered scheme begins to fall below that achieved by the periodically triggered system. This suggests that event-triggering, by itself, is not a scalable solution to networked feedback control systems. We firmly believe that the results indicate that broadcasting in such NCS requires a hybrid scheme that judiciously alternates between time-triggered and event-triggered broadcast strategies. Future work will investigate such hybrid schemes in a more rigorous manner.

References

1. Tabuada, P. and Wang, X.: Preliminary results on state-triggered scheduling of stabilizing control tasks. Proceedings of IEEE Conference on Decision and Control (2006).
2. Shin, K. G.,: Real-time communications in a computer-controlled workcell. IEEE Transactions on Robotics and Automation **7** (1991) 105–113
3. Zuberi, K. M., and Shin, K. G.,: Scheduling messages on controller area network for real-time CIM applications. IEEE Transactions on Robotics and Automation **13** (1997) 310–314
4. Krtolica R., and Ozguner U.,: Stability of linear feedback systems with random communication delays. Proceedings of American Control Conference (1991)
5. Wong, W. S., and Brockett, R. W.,: Systems with finite communication bandwidth constraints - Part I: State estimation problems. IEEE Transactions on Automatic Control **42** (1997) 1294–1299
6. Wong, W. S., and Brockett, R. W.,: Systems with finite communication bandwidth constraints - Part II: Stabilization with limited information feedback. IEEE Transactions on Automatic Control **44** (1999) 1049-1053
7. Walsh, G., Ye, H., and Bushnell, L.: Stability analysis of networked control systems. IEEE Transactions on Control Systems Technology, **10** (2002) 3 438–446
8. Zhang, W., Branicky, M., and Phillips, S.,: Stability of networked control systems. IEEE Control Systems Magazine, **21** (2001) 84–99
9. Sinopoli, B., Sharp, C., Schenato, L., Schaffert, S., and Sastry, S.,: Distributed control applications within sensor networks. Proceedings of the IEEE **91** (2003) 8 1235–1246
10. Gupta, P., and Kumar, P.,: The capacity of wireless networks. IEEE Transactions on Information Theory **46** (2000) 2 388–404
11. Arzen, K.,: A simple event-based pid controller. Proceedings of the 14th IFAC World Congress (1999).
12. Hristu-Varsakelis, D., and Kumar, P.,: Interrupt-based feedback control over a shared communication medium. Proceedings of the IEEE Conference on Decision and Control (2002)
13. Astrom, K., and Bernhardsson, B.: Comparison of riemann and lebesgue sampling for first order stochastic systems. Proceedings of the IEEE Conference on Decision and Control (1999)
14. Wang, X., and Lemmon, M.,:, Event-triggered Broadcasting across Distributed Networked Control Systems. submitted to American Control Conference (2008)
15. Wang, X., and Lemmon, M.,: Self-triggered feedback control systems with finite-gain L_2 stability. submitted to IEEE Transactions on Automatic Control (2007)