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Weakly Coupled Event Triggered Output Feedback System in Wireless Networked Control Systems

Lichun 'Lucinda' Li · Michael Lemmon

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Abstract This paper examines output feedback control of wireless networked control systems where there are separate links between the sensor-to-controller and controller-to-actuator. The proposed triggering events only rely on local information so that the transmissions from the sensor and controller subsystems are not necessarily synchronized. This represents an advance over recent work in event-triggered output feedback control where transmission from the controller subsystem was tightly coupled to the receipt of event-triggered sensor data. The paper presents an upper bound on the optimal cost attained by the closed-loop system. Simulation results demonstrate that transmissions between sensors and controller subsystems are not tightly synchronized. These results are also consistent with derived upper bounds on overall system cost.

Keywords Weakly coupled transmissions · Event triggering · Output feedback control · Networked control systems

1 Introduction

Large-scale wireless networked control systems (WNCS) are invaluable in many civil and military applications for monitoring and controlling in complex environment. An important issue for large-scale WNCS concerns energy efficiency. Sensor nodes need to operate on an extremely frugal energy budget, since they are battery driven and since battery replacement is not an option for large-scale WNCS with thousands of physically embedded nodes. To conserve power, it is important to manage wireless communication as such communication is a major source of power consumption [17]. There has been a great

Lichun(Lucinda) Li and Michael Lemmon
Department of Electrical Engineering, University of Notre Dame, IN 46556, USA
Tel.: 1-574-631-3736,1-574-631-8796
E-mail: lli3,lemmon@nd.edu
Web: www.nd.edu/∼lli3, ∼lemmon

deal of prior work seeking to conserve power [19,1] through energy efficient networking protocols. Another way of conserving power, however, is to make the application power aware, and attempt to minimize the application's use of the communication network, while still maintaining a desired level of control system performance. One recent method for realizing this goal is known as *event-triggered* sampling.

Event triggering can be seen as a communication protocol where information is transmitted only if some event occurs. In particular, information is transmitted when a measure of data 'novelty' exceeds a specified threshold. In contrast to more commonly used periodic transmission schemes, event-triggering tends to generate traffic patterns that are *sporadic* in nature. Prior experimental results have demonstrated that event-triggering can use fewer communication resources than periodic transmission schemes with comparable performance levels [22,10,21,8,14]. The reason for this more efficient use of communication resources is that event-triggering makes use of on-line information in making transmission decisions. This method, therefore, can adapt its usage of the communication in channel to the importance of the data it must transmit.

Most prior work in the event triggering literature discusses state feedback control and state estimation. This work has traditionally assumed a single feedback link in the system. It has only been very recently that researchers have turned to study event-triggered output feedback control where there are separate communication channels from sensor to controller and controller to actuator. If we design triggering events for both communication channels, an interesting question to ask is how these two triggering events are coupled with each other.

Some of the work in event triggered output feedback systems hid this question by assuming that only part of the control loop was closed over communication channel, i.e. either sensor-to-controller link or controller-to-actuator link is connected directly [11,25,16]. Another work in [6] assumed very strong coupling between the triggering rules of sensor-to-controller link and controller-to-actuator link. They required that the transmission in one link triggered the transmission in the other link, so transmissions in both communication channels are synchronized.

This synchronization is not necessary. This paper examines a *weakly coupled* event triggered system. We study the optimal event-triggers to minimize the mean square cost of the system state discounted by the communication cost in both links. By realizing that the remote state estimate in controller is orthogonal to the remote state estimation error, the optimal cost of the output feedback system turns out to be bounded from above by the sum of the optimal costs of a state estimation problem and a state feedback problem. The state estimation problem is to minimize the mean square state estimate error discounted by the communication cost in the controller-to-actuator link, and the state feedback problem is to minimize the mean square state estimate discounted by the communication cost in controller-to-actuator link.

This paper first gives the optimal event-trigger in the sensor-to-controller link for a state estimation problem, which is minimizing the mean square state estimation error discounted by the communication cost in sensor-to-controller link, and then, based on this optimal event trigger in the sensor-to-controller link, calculates the optimal event-trigger in the controller-to-actuator link for a state feedback problem , which is minimizing the mean square state estimate discounted by the communication cost in controller-to-actuator link. It turns out that the optimal cost of the output feedback control system is bounded from above by the sum of the optimal costs of the state estimation problem and the state feedback problem. Because the optimal event-triggers are more difficult to calculate as the system state's dimension increases, this paper then derives suboptimal event triggers and the upper bound on the cost.

This paper is based on our results in [12] and [13], and is organized as the following: section 2 gives the problem statement, section 3 states that the closed loop cost is the sum of a state estimation cost and a state feedback cost, section 4 and 5 describes how to design the optimal triggering sets and suboptimal triggering sets, respectively, the simulation results are presented in section 6, and we conclude this paper in section 7.

2 Problem Statement

Consider a networked control system with its control loop closed all over networks. A block diagram of the closed loop system is shown in Figure 1. This closed loop system consists of four components: a *plant subsystem*, a *sensor subsystem*, a *controller subsystem* and an *actuator subsystem.* The control loop, i.e. from sensor subsystem to controller subsystem and from controller subsystem to actuator subsystem, is closed over a communication network. We assume there are no delays or dropouts in the networks.

2.1 Plant subsystem

The plant subsystem consists of two parts: a plant and a sensor. The plant's state and sensor's measured output satisfy the following difference equation

$$\begin{aligned} x(k) &= Ax(k-1) + Bu_a(k-1) + w(k-1), \\ y(k) &= Cx(k) + v(k), \end{aligned}$$

for $k = 1, 2, \cdots$. Let $\mathbb{R}^n$ indicate the n dimensional real space, and $\mathbb{Z}^+$ indicate the set of all non-negative integers. In the difference equation,

- $x : \mathbb{Z}^+ \to \mathbb{R}^n$ is the system state with initial state $x(0)$ being a Gaussian random variable with mean μ_0 and variance Π_0.
- $w \in \mathbb{R}^n$ is a zero mean white Gaussian noise process with variance W.
- $v \in \mathbb{R}^p$ is another zero mean white Gaussian noise process with variance V. The initial state $x(0)$, w and v are independent.

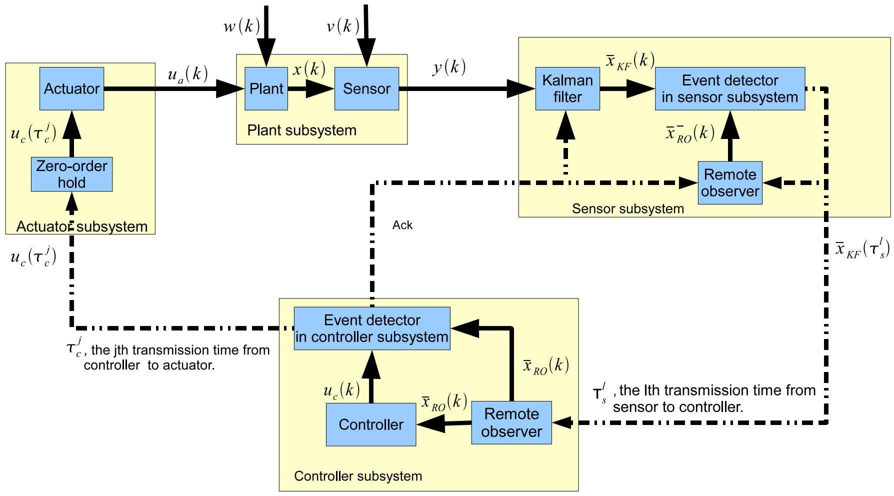

Fig. 1 Structure of the event triggered output feedback control systems

- $u_a : \mathbb{Z}^+ \to \mathbb{R}^m$ is the actual control input applied to the plant.
- The triplet (A, B, C) is controllable and observable.
- $y : \mathbb{Z}^+ \to \mathbb{R}^p$ is the measurement of the plant which is fed into the sensor subsystem.

2.2 Sensor subsystem

The sensor subsystem uses sensor measurements to decide when to transmit information to the controller subsystem. The sensor subsystem consists of a *Kalman filter*, a *remote observer* and an *event detector in sensor subsystem.*

The *Kalman filter* generates a filtered state $\overline{x}_{KF} : \mathbb{Z}^+ \to \mathbb{R}^n$ that minimizes the weighted mean square estimation error (MSEE), i.e.

$$\overline{x}_{KF}(k) = \min_{\overline{x}_{KF}(k)} E\left[\|x(k) - \overline{x}_{KF}(k)\|_Z^2 \mid \{y(0), y(1), \cdots, y(k)\}\right]$$

where $Z \geq 0$ is a symmetric weighting matrix, and $\|\theta\|_Z^2 = \theta^T Z \theta$. For the process under study the filter equation is

$$\overline{x}_{KF}(k) = A\overline{x}_{KF}(k-1) + Bu_a(k-1) + L\left[y(k) - C(A\overline{x}_{KF}(k-1) + Bu_a(k-1))\right],$$

where $L = AXC^T(CXC^T + V)^{-1}$, and X satisfies the discrete linear Riccati equation

$$AXA^T - X - AXC^T(CXC^T + V)^{-1}CXA^T + W = 0.$$

The steady state estimation error $\overline{e}_{KF}(k) = x(k) - \overline{x}_{KF}(k)$ is a Gaussian random variable with zero mean and variance

$$\mathbb{E}(\overline{e}_{KF}\overline{e}_{KF}^T) = Q = (I - LC)X.$$

While the Kalman filter generates the most 'knowledgable' state estimate, the *remote observer in sensor subsystem* duplicates the remote state estimate in the controller subsystem. With these two state estimates, we know that how far away the remote state estimate is from the most 'knowledgable' state estimate $\overline{x}_{KF}$. If the remote state estimate is too far away from $\overline{x}_{KF}$, then $\overline{x}_{KF}$ should be transmitted. To duplicate the remote state estimate, the *remote observer in sensor subsystem* doesn't need direct access to the controller subsystem. Instead, it only needs model information and information transmitted to the controller subsystem to duplicate the remote state estimate. It is obvious that both information is available to the *remote observer in sensor subsystem*. Now, let us see how the remote observer in sensor subsystem works. At step k, before the event detector in sensor subsystem decides whether to transmit or not, the remote observer in sensor subsystem produces an a priori remote state estimate $\overline{x}_{RO}^{-}(k)$ which will be described in detail when we introduce the remote observer in controller subsystem. The a priori remote state estimate $\overline{x}_{RO}^{-}(k)$ together with the filtered state $\overline{x}_{KF}(k)$ is then handed to the event detector in sensor subsystem to decided whether to transmit the filtered state $\overline{x}_{KF}(k)$ or not at step k.

The *event detector in the sensor subsystem* uses the *a priori gap*

$$e_{KF,RO}^{-}(k) = \overline{x}_{KF}(k) - \overline{x}_{RO}^{-}(k), \tag{1}$$

to decide whether or not to transmit $\overline{x}_{KF}(k)$ to the controller subsystem. The event detector in the sensor subsystem compares the a priori gap $e_{KF,RO}^{-}(k)$ with a triggering set $S_s \subseteq \mathbb{R}^n$. If the gap is inside the triggering set S_s, then no data is transmitted. Otherwise, the filtered state $\overline{x}_{KF}(k)$ is sent to the controller subsystem. The lth transmission time from sensor subsystem to controller subsystem is denoted as τ_s^l.

2.3 Controller subsystem

The controller subsystem produces the control input, and decides when to transmit the current control input to the actuator subsystem based on the history information received from the sensor subsystem. The controller subsystem has three components: a *remote observer*, a *controller*, and a *event detector in the controller subsystem*.

The *remote observer* generates the remote state estimate $\overline{x}_{RO}(k)$ to minimize the MSEE based on all the a posteriori history information up to step k. By 'a posteriori history information', we mean the history information known by the controller subsystem after the event detector in sensor subsystem decides whether to transmit at step k. Let $l(k) = \max\left\{l : \tau_s^l \le k\right\}$ indicate the latest transmission time from the sensor subsystem to the controller subsystem. The a posteriori history information $\mathbf{H}(k)$ is

$$\mathbf{H}(k) = \left\{\overline{x}_{KF}(\tau_s^1), \overline{x}_{KF}(\tau_s^2), \cdots, \overline{x}_{KF}(\tau_s^{l(k)}), u_a(0), u_a(1), \ldots, u_a(k-1)\right\},$$

for $k = 0, 1, \ldots$. Correspondingly, we also define the a priori history information, the history information know by the controller subsystem before the event detector in sensor subsystem decides whether to transmit at step k, $\mathbf{H}^-(k)$ as

$$\mathbf{H}^-(k) = \left\{ \overline{x}_{KF}(\tau_s^1), \overline{x}_{KF}(\tau_s^2), \cdots, \overline{x}_{KF}(\tau_s^{l(k-1)}), u_a(0), u_a(1), \ldots, u_a(k-1) \right\}.$$

for $k = 0, 1, \ldots$ with $\mathbf{H}^-(0) = \emptyset$. To minimize the MSEE, the a posteriori remote state estimate satisfies (see section 3.2.4 in [23])

$$\overline{x}_{RO}(k) = E(x(k)|\mathbf{H}(k)),$$

and the a priori remote state estimate $\overline{x}_{RO}^-(k)$ satisfy

$$\overline{x}_{RO}^-(k) = E(x(k)|\mathbf{H}^-(k)).$$

Now, let's examine how the remote state estimate evolves. The initial condition is

$$\overline{x}_{RO}^-(0) = E(x(k)|\mathbf{H}^-(0)) = \mu_0.$$

If information is transmitted from sensor to controller at step k, the history information at step k is $\mathbf{H}(k) = \{\mathbf{H}^-(k), \overline{x}_{KF}(k)\}$, and the a posteriori remote state estimate is

$$\begin{aligned}
\overline{x}_{RO}(k) &= E(x(k)|\mathbf{H}^-(k), \overline{x}_{KF}(k)) \\
&= E(\overline{x}_{KF}(k) + \bar{e}_{KF}(k)|\mathbf{H}^-(k), \overline{x}_{KF}(k)) \\
&= \overline{x}_{KF}(k),
\end{aligned}$$

where the second equality holds because $\bar{e}_{KF}(k)$ is uncorrelated with $\mathbf{H}(k)$.

If information is not transmitted from sensor to controller at step k, the history information at step k is $\mathbf{H}(k) = \mathbf{H}^-(k)$, and the a posteriori remote state estimate is the same as the a priori remote state estimate, i.e.

$$\begin{aligned}
\overline{x}_{RO}(k) &= E(x(k)|\mathbf{H}(k)) = E(x(k)|\mathbf{H}^-(k)) \\
&= \overline{x}_{RO}^-(k).
\end{aligned}$$

The a priori state estimate evolves as the following:

$$\begin{aligned}
\overline{x}_{RO}^-(k) &= E(x(k)|\mathbf{H}^-(k)) \\
&= E(Ax(k-1) + Bu_a(k-1) + w(k-1)|\mathbf{H}^-(k)) \\
&= AE(x(k-1)|\mathbf{H}^-(k)) + Bu_a(k-1) \\
&= A\overline{x}_{RO}(k-1) + Bu_a(k-1),
\end{aligned}$$

where the third equality holds because $w(k-1)$ is independent from $\mathbf{H}^-(k)$.

To conclude the above discussion about the *remote observer*, we say that the minimum mean square remote state estimate based on $\mathbf{H}(k)$ takes the form of

$$\overline{x}_{RO}^{-}(k) = A\overline{x}_{RO}(k-1) + Bu_a(k-1), \text{ with } \overline{x}^{-}(0) = \mu_0 \tag{2}$$

$$\overline{x}_{RO}(k) = \begin{cases} \overline{x}_{RO}^{-}(k), \text{ if } e_{KF,RO}^{-}(k) \in S_s; \\ \overline{x}_{KF}(k), \text{ otherwise }, \end{cases} \tag{3}$$

where $e_{KF,RO}^{-}(k)$ is the a priori gap between remote state estimate and the filtered state described in equation (1).

Remark 1 The closed-loop system structure in this paper is similar to that in [18] except that controller and actuator in this paper are also connected through a network. These two papers share the same remote observer structure, and show that the remote observer described by equation (2) and (3) minimize the MSEE based on the a posteriori history information. The difference is that we don't have any assumption of the triggering set while [18] assumed that the triggering set was symmetric.

Remark 2 If the controller is assumed to know the triggering event in the sensor subsystem (which is not the case in this paper), both the a priori and a posteriori history information should include the event triggering information in the sensor subsystem, and the minimum MSEE won't be realized by the form given by (2) and (3). An approximated minimum MSEE assuming that the controller knows the triggering event in sensor subsystem was presented in section 2 of [20].

The a posteriori state estimate $\overline{x}_{RO}(k)$ is, then, fed into the controller. The *controller* generates a control input

$$u_c(k) = K\overline{x}_{RO}(k),$$

where K is the controller gain. Notice that this control input u_c is different from the actual control input applied to the plant u_a.

Both the actual control input $u_a(k-1)$ and the a posteriori remote state estimate $\overline{x}_{RO}(k)$ are monitored by the *event detector in the controller subsystem*. It compares the augmented vector $[\overline{x}_{RO}(k)\ u_a(k-1)]^T$ with a triggering set S_c. If the augmented vector $[\overline{x}_{RO}(k)\ u_a(k-1)]^T$ lies outside the triggering set $S_c \subseteq \mathbb{R}^{m+n}$, the current control input $u_c(k)$ is transmitted to the actuator subsystem. Otherwise, no data is transmitted. The jth transmission time from controller to actuator is denoted by τ_c^j.

When the event detector in the controller subsystem decides to transmit, an *acknowledgement* is transmitted to the sensor subsystem. The main purpose of this acknowledgement is to synchronize the actual control input $u_a(k)$ in both the sensor subsystem and the controller subsystem. Once the sensor subsystem receives the acknowledgement, it uses the current remote state estimate $\overline{x}_{RO}(k)$ generated by the remote observer in sensor subsystem to obtain the new actual control input $u_a(k)$. In general the ack message is much shorter than the actual measurement data transmitted over the channel. It is therefore reasonable to assume this ack message as been heavily coded so the probability of failed reception is nearly zero.

2.4 Actuator subsystem

The actuator subsystem has two parts: a *zero order hold* and an *actuator*. Let $u_a(k)$ denote the actual control input applied to the plant. When $u_c(\tau_c^j)$ is transmitted, the actuator subsystem updates $u_a(k)$ to be $u_c(\tau_c^j)$, and holds this value until the next transmission occurs. $u_a(k)$, therefore, takes the form of

$$u_a(k) = u_c(\tau_c^j), \forall k = \tau_c^j, \dots, \tau_c^{j+1} - 1. \tag{4}$$

2.5 Average cost criterion

The average cost is defined as

$$J(S_s, S_c) = \lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N-1} E\left(c(x(k), S_s, S_c)\right),$$

where the cost function

$$c(x(k), S_s, S_c) = \|x(k)\|_Z^2 + \lambda_s 1(e^-_{KF,RO}(k) \notin S_s) + \lambda_c 1\left(\begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix} \notin S_c\right),$$

λ_s and λ_c are the communication prices for transmissions over the sensor-to-controller link and controller-to-actuator link, respectively. $1(\cdot)$ is the characteristic function satisfying

$$1(S) = \begin{cases} 1, \text{ if } S \text{ is true;} \\ 0, \text{ otherwise.} \end{cases}$$

Our objective is to design the triggering sets S_s and S_c to minimize the average cost $J(S_s, S_c)$, i.e.

$$J^* = \min_{S_s, S_c} J(S_s, S_c).$$

3 State estimation cost and control cost

With the problem setup given in the last section, this section shows that the average cost J can be expressed as the sum of the state estimation cost and the control cost.

Let $\overline{e}_{RO}(k) = x(k) - \overline{x}_{RO}(k)$ be the remote state estimation error. The key point for rewriting the average cost as the sum of the state estimation cost and the control cost is that the remote state estimation error $\overline{e}_{RO}(k)$ is orthogonal to the remote state estimate $\overline{x}_{RO}(k)$. This property is stated in the next lemma.

Lemma 1 *$\overline{e}_{RO}(k)$ is orthogonal to the remote state estimate $\overline{x}_{RO}(k)$ for all $k \in \mathbb{Z}^+$.*

Proof Since $\overline{x}_{RO}(k)$ is a minimum mean square estimate of the system state $x(k)$ (as shown in Section 2.3), according to Theorem 3.1 of [23], the estimation error $\overline{e}_{RO}(k)$ is orthogonal to $\overline{x}_{RO}(k)$. □

Based on the Lemma 1, it is easy to show that the average cost is the sum of state estimation cost and the control cost.

Theorem 1 *The average cost*

$$J(S_s, S_c) = J_s(S_s, \{\overline{e}_{RO}(k)\}_{k=0}^{\infty}) + J_c(S_c, \{\overline{x}_{RO}(k)\}_{k=0}^{\infty}),$$

where

$$J_s(S_s, \{\overline{e}_{RO}(k)\}_{k=0}^{\infty}) = \lim_{M\to\infty} \frac{1}{M} \sum_{k=0}^{M-1} E\left[\|\overline{e}_{RO}(k)\|_Z^2 + \lambda_s 1(e_{KF,RO}^{-}(k) \notin S_s)\right],$$

$$J_c(S_c, \{\overline{x}_{RO}(k)\}_{k=0}^{\infty}) = \lim_{M\to\infty} \frac{1}{M} \sum_{k=0}^{M-1} E\left[\|\overline{x}_{RO}(k)\|_Z^2 + \lambda_c 1\left(\begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k) \end{bmatrix} \notin S_c\right)\right].$$

Proof According to Lemma 1, the average cost $J(S_s, S_c)$ is rewritten as

$$\begin{aligned} J(S_s, S_c) = &\lim_{M\to\infty} \frac{1}{M} \sum_{k=0}^{M-1} E\Big(\|\overline{e}_{RO}(k)\|_Z^2 + \lambda_s 1(e_{KF,RO}^{-}(k)) \\ &+\|\overline{x}_{RO}(k)\|_Z^2 + \lambda_c 1\left(\begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix} \notin S_c\right)\Big) \\ =&J_s(S_s, \{\overline{e}_{RO}(k)\}_{k=0}^{\infty}) + J_c(S_c, \{\overline{x}_{RO}(k)\}_{k=0}^{\infty}). \end{aligned}$$

□

$J_s(S_s, \{\overline{e}_{RO}(k)\}_{k=0}^{\infty})$ relies on the remote state estimation error and the communication price between sensor and controller, and hence is called the *state estimation cost*. $J_c(S_c, \{\overline{x}_{RO}(k)\}_{k=0}^{\infty})$ relies on the remote state estimate and the communication price between controller and actuator, and hence is called the *control cost*.

Remark 3 Both the state estimation cost and the control cost depend on the triggering set in sensor subsystem S_s. It is easy to see that the state estimation cost J_s relies on S_s. The control cost J_c also relies on S_s, because J_c must be computed with respect to the probability distribution of the remote state estimate, $\overline{x}_{RO}(k)$. From equation (3), we can see that the distribution of $\overline{x}_{RO}(k)$ is a function of S_s, the triggering set in sensor subsystem. Thereby, the control cost $J_c(S_c, \{\overline{x}_{RO}(k)\}_{k=0}^{\infty})$ relies on S_s, and hence is coupled with

$J_s(S_s, \{\overline{e}_{RO}(k)\}_{k=0}^{\infty})$. To emphasize the dependence between the state estimation cost and the control cost on the triggering set in sensor subsystem S_s, we rewrite the state estimation cost and the control cost as

$$J_s(S_s) = J_s(S_s, \{\overline{e}_{RO}(k)\}_{k=0}^{\infty}),$$
$$J_c(S_c, S_s) = J_c(S_c, \{\overline{x}_{RO}(k)\}_{k=0}^{\infty}),$$

respectively.

Let $S_s^\dagger$ be the optimal sensor triggering set that minimizes J_s, and the corresponding optimal cost is $J_s^\dagger$. Let $S_c^\dagger$ be the controller's event-triggering strategy that minimizes the controller cost J_c assuming the sensor uses the event-trigger $S_s^\dagger$, and the corresponding controller's cost becomes $J_c^\dagger(S_s^\dagger)$. Since J_s and J_c are coupled, we can see that the minimum cost J^* is bounded above by

$$J^* \leq J(S_s^\dagger, S_c^\dagger) = J_s^\dagger + J_c^\dagger(S_s^\dagger). \tag{5}$$

Therefore, our design approach is to calculate the optimal triggering set in sensor subsystem $S_s^\dagger$ first, and then to compute the optimal triggering set in controller subsystem $S_c^\dagger$ based on the optimal triggering set in sensor subsystem $S_s^\dagger$. This approach is described in section 4. There are, however, no closed form solutions for the optimal triggering sets, and numerically computing the optimal triggering set becomes very difficult as state dimension increases. Due to these difficulties, we then show how to efficiently compute suboptimal triggering sets in both sensor and controller subsystems. The design of suboptimal triggering sets are given in section 5.

4 The Optimal Triggering Sets

This section first provides the optimal triggering set $S_s^\dagger$ in sensor subsystem, then gives the optimal triggering set $S_c^\dagger$ in controller subsystem based on $S_s^\dagger$. This section concludes with a theorem stating that the overall cost induced by $S_s^\dagger$ and $S_c^\dagger$ is bounded from above by a explicit upper bound that experiments in later section suggest are relatively tight.

4.1 Optimal triggering set in sensor subsystem

Before talking about the optimal triggering set for the sensor subsystem, let us first analyze the remote state estimation error $\overline{e}_{RO}$ to simplify the optimal problem to minimize $J_s(S_s)$ with respect to S_s. Let $e_{KF,RO}(k) = \overline{x}_{KF}(k) - \overline{x}_{RO}(k)$ be the gap between filtered state and the remote state estimate. We notice that

$$\overline{e}_{RO}(k) = \overline{e}_{KF}(k) + e_{KF,RO}(k), \tag{6}$$

and the filtered state error $\overline{e}_{KF}(k)$ is orthogonal to the gap between filtered state and the remote state estimate $e_{KF,RO}(k)$. This property is stated and proved in the following lemma.

Lemma 2 *The filtered state error, $\overline{e}_{KF}(k)$ is orthogonal to $e_{KF,RO}(k)$, the gap between filtered state and the remote state estimate.*

Proof Since Kalman filter is the minimum mean square estimate of the s-tate based on all history measurements $\{y(0), \dots, y(k)\}$, the estimation error $\overline{e}_{KF}(k)$ is orthogonal to any function of the history measurements $\{y(0), \dots, y(k)\}$. Since both $\overline{x}_{KF}(k)$ and $\overline{x}_{RO}(k)$ (see equation (2) and (3)) are functions of history measurements $\{y(0), \dots, y(k)\}$, $\overline{e}_{KF}(k)$ is orthogonal to $\overline{x}_{KF}(k) - \overline{x}_{RO}(k)$, i.e. $e_{KF,RO}(k)$. □

From equation (6) and lemma 2, the state estimation cost $J_s(S_s)$ takes another form which is stated in the next lemma.

Lemma 3

$$J_s(S_s) = tr(QZ) + \hat{J}_s(S_s), \tag{7}$$

where

$$\hat{J}_s(S_s) = \lim_{M \to \infty} \frac{1}{M} \sum_{k=0}^{M-1} E\left[c_s(e^{-}_{KF,RO}(k), S_s)\right],$$

and

$$\begin{aligned} c_s(e^{-}_{KF,RO}(k), S_s) &= \|e_{KF,RO}(k)\|_Z^2 + \lambda_s 1(e^{-}_{KF,RO}(k) \notin S_s) \\ &= \|e^{-}_{KF,RO}(k)\|_Z^2 1(e^{-}_{KF,RO}(k) \in S_s) + \lambda_s 1(e^{-}_{KF,RO}(k) \notin S_s). \end{aligned}$$

It is easy to see that the optimal triggering set $S_s^\dagger$ that minimizes $\hat{J}_s(S_s)$ also minimizes the state estimation cost $J_s(S_s)$.

Now, we are ready to analyze the optimal triggering set $S_s^\dagger$ in sensor subsystem. This optimal problem is an average cost optimal problem of a discrete-time Markov control process. According to Theorem 5.5.4 of [9], we have the following lemma.

Lemma 4 *If there exists a bounded function $h_s : \mathbb{R}^n \to \mathbb{R}$ and a finite number J'_s such that*

$$J'_s + h_s(e^{-}_{KF,RO}(k)) = G_s\left(e^{-}_{KF,RO}(k)\right) \tag{8}$$

where

$$G_s(\theta) = \min_{S_s}\left\{E(h_s(e^{-}_{KF,RO}(k+1))|e^{-}_{KF,RO}(k) = \theta) + c_s(\theta, S_s)\right\},$$

then the optimal average cost of remote state estimation is

$$J_s^\dagger = J'_s + tr(QZ), \tag{9}$$

and the optimal triggering set in sensor subsystem

$$S_s^\dagger = \Big\{\theta : E(h_s(e^-_{KF,RO}(k+1))|e_{KF,RO}(k) = \theta) + \|\theta\|^2_Z \\ \leq \lambda_s + E(h_s(e^-_{KF,RO}(k+1))|e_{KF,RO}(k) = 0)\Big\}. \tag{10}$$

Remark 4 [24] studied the same optimal problem assuming that the system state was directly measured. Their triggering event is similar to ours. The difference is that in [24], besides the triggering event $e^-_{KF,RO} \notin S_s^\dagger$, the transmission between the sensor subsystem and controller subsystem is also triggered by $\|e^-_{KF,RO}\| > c$ for some big enough positive constant c.

4.2 Optimal triggering set in controller subsystem

Using the same technique, we derive the optimal triggering set in the controller subsystem assuming that $S_s^\dagger$ is used as the triggering set in sensor subsystem.

Lemma 5 *Define C_c as*

$$C_c\left(\overline{x}_{RO}(k), u_a(k-1), S_c\right) = \|\overline{x}_{RO}(k)\|^2_Z + \lambda_c 1\left(\begin{bmatrix}\overline{x}_{RO}(k)\\ u_a(k)\end{bmatrix} \notin S_c\right).$$

Given $S_s^\dagger$, if there exists a bounded function $h_c : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$, and a bounded function $J'_c : \mathbb{S}^n \to \mathbb{R}$ ($\mathbb{S}^n$ indicates the collection of all subsets of $\mathbb{R}^n$) such that

$$J'_c(S_s^\dagger) + h_c(\overline{x}_{RO}(k), u_a(k-1)) = \min_{S_c}\{C_c(\overline{x}_{RO}(k), u_a(k-1), S_c) \\ + E\left(h_c(\overline{x}_{RO}(k+1), u_a(k))|\overline{x}_{RO}(k), u_a(k-1), S_c\right)\}, \tag{11}$$

then

$$J_c^\dagger(S_s^\dagger) = J'_c(S_s^\dagger), \tag{12}$$

and the optimal triggering set in controller subsystem is

$$S_c^\dagger = \left\{\begin{bmatrix}\theta\\ \eta\end{bmatrix} : E\left[h_c\left(\begin{bmatrix}\overline{x}_{RO}(k+1)\\ u_a(k)\end{bmatrix}\right)\middle|\begin{bmatrix}\overline{x}_{RO}(k)\\ u_a(k-1)\end{bmatrix} = \begin{bmatrix}\theta\\ \eta\end{bmatrix}\right] \leq \\ E\left[h_c\left(\begin{bmatrix}\overline{x}_{RO}(k+1)\\ u_a(k)\end{bmatrix}\right)\middle|\begin{bmatrix}\overline{x}_{RO}(k)\\ u_a(k-1)\end{bmatrix} = \begin{bmatrix}\theta\\ K\theta\end{bmatrix}\right] + \lambda_c\right\}.$$

4.3 Upper bound on the cost of the optimal triggering sets

From the results in equation (5), Lemma 4 and 5, we obtain the optimal triggering sets in sensor and controller subsystems, and an upper bound on the optimal cost.

Theorem 2 *The optimal triggering set in sensor subsystem $S_s^\dagger$ defined in theorem 4 minimizes $J_s(S_s)$, and the optimal triggering set in controller subsystem $S_c^\dagger$ defined in theorem 5 minimizes $J_c(S_c, S_s^\dagger)$.*

The optimal cost of the closed loop system J^* satisfies

$$J^* \leq J_s^\dagger + J_c^\dagger(S_s^\dagger),$$

where $J_s^\dagger$ and $J_c^\dagger(S_s^\dagger)$ are described in equation (9) and (12), respectively.

Remark 5 Although the equations (9) and (12) can be solved iteratively by value iteration [7] or algorithm iteration [15], computing the optimal policy still becomes prohibitive as the dimension of the system state increases.

With the concern about the computational complexity of the optimal triggering sets, we turn to a more computationally tractable approach for determining approximations to the optimal event triggers in the next section.

5 Suboptimal Triggering Sets

In this section, computationally tractable methods for computing suboptimal triggering sets of both sensor and controller subsystems are derived, and an upper bound on the overall cost achieved by the suboptimal triggering sets is provided.

5.1 Suboptimal triggering set in sensor subsystem

To find the suboptimal triggering set in the sensor subsystem, we first give the following lemma which works as the basis of searching for the suboptimal triggering set in the sensor subsystem and an upper bound on its cost. This is a direct results from Theorem 1 of [4] and Lemma 1 of [3].

Lemma 6 *Given the triggering set S_s, if there exists a bounded function $f_s : \mathbb{R}^n \to \mathbb{R}$ and a finite constant $\overline{J}_s$ such that for any $\theta \in \mathbb{R}^n$,*

$$\overline{J}_s + f_s(\theta) \geq E\left(f_s(e_{KF,RO}^-(k+1)) | e_{KF,RO}^-(k) = \theta, S_s\right) + c_s(\theta, S_s) \tag{13}$$

then

$$\hat{J}_s(S_s) \leq \overline{J}_s. \tag{14}$$

Based on lemma 6, we can identify a suboptimal triggering set which is in quadratic form. Moreover, the upper bound on this suboptimal triggering set is also given.

Lemma 7 *Given a quadratic triggering set*

$$S_s = \{e_{KF,RO}^-(k) : \|e_{KF,RO}^-(k)\|_{H_s}^2 \leq \lambda_s - \zeta_s\}, \tag{15}$$

where the $n \times n$ matrix $H_s \geq 0$ satisfies the Lyapunov inequality

$$\frac{A^T H_s A}{1+\delta_s^2} - H_s + \frac{Z}{1+\delta_s^2} \leq 0, \tag{16}$$

for some $\delta_s^2 \geq 0$, and $\zeta_s = \frac{\delta_s^2 \lambda_s + tr(H_s R)}{1+\delta_s^2}$, where

$$R = L(CAQA^T C^T + CWC^T + V)L^T,$$

then $J_s(S_s)$ is bounded from above by

$$J_s(S_s) \leq \min\{tr(H_s R) + \zeta_s, \lambda_s\} + tr(QZ) \tag{17}$$

Proof To find an upper bound on the cost of triggering set defined in equation (15), we need to find a bounded function f_s and a finite constant $\overline{J}_s$ such that equation (13) is satisfied. With lemma 6 and 3, we can derive that $J_s(S_s) \leq \overline{J}_s + tr(QZ)$.

Now, let's define f_s as

$$f_s(e^-_{KF,RO}(k)) = \min\{\|e^-_{KF,RO}(k)\|^2_{H_s} + \zeta_s, \lambda_s\},$$

and $\overline{J}_s$ as

$$\overline{J}_s = E(f_s(e^-_{KF,RO}(k+1))|e_{KF,RO}(k) = 0). \tag{18}$$

In the case of $\|e^-_{KF,RO}(k)\|^2_{H_s} \leq \lambda_s - \zeta_s$, no transmission occurs at step k, so the right hand side of equation (13) satisfies the following equations.

$$\begin{aligned}
&E\left(f_s(e^-_{KF,RO}(k+1))|e^-_{KF,RO}(k), S_s\right) + c_s\left(e^-_{KF,RO}, S_s\right) \\
=&E\left(f_s(e^-_{KF,RO}(k+1))|e_{KF,RO}(k) = e^-_{KF,RO}(k)\right) + \|e^-_{KF,RO}\|^2_Z \\
\leq&\|e^-_{KF,RO}\|^2_{A^T H_s A} + tr(H_s R) + \zeta_s + \|e^-_{KF,RO}\|^2_Z \\
\leq&\|e^-_{KF,RO}(k)\|^2_{H_s} + \|e^-_{KF,RO}(k)\|^2_{A^T H_s A - H_s + Z} + \zeta_s + tr(H_s R) \\
\leq&\|e^-_{KF,RO}(k)\|^2_{H_s} + \zeta_s + \delta_s^2(\lambda_s - \zeta_s) + tr(H_s R) \\
=&f_s(e^-_{KF,RO}(k)) + \zeta_s \\
\leq&f_s(e^-_{KF,RO}(k)) + \overline{J}_s.
\end{aligned}$$

The second step is taken by the fact that $E(\min(f,g)) \leq \min(E(f), E(g))$, the fourth step is derived from equation (16) and the fact that $\|e^-_{KF,RO}(k)\|^2_{H_s} \leq \lambda_s - \zeta_s$, and the fifth step is derived from how we define the ζ_s.

In the case of $\|e^-_{KF,RO}(k)\|^2_{H_s} > \lambda_s - \zeta_s$, transmission occurs. The right side of inequality (13) satisfies

$$\begin{aligned}
&E\left(f_s(e^-_{KF,RO}(k+1))|e^-_{KF,RO}(k), S_s\right) + c_s\left(e^-_{KF,RO}, S_s\right) \\
=&E\left(f_s(e^-_{KF,RO}(k+1))|e_{KF,RO}(k) = 0\right) + \lambda_s \\
=&\overline{J}_s + f_s(e^-_{KF,RO}(k))
\end{aligned}$$

Since the inequality (13) holds in any condition, from Lemma 6, we know that $\hat{J}_s(S_s)$ is bounded from above by $\overline{J}_s$ defined in (18), i.e.

$$\hat{J}_s(S_s) \leq \overline{J}_s.$$

From the fact that $E(\min(f,g)) \leq \min(E(f), E(g))$, it's easy to show that

$$\overline{J}_s \leq \min\{tr(H_sR) + \zeta_s, \lambda_s\}.$$

From lemma 3, we have

$$J_s(S_s) = \hat{J}_s(S_s) + tr(QZ) \leq \min\{tr(H_sR) + \zeta_s, \lambda_s\} + tr(QZ).$$

□

Remark 6 (i) For any A and Z, there must exist a positive definite matrix H_s and a constant δ_s such that equation (16) holds. It is easy to see that we can always choose δ_s^2 big enough (e.g square of the largest singular value of A) so that $\frac{A}{\sqrt{1+\delta_s^2}}$ is stable, and hence for any positive definite Z, there is a positive definite matrix H_s satisfying equation (16) (Theorem 10.12 of [2]).

(ii) Computing H_s, and hence the suboptimal triggering set S_s in (15), is easy, since Equation (16) is a linear matrix inequality with respect to matrix H_s when we fix constant δ_s.

(iii) According to our experience, to obtain as small as possible upper bound on the suboptimal cost, δ_s should be chosen as small as possible while equation (16) is feasible.

(iv) Compared with the work in [5], our suboptimal triggering set in sensor subsystem can be applied to unstable systems while [5] required that the matrix A was Hurwitz. But we only provide an upper bound on $J_s(S_s)$ while [5] provided both an upper bound on $J_s(S_s)$ and a lower bound on the optimal cost J_s^*, and hence characterized how far away the suboptimal cost was from the optimal cost.

5.2 Suboptimal triggering set in controller subsystem

Similar to the derivation of the suboptimal triggering set in sensor subsystem, we first give a lemma working as the basis of finding an upper bound on the suboptimal triggering set in the controller subsystem.

Lemma 8 *Given any S_c, if there exists a function $f_c : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ bounded from below and a finite constant $\overline{J}_c'$ such that*

$$\begin{aligned} \overline{J}_c' + f_c\left(\overline{x}_{RO}(k), u_a(k-1)\right) \geq C_c\left(\overline{x}_{RO}(k), u_a(k-1), S_c\right) \\ + E\left[f_c\left(\overline{x}_{RO}(k+1), u_a(k)\right) | \overline{x}_{RO}(k), u_a(k-1)\ , S_c\right], \end{aligned} \tag{19}$$

then $J_c(S_c) \leq \overline{J}_c'$.

Now, let us give the suboptimal triggering set in controller subsystem and its upper bound in the following Lemma.

Lemma 9 *Let S_s in Equation (15) be the triggering set in sensor subsystem.* $A_u = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix}$, $A_c = \begin{bmatrix} A+BK & 0 \\ K & 0 \end{bmatrix}$, $Z_a = \begin{bmatrix} Z & 0 \\ 0 & 0 \end{bmatrix}$, *and* $H_s = P_{Hs}^T P_{Hs}$. *Given a quadratic triggering set of controller subsystem*

$$S_c = \left\{ \begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix} : \left\| \begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix} \right\|_{H_c}^2 + \zeta_c \le \|\overline{x}_{RO}(k)\|_Z^2 + \lambda_c \right\}, \quad (20)$$

where $H_c \ge Z_a$ and controller gain K satisfy

$$A_u^T H_c A_u + (1+\delta_c^2)(Z_a - H_c) \le 0, \quad (21)$$

$$A_c^T H_c A_c + (1-\rho_c^2)(Z_a - H_c) \le 0, \quad (22)$$

for some constant $\delta_c^2 \ge 0$ and $0 \le \rho_c^2 \le 1$, and

$$\zeta_c = \frac{\delta_c^2 + \rho_c^2 - 1}{\delta_c^2 + \rho_c^2} \lambda_c, \quad (23)$$

the controller cost satisfies

$$J_c(S_s, S_c) \le \overline{J}_c(S_c, S_s) = \frac{\delta_c^2}{\delta_c^2 + \rho_c^2} \lambda_c + \overline{\sigma}((P_{Hs}^T)^{-1} H_{c,lu} P_{Hs}^{-1})(\lambda_s - \zeta_s), \quad (24)$$

where $\overline{\sigma}(\cdot)$ indicates the greatest singular value, and $H_{c,lu}$ is the left upper $n \times n$ sub-matrix of H_c.

Proof According to Lemma 8, as long as we can find a function f_c bounded from below such that the inequality (19) is satisfied with $\overline{J}_c' = \overline{J}_c$, Lemma 9 is true.

Let's define f_c as

$$f_c(\overline{x}_{RO}(k), u_a(k-1)) = \left\| \begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix} \right\|_{H_c}^2 + \zeta_c.$$

First, we consider the case when $\left\| \begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix} \right\|_{H_c}^2 + \zeta_c \le \|\overline{x}_{RO}(k)\|_Z^2 + \lambda_c$. In this case, the controller subsystem doesn't transmit. The right hand side of equation (19) is

$$\begin{aligned} &\le \begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix}^T A_u^T H_c A_u \begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix} + \overline{\sigma}((P_{Hs}^T)^{-1} H_{c,lu} P_{Hs}^{-1})(\lambda_s - \zeta_s) + \zeta_c \\ &\quad + \begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix}^T Z_a \begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix} \\ &\le \overline{J}_c + f_c\left(\begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix} \right) \end{aligned}$$

The first inequality is from equation (15), and the second inequality is from equation (21) and (23).

The second case is when $\left\| \begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix} \right\|_{H_c}^2 + \zeta_c > \|\overline{x}_{RO}(k)\|_Z^2 + \lambda_c$. In this case, the controller subsystem transmits information. So the right hand side of equation (19) is

$$
\begin{aligned}
&\leq \begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix}^T A_c^T H_c A_c \begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix} + \overline{\sigma}((P_{Hs}^T)^{-1} H_{c,lu} P_{Hs}^{-1})(\lambda_s - \zeta_s) + \zeta_c \\
&\quad + \begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix}^T Z_a \begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix} + \lambda_c \\
&\leq \overline{J}_c + f\left(\begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix} \right).
\end{aligned}
$$

The first inequality is from Equation (15), and the second inequality is from Equation (22) and (23).

Since in both cases, equation (19) holds, we conclude that the control cost of the suboptimal triggering set $J_c(S_c, S_s)$ is bounded from above by $\overline{J}_c(S_c, S_s)$. □

5.3 Upper bound on the cost of suboptimal triggering sets

From the results in Lemma 1, 7 and 9, we have the following theorem.

Theorem 3 *Given the triggering set in sensor subsystem S_s defined in equation (15) and the triggering set in controller subsystem S_c defined in equation (20), the average cost $J(S_s, S_c)$ given by the two weakly coupled triggering sets satisfies*

$$J(S_s, S_c) \leq \overline{J}_s(S_s) + \overline{J}_c(S_c, S_s),$$

where $J_s(S_s)$ and $J_c(S_c, S_s)$ are defined in equation (17) and (24), respectively.

6 Simulation Results

This section uses an example to demonstrate theorem 3. We first calculate the triggering sets S_s and S_c according to equation (15) and (20), and search for the controller gain K such that inequality (22) is satisfied. The system, then, is run using three different transmission rules: weakly coupled event triggered transmission, synchronized event triggered transmission and periodic transmission. The average costs are compared. We also show that the number of transmission times in sensor subsystem, the number of transmission times in controller subsystem, and the number of times when both sensor and controller transmit (concurrent transmission times) to illustrate that the transmission in the sensor subsystem doesn't necessarily trigger the transmission in the controller subsystem, or vice versa.

Let's consider the following system

$$x(k) = \begin{bmatrix} 0.4 & 0 \\ 0 & 1.01 \end{bmatrix} x(k-1) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_a(k-1) + w(k-1)$$
$$y(k) = \begin{bmatrix} 0.1 \; 1 \end{bmatrix} x(k) + v(k).$$

The variances of the system noises are $W = \begin{bmatrix} 0.2 \; 0.1 \\ 0.1 \; 0.2 \end{bmatrix}$, and $V = 0.3$. The weight matrix Z is chosen to be an identity matrix.

Given $\delta_s^2 = 1.5$, $\lambda_s = 3$, $\delta_c^2 = 1.02$ and $\rho_c = 0.3$, we can obtain the triggering set in sensor subsystem S_s as

$$S_s = \left\{ e^-_{KF,RO} : e^{-T}_{KF,RO} \begin{bmatrix} 2.5641 & 0 \\ 0 & 4.0543 \end{bmatrix} e^-_{KF,RO} \leq 0.8414 \right\}, \quad (25)$$

and the triggering set in controller subsystem S_c as

$$S_c = \left\{ \begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix} : \begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix}^T \begin{bmatrix} 1.3315 & -0.2836 & -0.3512 \\ -0.2836 & 3.6377 & 2.6808 \\ -0.3512 & 2.6808 & 13.7606 \end{bmatrix} \begin{bmatrix} \overline{x}_{RO}(k) \\ u_a(k-1) \end{bmatrix} \right.$$
$$\left. \leq 0.9008\lambda_c \right\}, \quad (26)$$

and the controller gain $K = [-0.1967 \;\; -0.3133]$.

The closed loop system was run for 3000 steps using different transmission rules with the communication price from controller to actuator varying from 0 to 200. When we run the system, there was one step delay in the communication network. We first run the system with triggering sets S_s and S_c defined in (25) and (26). After that, the system was run using a synchronized transmission rule, with which the transmissions from sensor to controller were only triggered by $e^-_{KF,RO}(k) \notin S_s$, and the transmissions from controller to actuator were triggered by either $[\overline{x}_{RO}(k) \; u_a(k-1)]^T \notin S_c$ or transmissions from sensor to controller. Finally, we used the average periods in weakly coupled event triggered transmission experiment as the periods to run the system using periodic transmission. The average costs are given in Figure 2.

Figure 2 describes the average costs with respect to λ_c, the communication price from controller to actuator. Stars indicate the average cost using weakly coupled transmission rule which gives the least average cost during the costs using different triggering rules. Squares denote the average cost using synchronized transmission rule which gives the second least average cost, and its difference from the average cost of weakly coupled transmission increases as λ_c increases. That's because when λ_c is small, the transmission time instants from controller to actuator incurred by $[\overline{x}_{RO}(k) \; u_a(k-1)]^T \notin S_c$ (weakly coupled transmission) and the transmission time instants from controller to actuator incurred by either $[\overline{x}_{RO}(k) \; u_a(k-1)]^T \notin S_c$ or transmissions from

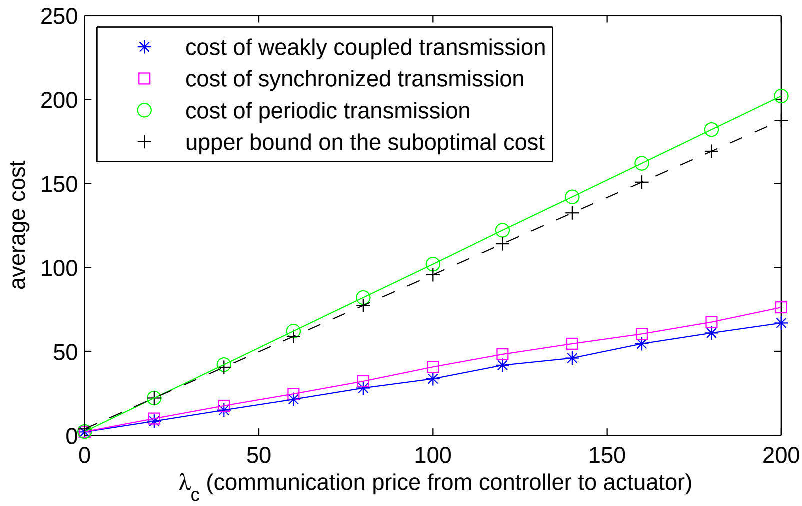

Fig. 2 Average costs using weakly coupled transmission, synchronized transmission and periodic transmission which different communication price from controller to actuator

sensor to controller (synchronized transmission) are very similar to each other. An extreme case is when $\lambda_c = 0$. As λ_c increases, the transmission time instants from controller to actuator using weakly coupled transmission rule become more and more different from the transmission time instants from controller to actuator using synchronized transmission rule, and hence the difference between the average costs using weakly coupled transmission rule and synchronized transmission rule also increases. Circles indicate the average cost using periodic transmission. Figure 2 shows that the average cost using periodic transmission is always greater than the average cost using event triggered transmission no matter whether the transmission is weakly coupled (stars) or synchronized (squares). Crosses are the upper bounds on the average cost of weakly coupled transmission calculated according to Theorem 3. These crosses are always above the average cost of weakly coupled transmission (stars), which demonstrates Theorem 3. We also notice that both the upper bound (crosses) and the actual cost (stars) increases linearly with respect to λ_c, and the ratio of the upper bound (crosses) to the actual cost (stars) is about 3.

Figure 3 shows that the transmission in the sensor subsystem doesn't always trigger the transmission in controller subsystem, or vice versa. The x-axis of this plot is the communication price from controller to actuator λ_c, and the y-axis indicates the transmission times. We can see that the number of concurrent transmission times (circles) is always less or equal to both the numbers of transmission times in sensor subsystem (stars) and controller subsystem (crosses), which indicates that the transmission in sensor subsystem doesn't always trigger the transmission in controller subsystem, or vice versa.

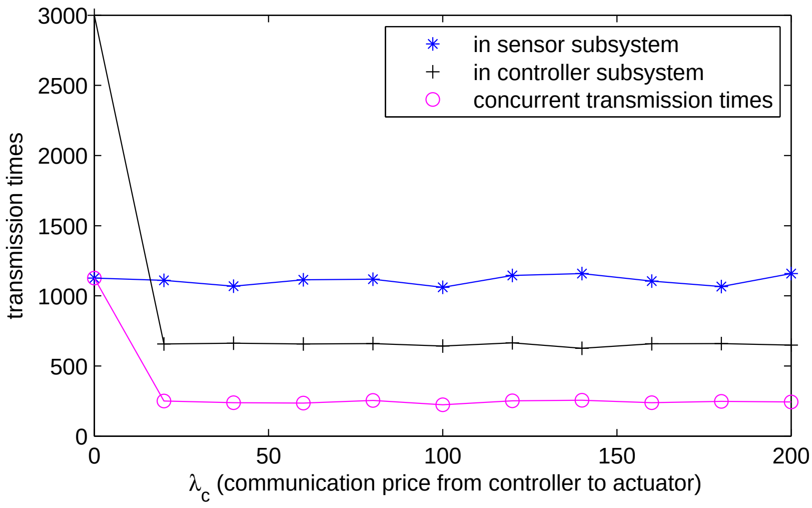

Fig. 3 Transmission times in sensor subsystem, controller subsystem and the concurrent transmission times using weakly coupled transmission rule

7 Conclusion

This paper presents weakly coupled triggering events in event triggered output feedback system with the whole control loop closed over wireless network. By 'weakly coupled', we mean that the triggering events in both sensor and controller only use local information to decide when to transmit data, and the transmission in one link doesn't necessarily trigger the transmission in other link. We also show that with the triggering events and controller we designed, the cost of the closed loop system is bounded from above, and an explicit upper bound on the cost is obtained. Our simulation results show that the proposed triggering events are weakly coupled and the upper bound on the cost of the closed loop system is relatively tight when the communication price λ_s from the sensor subsystem to the controller subsystem is low.

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