Efficiently Attentive Event-Triggered Control Systems with Limited Bandwidth

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Abstract

Wireless networked control systems have limited bandwidth, which means that each transmitted packet has a finite number of bits, and always arrives at its destination with non-negligible delay. This paper derives a bound on an event-triggered system's stabilizing "instantaneous" bit-rate when the sampled signal is dynamically quantized. This instantaneous bit-rate is a time-varying function whose average value can be made small by requiring the instantaneous bit-rate get smaller as the system state approaches the origin. We refer to this property as *efficient attentiveness*. This paper provides sufficient conditions guaranteeing the instantaneous bit-rate's efficient attentiveness. Our numerical example illustrates the results, and indicates a tradeoff between inter-sampling interval and instantaneous bit-rate.

Index Terms

event-triggering, efficient attentiveness, dynamic quantization, stabilizing bit-rate.

I. Introduction

Wireless sensor-actuator networks are networked control systems whose actuators/controllers and sensors communicate over a wireless communication network. These communication networks are digital networks with finite capacity. This means that transmitted packets consist of a finite number of bits and always arrive at their destination with a non-negligible delay. The

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resulting quantization error and message latency has a negative impact on overall closed-loop system performance [1] and hence must be considered when designing any wireless sensor-actuator network.

Event triggering is a recent approach to sampled-data control in which sampled feedback measurements are transported over the feedback channel in a sporadic manner. In event-triggered systems, the system state is sampled and transmitted back to the actuator or controller when the difference between the current state and the last transmitted state exceeds a specified threshold. It has been experimentally demonstrated [2]–[8] that event-triggered system can greatly reduce the average rate at which the control system accesses the feedback channel over periodically-sampled systems with comparable performance levels.

While event-triggered systems have the *potential* to reduce the inter-sampling rate, they can also increase that rate if improperly designed. The following example illustrates this issue. Consider a cubic system

$$\dot{x} = x^3 + u; \quad x(0) = x_0$$

where $u = -3\hat{x}_k^3$ for $t \in [s_, s_{k+1})$ where s_k is the kth consecutive sampling instant and $\hat{x}_k = x(s_k)$. Let us now consider two different event-triggers. The trigger $\mathbf{E}_1$ generates a sampling instant s_{k+1} when $|x(t) - \hat{x}_k| = 0.5|x(t)|$ and the second trigger $\mathbf{E}_2$ generates a sampling instant when $|x(t) - \hat{x}_k| = 0.5|x^4(t)|$. For both event-triggers we find the system is locally asymptotically stable for all $|x_0| \leq 1$. But if one examines the inter-sampling intervals for these systems in Figure 1, it should be apparent that the inter-sampling intervals generated by trigger $\mathbf{E}_1$ get longer as the system approaches its equilibrium. On the other hand, the trigger $\mathbf{E}_2$ results in a sequence of inter-sampling times that get shorter as the system approaches the origin.

Since the above example focuses on regulation about the origin, one would clearly want the inter-sampling interval to be longest when the system is close to the origin. The ”interesting” feedback information that mandates use of the feedback channel should occur when the system is perturbed away from the origin, not when the system is resting in the neighborhood of the origin. This ”desired” behavior is exhibited by event-trigger $\mathbf{E}_1$, but as shown in Figure 1, the opposite trend is exhibited by event-trigger $\mathbf{E}_2$. Event trigger $\mathbf{E}_1$ is said to be *efficiently attentive* because the control system is more efficient in its use of the channel when the system state is close to the origin.

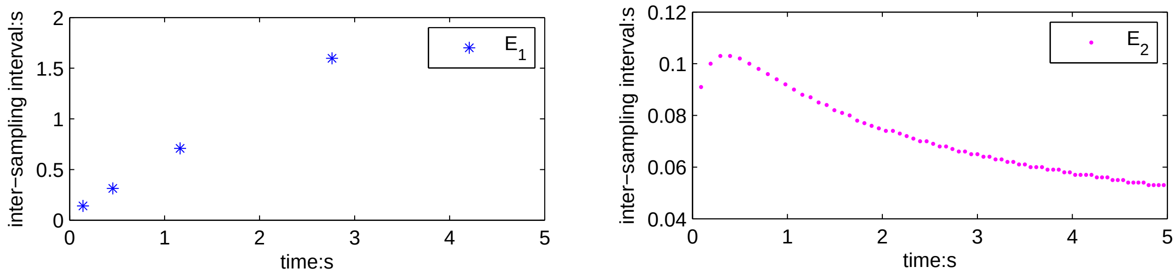

Fig. 1. Inter-sampling intervals for two different types of event-triggers

One reason why there has been such great interest in the inter-sampling interval is that it can be taken as a measure of channel usage. Much of this prior work, however, [4], [9]–[11] has ignored delays or quantization errors. For this reason, results bounding the inter-sampling interval only provide a partial picture of an event-triggered system's network usage. Recent work has begun to consider constant bounded delays [12]–[15], but this paper shows that delays preserving input-to-state stability (ISS) are state-dependent, thereby suggesting that the "bit-rates" required to support event-triggered systems are time-varying. If, in fact, one can assure that these "instantaneous" bit-rates are efficiently attentive, then one reduces the average bit rate over systems using constant bounded delays. The main results in this paper establish sufficient conditions for an event-triggered system's instantaneous bit-rate to be efficiently attentive. We then provide simulation examples to illustrate the value of these results. In the examples, we also see a tradeoff between the inter-sampling interval and the instantaneous bit-rate. This tradeoff indicates that we should not only focus on lengthening the inter-sampling interval, but also need to guarantee the channel bandwidth satisfies the required instantaneous bit-rate.

This paper is an extended version of our prior work in [16]–[18], and is organized as follows. Section II and III gives the mathematical preliminaries and problem statement. Section IV discusses how to design the triggering event and the quantization map in event-triggered systems. Section V provides the acceptable delay preserving ISS, and Section VI presents sufficient conditions for efficiently attentive bit-rates. Numerical example and conclusion are given in Section VII and VIII, respectively.

II. NOTATION AND INPUT-TO-STATE STABILITY

A. Notation

Throughout this paper, the n dimensional real space will be denoted as $\mathbb{R}^n$ and the set of non-negative reals will be denoted as $\mathbb{R}^+$. The infinity (supremum) norm of a vector $x \in \mathbb{R}^n$ will be denoted as $\|x\|$. The $\mathcal{L}$-infinity norm of a function $x(\cdot) : \mathbb{R}^+ \to \mathbb{R}^n$ is defined as $\|x\|_{\mathcal{L}_\infty} = \operatorname{ess\,sup}_{t \geq 0} \|x(t)\|$. This function is said to be essentially bounded if $\|x\|_{\mathcal{L}_\infty} < \infty$ and the linear space of all essentially bounded real-valued functions will be denoted as $\mathcal{L}_\infty$. A subset $\Omega \subset \mathbb{R}^n$ is said to be compact if it is closed and bounded.

A function $\alpha(\cdot) : \mathbb{R}^+ \to \mathbb{R}^+$ is class $\mathcal{K}$ if it is continuous, strictly increasing and $\alpha(0) = 0$. It is said to be of $\mathcal{K}_\infty$, if $\alpha(s) \to \infty$ as $s \to \infty$. A function $\beta : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is class $\mathcal{K}L$ if $\beta(\cdot, t)$ is class $\mathcal{K}$ for each fixed $t \geq 0$ and $\beta(r, t)$ decreases to 0 as $t \to \infty$ for each fixed $r \geq 0$.

Lemma II.1. *Let $g : [0, \sigma] \to \mathbb{R}^+$ be a continuous, positive definite function satisfying $\lim_{s \to 0} < \infty$. There must exist continuous, positive definite, increasing functions $\underline{h}$ and $\overline{h}$ defined on $[0, \sigma]$ such that*

$$\underline{h}(s) \leq g(s) \leq \overline{h}(s), \forall s \in [0, \sigma],$$
$$\lim_{s \to 0} g(s) = \lim_{s \to 0} \underline{h}(s) = \lim_{s \to 0} \overline{h}(s).$$

Proof: See Lemma 4.3 in [19]. ■

Let Ω be a compact subset of $\mathbb{R}^n$. We say $f(\cdot) : \Omega \to \mathbb{R}^n$ is locally Lipschitz on Ω if for any $x, y \in \Omega$, there exists a constant $L \geq 0$ such that

$$\|f(x) - f(y)\| \leq L\|x - y\|$$

B. Input-to-state stability

Consider a system whose state trajectory $x(\cdot) : \mathbb{R}^+ \to \mathbb{R}^n$ satisfies the initial value problem,

$$\dot{x}(t) = f(x(t), w(t)), \quad x(0) = x_0 \tag{1}$$

where $w(\cdot) : [0, \infty) \to \mathbb{R}^m$ is an essentially bounded signal. Let $x = 0$ be an equilibrium point for (1) with $w(t) \equiv 0$, and $\Upsilon \subset \mathbb{R}^n$ be a domain containing $x = 0$. Let $V : \Upsilon \to \mathbb{R}$ be a

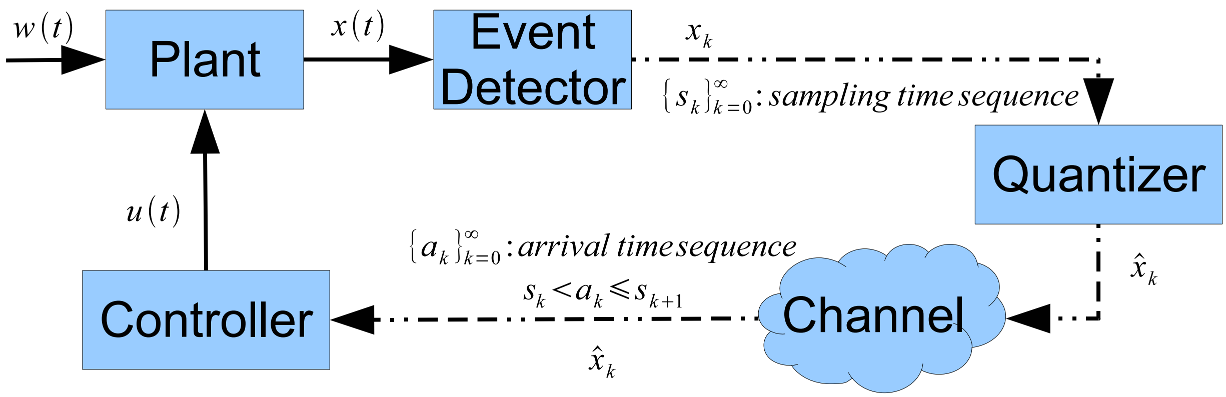

Fig. 2. Event-triggered control system with quantization

continuously differentiable function such that

$$\underline{\alpha}(\|x\|) \leq V \leq \overline{\alpha}(\|x\|), \tag{2}$$

$$\frac{\partial V}{\partial x} f(x, w) \leq -\alpha(\|x\|) + \gamma(\|w\|), \tag{3}$$

for all $(x, w) \in \Upsilon \times \mathbb{R}^m$, where $\underline{\alpha}$, $\overline{\alpha}$ are class $\mathcal{K}_\infty$ functions, and α, γ are class $\mathcal{K}$ functions, then the system (1) is input-to-state stable (ISS). The function V is called *ISS-Lyapunov* function.

III. Problem Statement

The system under study is a wireless networked event-triggered control system with quantization shown in Figure 2.

Consider the following plant whose state satisfies the following differential equation.

$$\dot{x}(t) = f(x(t), u(t), w(t)), \quad x(0) = x_0 \tag{4}$$

where $f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^q \to \mathbb{R}^n$ is locally Lipschitz in all three variables with $f(0, 0, 0) = 0$. The disturbance $w(\cdot) : \mathbb{R}^+ \to \mathbb{R}^q$ is an $\mathcal{L}_\infty$ disturbance with $\|w\|_{L_\infty} = \bar{w}$. The control signal $u(\cdot) : \mathbb{R}^+ \to \mathbb{R}^m$ is generated by the *controller* in figure 2.

The system state, $x(t)$, at time t is measured by the *event detector* which triggers the transmission once upon the violation of an event $\mathbf{E}$. Formally, event $\mathbf{E}(\cdot) : \mathbb{R}^n \to \{\text{true}, \text{false}\}$ maps the current state onto either "TRUE" or "FALSE". The kth transmission time s_k satisfies

$$s_k = \min\{t : \mathbf{E}(x(t)) \text{ is false and } t > s_{k-1}\}, \tag{5}$$

and the inter-sampling interval τ_k is defined as $\tau_k = s_{k+1} - s_k$. Let x_k indicate the system state $x(s_k)$ at s_k. Once the event detector decides to transmit, the *quantizer* converts this continuous valued state x_k into N_k bit representation $\hat{x}_k$ with quantization error to be $\Delta_k = |\hat{x}_k - x(s_k)|$. Notice that both N_k and Δ_k can be time varying and state dependent. The quantized state $\hat{x}_k$ is, then, transmitted to the controller with delay d_k. The *instantaneous bit-rate* r_k is, then, defined as $r_k = \frac{N_k}{d_k}$, and the arrival time a_k of the kth transmission, then, satisfies $a_k = s_k + d_k$. We say that the transmission and arrival sequences are admissible if $s_k < a_k \leq s_{k+1}$ for $k = 0, 1, \dots, \infty$.

Upon the arrival of the kth quantized state, $\hat{x}_k$, at the controller, a control input is computed and then held until the next quantized state is received. In other words, the control signal takes the form

$$u(t) = u_k = K(\hat{x}_k) \tag{6}$$

for $t \in [a_k, a_{k+1})$. The function $K(\cdot) : \mathbb{R}^n \to \mathbb{R}^m$ is locally Lipschitz, and satisfies $K(0) = 0$. As has been done in [4], this paper assumes that K is chosen so the system

$$\dot{x}(t) = f(x(t), K(x(t) + e(t)), w(t)), \tag{7}$$

is locally input-to-state stable with respect to the signals $e, w \in \mathcal{L}_\infty$. This means, of course, that there exists a function $V(\cdot) : \Upsilon \to \mathbb{R}^+$ satisfying

$$\underline{\alpha}(\|x\|) \leq V \leq \overline{\alpha}(\|x\|) \tag{8}$$

$$\frac{\partial V}{\partial x} f(x, w) \leq -\alpha(\|x\|) + \gamma_1(\|e\|) + \gamma_2(\|w\|), \tag{9}$$

for all $x \in \Upsilon$ where $\Upsilon \subseteq \mathbb{R}^n$ is a domain containing the origin, $\underline{\alpha}$, $\overline{\alpha}$, α, γ_1 and γ_2 are class $\mathcal{K}$ functions.

Our objective is to design the event detector and the quantizer such that the system defined in (4), (5) and (6) is ISS and efficiently attentive.

Definition III.1. *The system defined in (4), (5) and (6) is said to be* efficiently attentive *if there exists a continuous, positive definite function* $\underline{h}(s_1, s_2)$ *satisfying* $\lim_{s \to 0} \underline{h}(s_1, s_2) > 0$ *and decreasing with respect to both variables, and a continuous, positive definite function* $\overline{h}(s_1, s_2)$ *satisfying* $\lim_{s \to 0} \overline{h}(s_1, s_2) < \infty$ *and increasing with respect to both variables such that*

- *the inter-sampling interval* τ_k *is bounded from below by* $\underline{h}(\|\hat{x}_{k-1}\|, \|\hat{x}_k\|)$*, i.e.*

$$\tau_k \geq \underline{h}(\|\hat{x}_{k-1}\|, \|\hat{x}_k\|). \tag{10}$$

- *the instantaneous bit-rate r_k is bounded from above by $\bar{h}(\|\hat{x}_{k-1}\|, \|\hat{x}_k\|)$, i.e.*

$$r_k \leq \bar{h}(\|\hat{x}_{k-1}\|, \|\hat{x}_k\|). \tag{11}$$

Remark III.2. *The bounds on τ_k and r_k depend on both $\hat{x}_{k-1}$ and $\hat{x}_k$, because $\hat{x}_k$ is transmitted with some delay, and both $\hat{x}_{k-1}$ and $\hat{x}_k$ are used to produce control input and hence influence the system dynamic during intervals $[s_k, a_k)$ and $[a_k, s_{k+1}]$, respectively.*

Remark III.3. *The requirement (10) on τ_k indicates how often the event-triggered control system uses communication resources. Once requirement (10) is satisfied, the event-triggered control system will use the communication less frequently as the system state get closer to the origin.*

Remark III.4. *The requirement (11) on r_k indicates how much communication resources (i.e. bandwidth) is used by the event-triggered control system in each transmission. Equation (11) requires the event-triggered control system uses fewer communication resources as the system state gets closer to the origin.*

To guarantee ISS and efficient attentiveness, we first provide the event detector and quantizer guaranteeing ISS assuming there is no delay, then study the acceptable delay based on the designed event detector and quantizer, and finally give the sufficient condition of efficient attentiveness.

IV. Event detector and quantizer guaranteeing ISS without delay.

Let us first look at the dynamic behavior of the closed loop system. Let $e_k(t) = x(t) - \hat{x}_k$ be the *gap* between the current state and the kth quantized state. From equation (8) and (9), it is easy to get the following lemma.

Lemma IV.1. *Given the event-triggered system in (4-5) whose controller (6) satisfies is locally ISS and satisfies (8-9). If for all $t \in [a_k, a_{k+1})$ and all $k = 0, 1, \cdots, \infty$,*

$$\|e_k(t)\| \leq \xi(\|x(t)\|, \bar{w}) = \gamma_1^{-1}(c\alpha(\|x(t)\|) + \gamma_3(\bar{w})), \tag{12}$$

where $c \in (0, 1)$ and γ_3 is a $\mathcal{K}$ function, then the system is locally ISS.

Proof: Apply equation (12) into equation (9), we have

$$\frac{\partial V}{\partial x} f(x, w) \leq -(1-c)\alpha(\|x\|) + \gamma_2(\bar{w}) + \gamma_3(\bar{w}), \forall t \geq 0.$$

Since $c \in (0,1)$ and $\gamma_2(\bar{w}) + \gamma_3(\bar{w})$ is a class $\mathcal{K}$ function of $\bar{w}$, the system is ISS. ■

Suppose that $\xi(s,\bar{w})$ in (12) is locally Lipschitz for all $s \in \Upsilon$ with the Lipschitz constant L^{ξ}. Let

$$\underline{\xi}(s,\bar{w}) = \frac{1}{L^{\xi}+1}\xi(s,\bar{w}). \tag{13}$$

It can be shown that

Corollary IV.2. *Given the event-triggered system in (4-5) whose controller (6) satisfies is locally ISS and satisfies (8-9). If* $\|e_k(t)\| \leq \underline{\xi}_k = \underline{\xi}(\|\hat{x}_k\|,\bar{w})$ *in (13) , for all* $t \in [a_k, a_{k+1})$ *and all* $k = 0, 1, \cdots, \infty$*, then the event-triggered system is ISS.*

Proof:

$$\begin{aligned}
\|e_k(t)\| &\leq \frac{1}{L^{\xi}+1}\xi(\|\hat{x}_k\|,\bar{w}) \\
&\leq \frac{1}{L^{\xi}+1}\xi(\|x(t)\| + \|e_k(t)\|,\bar{w}). \\
&\leq \frac{1}{L^{\xi}+1}\xi(\|x(t)\|,\bar{w}) + \frac{L^{\xi}}{L^{\xi}+1}\|e_k(t)\| \\
\Rightarrow \|e_k(t)\| &\leq \xi(\|x(t)\|,\bar{w})
\end{aligned}$$

According to lemma IV.1, the above equation implies the system is ISS. ■

Now, let us choose our event-trigger as

$$\mathbf{E}: \ \ \|e_k(t)\| < \theta_k = \theta(\|\hat{x}_k\|,\bar{w}) = \rho_{\theta}\underline{\xi}(\|\hat{x}_k\|,\bar{w}), \tag{14}$$

where $\rho_{\theta} \in (0,1)$ is a given constant. Let Δ_k be the quantization error at the kth transmission time. To make sure the inter-sampling interval is strictly positive, we need to guarantee $\Delta_k < \theta_k$, so Δ_k is chosen as

$$\Delta_k = \rho_{\Delta}\theta(|\|\hat{x}_{k-1}\| - \theta_{k-1}|,\bar{w}), \tag{15}$$

where $\rho_{\Delta} \in (0,1)$ is a given constant. The proof of the following Lemma IV.3 shows that (15) implies $\Delta_k < \theta_k$. We assume that the controller knows the event-trigger and the quantization error. So, when the controller receives the kth packet, it knows that the state x satisfies $\|x - \hat{x}_{k-1}\| = \theta_{k-1}$. The set $\{x : \|x - \hat{x}_{k-1}\| = \theta_{k-1}\}$ is then uniformly quantized such that $\|x - \hat{x}_k\| \leq$

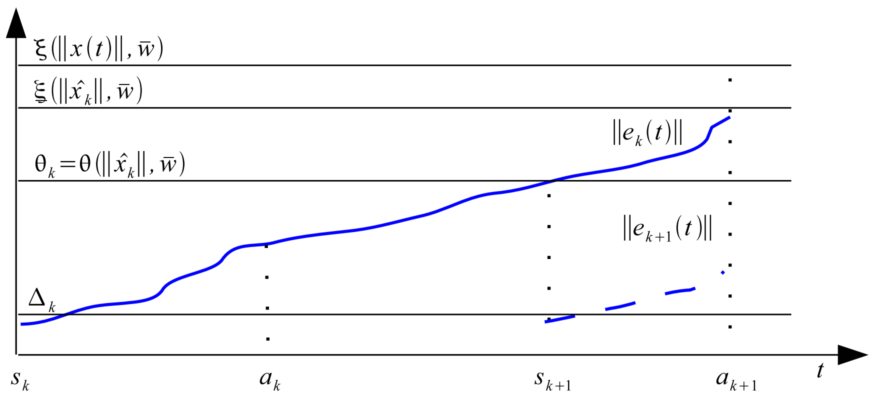

Fig. 3. A typical trajectory of $\|e_k(t)\|$

Δ_k, and the number of bits N_k transmitted at s_k satisfies

$$N_k = \lceil \log_2 2n \rceil + \left\lceil \log_2 \left\lceil \frac{\theta_{k-1}}{\Delta_k} \right\rceil^{n-1} \right\rceil. \tag{16}$$

Lemma IV.3. *Suppose there is no delay. The system defined in (4-6) with the event-trigger and quantization error defined as (14) and (15) is ISS.*

Proof: From equation (12-14), we see that $\theta(s, \bar{w})$ is a increasing function of s. Since $\|\hat{x}_k - \hat{x}_{k-1}\| = \theta_{k-1} \Rightarrow \|\hat{x}_k\| \geq |\|\hat{x}_{k-1}\| - \theta_{k-1}|$, we have

$$\|e_k(s_k)\| \leq \Delta_k < \theta(|\|\hat{x}_{k-1}\| - \theta_{k-1}|, \bar{w}) \leq \theta(\|\hat{x}_k\|, \bar{w}) = \theta_k.$$

Since $\|e_k(s_{k+1})\| = \theta_k$, according to the continuity of e_k, $\|e_k\| < \underline{\xi}_k$ for all $t \in [s_k, s_{k+1})$ and all $k = 0, 1, \cdots, \infty$. From Corollary IV.2, the system is ISS. ■

V. Acceptable Delays Preserving Input-to-State Stability

Corollary IV.2 indicates that bounding $e_k(t)$ during interval $[a_k, a_{k+1})$ is essential to preserve the input-to-state stability of the closed loop system. A typical trajectory of $\|e_k(t)\|$ is shown in Figure 3. At time s_k, system state is quantized and transmitted with the initial gap $\|e_k(s_k)\| \leq \Delta_k$. This gap gradually increases and finally hits θ_k, which generates the $k+1$-st sampling instant. Before the $k+1$-st quantized state arrives at the controller, the gap $\|e_k(t)\|$ keeps increasing. To guarantee ISS, the gap $\|e_k(t)\|$ should be bounded by $\underline{\xi}_k$.

The gap $e_k(t)$ has the following dynamic behavior.

$$\dot{e}_k(t) = f(\hat{x}_k + e_k(t), u_k, w(t)), \forall t \in [a_k, a_{k+1}). \tag{17}$$

Let L_k^x be the Lipschitz constant of f with respect to x during interval $[s_k, a_k]$. $\|f\|$ is bounded from above by

$$\|f(\hat{x}_k + e_k(t), u_k, w)\| \leq \bar{f}(\hat{x}_k, u_k, \bar{w}) + L_k^x \|e_k\|, \tag{18}$$

where

$$\bar{f}(\hat{x}_k, u_k, \bar{w}) = \|f(\hat{x}_k, u_k, 0)\| + L_k^w \bar{w}.$$

If the transmission and arrival sequences are admissible, the norm of the gap $\|e_k(t)\|$, then, satisfies

$$\frac{d\|e_k(t)\|}{dt} \leq \|\dot{e}_k(t)\| \leq \bar{f}(\hat{x}_k, u_{k-1}, \bar{w}) + L_k^x \|e_k(t)\|, \forall t \in [s_k, a_k).$$
$$\frac{d\|e_k(t)\|}{dt} \leq \|\dot{e}_k(t)\| \leq \bar{f}(\hat{x}_k, u_k, \bar{w}) + L_k^x \|e_k(t)\|, \forall t \in [s_{k+1}, a_{k+1}).$$

With comparison principle, $\|e_k(t)\|$ satisfies

$$\|e_k(t)\| \leq \frac{\bar{f}(\hat{x}_k, u_{k-1}, \bar{w})}{L_k^x}(e^{L_k^x(t-s_k)} - 1) + \Delta_k e^{L_k^x(t-s_k)}, \forall t \in [s_k, a_k). \tag{19}$$

$$\|e_k(t)\| \leq \frac{\bar{f}(\hat{x}_k, u_k, \bar{w})}{L_k^x}(e^{L_k^x(t-s_{k+1})} - 1) + \theta_k e^{L_k^x(t-s_{k+1})}, \forall t \in [s_{k+1}, a_{k+1}). \tag{20}$$

First, let us assume the transmission and arrival sequences are admissible. The acceptable delay preserving ISS is given by the following lemma.

Lemma V.1. *If the transmission and arrival sequences are admissible, i.e.* $d_k \leq \tau_k$ *for all* $k = 0, 1, \ldots, \infty$*, and the transmission delay* d_{k+1} *satisfies*

$$d_{k+1} \leq \bar{d}_{k+1} = \frac{1}{L_k^x} \ln\left(1 + \frac{L_k^x(\underline{\xi}_k - \theta_k)}{\bar{f}(\hat{x}_k, u_k, \bar{w}) + L_k^x \theta_k}\right), \tag{21}$$

then the system defined in (4), (5) and (6) with the event-trigger and quantization error defined as (14) and (15) is ISS.

Proof: We assume $a_k \leq s_{k+1} \leq a_{k+1}$ for all $k = 0, 1, \ldots, \infty$.

It is easy to see that during interval $[a_k, s_{k+1}]$, $\|e_k(t)\| \leq \theta_k < \underline{\xi}_k$.

For interval $[s_{k+1}, a_{k+1})$, from equation (20), we have

$$\|e_k(t)\| \leq \|e_k(a_{k+1})\| \leq \frac{\bar{f}(\hat{x}_k, u_k, \bar{w})}{L^x_k}(e^{L^x_k d_{k+1}} - 1) + \theta_k e^{L^x_k d_{k+1}}, \forall t \in [s_{k+1}, a_{k+1})$$

So, equation (13) is guaranteed by

$$\frac{\bar{f}(\hat{x}_k, u_k, \bar{w})}{L^x_k}(e^{L^x_k d_{k+1}} - 1) + \theta_k e^{L^x_k d_{k+1}} < \underline{\xi}_k \Leftrightarrow d_{k+1} \leq \bar{d}_{k+1}$$

Therefore, $\|e_k(t)\| < \underline{\xi}_k$ holds for all $t \in [a_k, a_{k+1})$ and all $k = 0, 1, \ldots, \infty$. According to Corollary IV.2, the closed loop system is ISS. ■

Next, we would like to find an upper bound on the delay such that the transmission arrival sequences are admissible.

Lemma V.2. *If the transmission delay d_k satisfies*

$$d_k \leq T_k = \frac{1}{L^x_k} \ln\left(1 + \frac{L^x_k(\theta_k - \Delta_k)}{\bar{f}(\hat{x}_k, u_{k-1}, \bar{w}) + L^x_k \Delta_k}\right), \tag{22}$$

then the transmission arrival sequences are admissible, i.e. $d_k \leq \tau_k$ for all $k = 0, 1, \ldots, \infty$.

Proof: First, we realize that $d_0 = 0 \leq \tau_0$.

Now, let us assume that $d_{k-1} \leq \tau_{k-1}$ holds, i.e. $a_{k-1} \leq s_k$. If $d_k > \tau_k$, then we have $a_{k-1} \leq s_k \leq s_{k+1} < a_k$. For interval $[s_k, s_{k+1}]$, from equation (19), we have

$$\|e_k(s_{k+1})\| \leq \frac{\bar{f}(\hat{x}_k, u_{k-1}, \bar{w})}{L^x_k}(e^{L^x_k \tau_k} - 1) + \Delta_k e^{L^x_k \tau_k}.$$

Since $\|e_k(s_{k+1})\| = \theta_k$, together with the equation above, we have $\tau_k \geq T_k$. From equation (22), we further have $\tau_k \geq d_k$. This contradicts the assumption $d_k > \tau_k$.

Therefore, $d_k \leq \tau_k$ for all $k = 0, 1, \ldots, \infty$. ■

Corollary V.3. *If $d_k \leq T_k$, then the inter-sampling interval τ_k is bounded from below by T_k, i.e.*

$$\tau_k \geq T_k, \tag{23}$$

where T_k is defined in (22).

Proof: The contrapositive of Lemma V.2 is

$$d_k > \tau_k \Rightarrow d_k > T_k. \tag{24}$$

Assume $T_k > \tau_k$. It is easy to see that

$$d_k > T_k \Rightarrow d_k > \tau_k. \tag{25}$$

From equation (24) and (25), we have

$$d_k > \tau_k \Leftrightarrow d_k > T_k,$$

which implies $\tau_k = T_k$. This contradicts the assumption $T_k > \tau_k$. So, $\tau_k \geq T_k$ ■

From Lemma V.1 and V.2, the main theorem is obtained.

Theorem V.4. *If the transmission delay d_k satisfies*

$$d_k \leq D_k = \min\{\bar{d}_k, T_k\}, \tag{26}$$

where $\bar{d}_k$ and T_k are given by (21) and (22) respectively, then the transmission and arrival sequences are admissible and the system defined in (4-6) with the event-trigger and quantization error defined as (14) and (15) is ISS.

Remark V.5. *Theorem V.4 indicates that when transmission delays are not negligible, we should not choose θ_k as large as possible to obtain as large as possible inter-sampling interval. Because as θ_k gets closer and closer to $\underline{\xi}_k$, $\bar{d}_k$ dominates the bound on the acceptable delay, and gets smaller and smaller according to (21).*

Remark V.6. *The results in Corollary V.3 and Theorem V.4 provide a basis to study the scheduling problem in large scale event-triggered control systems. Let us assume that there are several control systems which share a communication channel, and there is a scheduler which schedules the transmissions of all the control systems. Every time when a control system decides to transmit, from the results in Corollary V.3 and Theorem V.4, it tells the scheduler that N_k bits need to be transmitted in D_k seconds, and the next transmission time will be at least T_k seconds later. The scheduler, then, makes use of the information from all control systems to decide whether all transmission requirements are schedulable.*

This section uses the technique that we used in [5] to derive bounds on acceptable delay and inter-sampling interval. These results are further analyzed in the next section to explain how efficient attentiveness is achieved.

VI. Efficient attentiveness

This section studies the sufficient condition achieving efficient attentiveness. For the convenience of the rest of this section, we first define $\bar{f}_c(s)$ as a class $\mathcal{K}$ function satisfying

$$\|f(\hat{x}_k, K(\hat{x}_k), 0)\| \le \bar{f}_c(\|\hat{x}_k\|), \tag{27}$$

and we have the following corollary which is a direct result from Lemma II.1.

Corollary VI.1. *If*

$$\lim_{s\to 0}\frac{\bar{f}_c(s)}{\theta(s,0)} <\infty, \tag{28}$$

then there exist continuous, positive definite, increasing functions $h_1(s)$*,* $h_2(s)$ *such that* $\lim_{s\to 0} h_i(s) < \infty$ *for* $i = 1, 2$ *and*

$$\frac{\theta(s,\bar{w})}{\Delta(s,\bar{w})} \le h_1(s) \tag{29}$$

$$\frac{\bar{f}_c(s) + L_k^w \bar{w}}{\theta(s,\bar{w})} \le h_2(s). \tag{30}$$

Proof: If $\bar{w} \neq 0$, it is easy to show that

$$\lim_{s\to 0}\frac{\theta(s,\bar{w})}{\Delta(\theta(s,\bar{w}))} < \infty$$

$$\lim_{s\to 0}\frac{\bar{f}_c(s) + L_k^w \bar{w}}{\theta(s,\bar{w})} < \infty.$$

According to Lemma II.1, there must exist h_1 and h_2 which are continuous, positive definite, increasing and $\lim_{s\to 0} h_i(s) < \infty$ for $i = 1, 2$ such that equation (29) and (30) hold.

If $\bar{w} = 0$, we have

$$\lim_{s\to 0}\frac{\theta(s,0)}{\Delta(\theta(s,0))} = \lim_{s\to 0}\frac{\theta(s,0)}{\rho_\Delta \theta(|s-\theta(s,0)|,0)}$$

From (12) and (13), we know that $\theta(s,0)$ is a continuous function satisfying $\theta(0,0) = 0$. There must exist a constant c such that for all $s \in [0,c]$, $\theta(s,0)$ is either greater or equal to $0.5s$, or less than $0.5s$. If $\theta(s,0) \le 0.5s$, then

$$\lim_{s\to 0}\frac{\theta(s,0)}{\rho_\Delta \theta(|s-\theta(s,0)|,0)} \le \lim_{s\to 0}\frac{\theta(s,0)}{\rho_\Delta \theta(0.5s,0)} < \infty.$$

If $\theta(s,0) > 0.5s$, then

$$\lim_{s\to 0}\frac{\theta(s,0)}{\rho_\Delta \theta(|s-\theta(s,0)|,0)} \le \lim_{\theta(s,0)\to 0}\frac{\theta(s,0)}{\rho_\Delta \theta(3\theta(s),0)} < \frac{2}{3\rho_\Delta} < \infty.$$

Together with (28), according to Lemma II.1, equation (29) and (30) still hold. ■

A. Efficiently attentive inter-sampling interval

Now, we are ready to present a sufficient condition to achieve efficiently attentive inter-sampling interval.

Lemma VI.2. *If the assumption (28) is satisfied, then the inter-sampling interval τ_k is efficiently attentive, i.e. there exists a continuous, positive definite, decreasing function $\underline{h}(s_1, s_2)$ such that $\lim_{s\to 0} \underline{h}(s_1, s_2) > 0$ and equation (10) is satisfied.*

Proof: From Corollary V.3, we have $\tau_k \geq T_k$. Since $\ln(1+x) \geq \frac{x}{1+x}$, it is easy to show that

$$\begin{aligned} T_k \geq & \frac{\theta_k - \Delta_k}{\bar{f}(\hat{x}_k, u_{k-1}, \bar{w}) + L_k^x \theta_k} \\ = & \frac{1 - \Delta_k/\theta_k}{\bar{f}(\hat{x}_k, u_{k-1}, \bar{w})/\theta_k + L_k^x}. \end{aligned}$$

From the proof of Lemma IV.3, we know that $\Delta_k < \rho_\Delta \theta_k$. We further have

$$\tau_k \geq T_k \geq \frac{1 - \rho_\Delta}{\bar{f}(\hat{x}_k, u_{k-1}, \bar{w})/\theta_k + L_k^x}.$$

$\bar{f}$ is locally lipschitz with respect to $\hat{x}_k$, so

$$\begin{aligned} \frac{\bar{f}(\hat{x}_k, u_{k-1}, \bar{w})}{\theta_k} \leq & \frac{\bar{f}(\hat{x}_{k-1}, u_{k-1}, \bar{w}) + L_k^x \|\hat{x}_k - \hat{x}_{k-1}\|}{\theta_k} \\ \leq & \frac{\bar{f}_c(\|\hat{x}_{k-1}\|) + L_k^w \bar{w} + L_k^x \theta_{k-1}}{\Delta_k} \\ \leq & \frac{\bar{f}_c(\|\hat{x}_{k-1}\|) + L_k^w \bar{w}}{\theta_{k-1}} \frac{\theta_{k-1}}{\Delta_k} + L_k^x \frac{\theta_{k-1}}{\Delta_k} \end{aligned}$$

According to Corollary VI.1, there must exist continuous, positive definite, and increasing functions $h_1(\|\hat{x}_{k-1}\|)$ and $h_2(\|\hat{x}_{k-1}\|)$ such that

$$\frac{\bar{f}(\hat{x}_k, u_{k-1}, \bar{w})}{\theta_k} \leq h_1(\|\hat{x}_{k-1}\|) + L_k^x h_2(\|\hat{x}_{k-1}\|),$$

and hence we have

$$\tau_k \geq T_k \geq \frac{1 - c_1}{h_1(\|\hat{x}_{k-1}\|) + L_k^x h_2(\|\hat{x}_{k-1}\|) + L_k^x}.$$

Since h_1 and h_2 are increasing with respect to $\|\hat{x}_{k-1}\|$, L_k^x is increasing with respect to $\|\hat{x}_k\|$, and h_1, h_2 and L_k^x are all bounded from above as x approaches 0, we say that there exists a continuous, positive definite, decreasing function $\underline{h}(\|\hat{x}_{k-1}\|, \|\hat{x}_k\|)$ such that $\lim_{\|\hat{x}_{k-1}\|, \|\hat{x}_k\| \to 0} \underline{h}(\|\hat{x}_{k-1}\|, \|\hat{x}_k\|) > 0$, and equation (10) is satisfied. ■

B. Efficiently attentive instantaneous bit-rate

Since the instantaneous bit-rate r_k is defined as N_k/D_k, it is easy to see that the instantaneous bit-rate r_k is efficiently attentive if N_k is bounded from above by an increasing function, and D_k is bounded from below by a decreasing function. Both N_k and D_k will be studied in this subsection.

First, let us look at the number of bits N_k transmitted at step k.

Lemma VI.3. *There exists a continuous, positive definite increasing function $h'(\|\hat{x}_{k-1}\|)$ such that*

$$\lim_{s\to 0} h'(s) < \infty \tag{31}$$

$$N_k \leq h'(\|\hat{x}_{k-1}\|). \tag{32}$$

Proof: N_k satisfies

$$N_k = \lceil \log_2 2n \rceil + \left\lceil \log_2 \left\lceil \frac{\theta_{k-1}}{\Delta_k} \right\rceil^{n-1} \right\rceil.$$

According to Corollary VI.1, there exists a continuous, positive definite, increasing function $h_1(\|\hat{x}_{k-1}\|)$ such that

$$\lim_{s\to 0} h_1(s) < \infty$$
$$\frac{\theta_{k-1}}{\Delta_k} \leq h_1(\|\hat{x}_{k-1}\|).$$

Therefore, there must exist a continuous, positive definite increasing function $h'(\|\hat{x}_{k-1}\|)$ such that equation (31) and (32) hold. ■

Next, we would like to study D_k. Since $D_k = \min\{T_k, \bar{d}_k\}$ and T_k has be analyzed in Lemma VI.2, only $\bar{d}_k$ is examined here.

Lemma VI.4. *If the assumption (28) is satisfied, then there exists a continuous, positive definite decreasing function $h''(\|\hat{x}_{k-1}\|, \|\hat{x}_k\|)$ such that*

$$\lim_{s_1,s_2\to 0} h''(s_1, s_2) > 0 \tag{33}$$

$$\bar{d}_k \geq h''(\|\hat{x}_{k-1}\|, \|\hat{x}_k\|). \tag{34}$$

Proof: Since $\theta_k = \rho_\theta \underline{\xi}_k$, with the same steps in the proof of Lemma VI.2, we show that

$$\bar{d}_k \geq \frac{1-\rho_\theta}{\bar{f}(\hat{x}_{k-1}, u_{k-1}, \bar{w})/\underline{\xi}_k + L_k^x},$$

where

$$\frac{\bar{f}(\hat{x}_{k-1}, u_{k-1}, \bar{w})}{\underline{\xi}_k} \leq \frac{\bar{f}_c(\|\hat{x}_{k-1}\|) + L_k^w \bar{w}}{\theta_{k-1}}.$$

According to Corollary VI.1, there exist a continuous, positive definite, increasing function $h_2(\|\hat{x}_{k-1}\|)$ such that

$$\lim_{s\to 0} h_2(s) <\infty,$$
$$\frac{\bar{f}(\hat{x}_{k-1}, u_{k-1}, \bar{w})}{\underline{\xi}_k} \leq h_2(\|\hat{x}_{k-1}\|).$$

Together with the fact that L_k^x is increasing with respect to $\|\hat{x}_k\|$ and $\lim x \to 0 L_k^x < \infty$, we conclude that there exists a continuous, positive definite decreasing function $h''(s_1, s_2)$ such that equation (33) and (34) hold. ∎

From Lemma VI.2, VI.3 and VI.4, we have the following corollary.

Corollary VI.5. *If equation (28) are satisfied, the instantaneous bit-rate r_k is efficiently attentive, i.e. there exists a continuous, positive definite, increasing function $\bar{h}(s_1, s_2)$ such that $\lim_{s\to 0} \bar{h}(s_1, s_2) < \infty$ and equation (11) is satisfied.*

C. Efficiently attentive and ISS event-triggered control system

Now, we are at the step to make a conclusion. Event triggered control systems should be designed to achieve not only a desired system performance but also attention efficiency. To this purpose, we have the following theorem which is easily derived from Theorem V.4, Lemma VI.2, and Corollary VI.5.

Theorem VI.6. *If the transmission delay d_k is bounded from above by D_k given by (26), and the threshold function θ satisfy equation (28), then the system defined in (4-6) with the event-trigger and quantization error defined as (14) and (15) is input-to-state stable and efficiently attentive.*

Remark VI.7. *Equation (28) indicates that as the system state approaches 0, the threshold function θ decreases more slowly than the closed loop dynamic behavior of the state trajectory.*

Therefore, when x get closer to 0, it takes more time for the gap $e_k(t)$ to hit the threshold θ_k and $\underline{\xi}_k$, which leads to longer inter-sampling interval and acceptable delay.

VII. A Case Study

This section uses an example to explain how to design an event-triggered control system to achieve ISS and attention efficiency. After the event-triggered control system is well established, we will try different threshold functions to study the tradeoff between the inter-sampling interval and the instantaneous bit-rate.

Our experiment shows that the threshold function should be chosen as large as possible while the instantaneous bit-rate is lower than the channel capacity. It is also found that the event-triggered control system with its threshold function to be $0.6\underline{\xi}$ can tolerate 25 times of the delay tolerated by the system with the threshold function to be $0.99\underline{\xi}$, while only transmitting 2 times as frequently as the system with the threshold function to be $0.99\underline{\xi}$.

A. Design of event-triggered control systems

Consider the following nonlinear dynamic system.

$$\dot{x}_1 = -2x_1^3 + x_2^3 + w_1 \tag{35}$$

$$\dot{x}_2 = x_2^3 + u + w_2, \tag{36}$$

with $x_0 = [1\ 1]^T$. w is an L_∞ disturbance with $\|w\|_{L_\infty} = 0.1$. The control input u is chosen such that

$$u_k = -3\hat{x}_2^3, \forall t \in [a_k, a_{k+1}).$$

First, the input-to-state stability is studied for the dynamic system described in equation (35) and (36). Let $V = \frac{1}{4}x_1^4 + \frac{1}{4}x_2^4$ be a Lyapunov candidate function. Its derivative satisfies

$$\dot{V} = -2x_1^6 + x_1^3 x_2^3 - 2x_2^6 + x_1^3 w_1 + x_2^3 w_2 + 3(3x_2^2 e_2 - 3x_2 e_2^2 + e_2^3)x_2^3,$$

where $e_i = x_i(t) - \hat{x}_i(s_k)$ for $i = 1, 2$. Let $\Omega = \{x : \|x\| \leq 1\}$.

$$\dot{V} \leq -1.5x_1^6 - 1.5x_2^6 + 2\bar{w} + 3x_2^3(3x_2^2 e_2 - 3x_2 e_2^2 + e_3^3), \forall x \in \Omega.$$

If

$$\|e\| \leq \epsilon \|x\|^l + \bar{w}, \text{for all } l \geq 1,$$

we have

$$\begin{aligned}\dot{V} \leq & -1.5x_1^6 - 1.5x_2^6 + 2\bar{w} + 3(3\epsilon + 3\epsilon^2 + \epsilon^3)\|x\|^6 \\ & + 3\|x\|^5\bar{w} - 6\epsilon\|x\|^5\bar{w} - 3\|x\|^4\bar{w}^2 + 3\epsilon^2\|x\|^5\bar{w} + 3\epsilon\|x\|^4\bar{w}^2 + \|x\|^3\bar{w}^3 \\ \leq & -1.5x_1^6 - 1.5x_2^6 + 3(3\epsilon + 3\epsilon^2 + \epsilon^3)\|x\|^6 \\ & + 3(\bar{w}^3 + 3|\epsilon - 1|\bar{w}^2 + 3|\epsilon - 1|^2\bar{w}) + 2\bar{w}, \forall x \in \Omega.\end{aligned}$$

Let $\epsilon = 0.14$, and $\gamma(\bar{w}) = 3(\bar{w}^3 + 3|\epsilon - 1|\bar{w}^2 + 3|\epsilon - 1|^2\bar{w}) + 2\bar{w}$. We have

$$\dot{V} \leq -0.055\|x\|^6 + \gamma(\bar{w}).$$

Therefore, the system is input-to-state stable if

$$\|e_k(t)\| \leq \xi(\|x(t)\|) = 0.14\|x(t)\|^l + \bar{w}, \text{for all } l \geq 1. \tag{37}$$

According to equation (13), (14) and (15), we have

$$\theta(\|\hat{x}_k\|) = \rho_\theta(0.12\|\hat{x}_k\|^l + 0.87\bar{w}), \text{for some } \rho_\theta \in (0, 1). \tag{38}$$

$$\Delta(s) = \rho_\Delta\rho_\theta(0.12(s - \theta(s))^l + 0.87\bar{w}), \text{for some } \rho_\Delta \in (0, 1).$$

B. *Efficient attentiveness*

This subsection will design an experiment to demonstrate Theorem VI.6. According to this theorem, if $l \leq 3$, then the system is efficiently attentive. So, in this experiment, we will fix $\rho_\theta = 0.6$ and $\rho_\Delta = 0.6$, vary $l = 1, 4$, and run the system for 50 seconds to see how inter-sampling interval and instantaneous bit-rate change.

When $\bar{w} = 0$, the system performance, inter-sampling interval, and instantaneous bit-rate are shown in Figure 4. The left plots are the simulation results for $l = 1$. The top plot gives the system performance, and the system is asymptotically stable. The middle plot shows the inter-sampling interval which got longer and longer as the system state approached 0. The bottom plot shows the instantaneous bit-rate which was smaller and smaller as the state went to 0. So, we say the system is efficiently attentive. The right plots are the simulation results for $l = 4$. We see that the system is still ISS, but the inter-sampling interval became shorter and shorter and the instantaneous bit-rate got larger and larger as the system state went to the origin. Therefore, when $l = 4$, the system is not efficiently attentive.

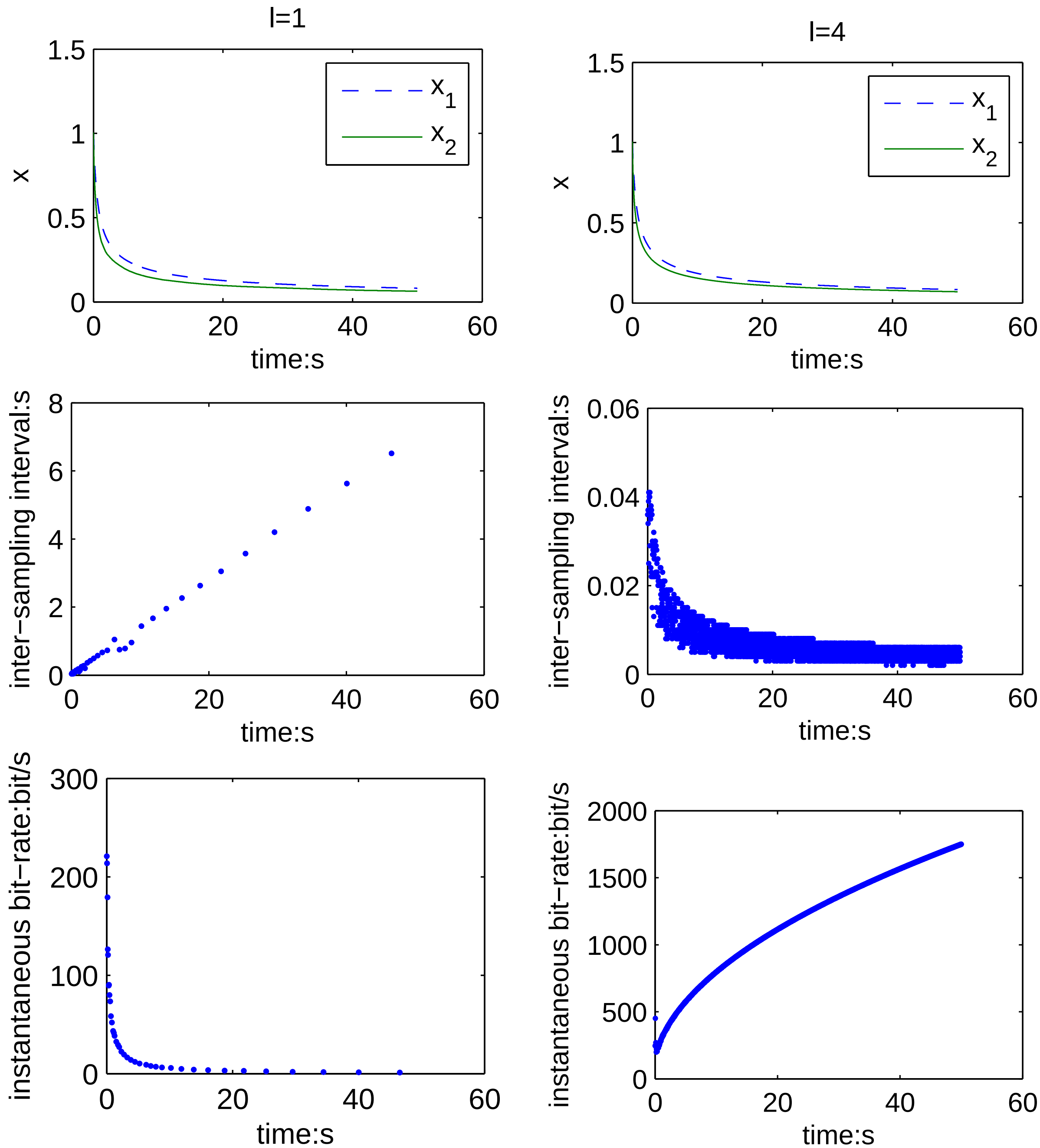

Fig. 4. Performance, inter-sampling interval and instantaneous bit-rate with $\rho_\theta = \rho_\Delta = 0.6$ and $l = 1, 4$ for the noise free case.

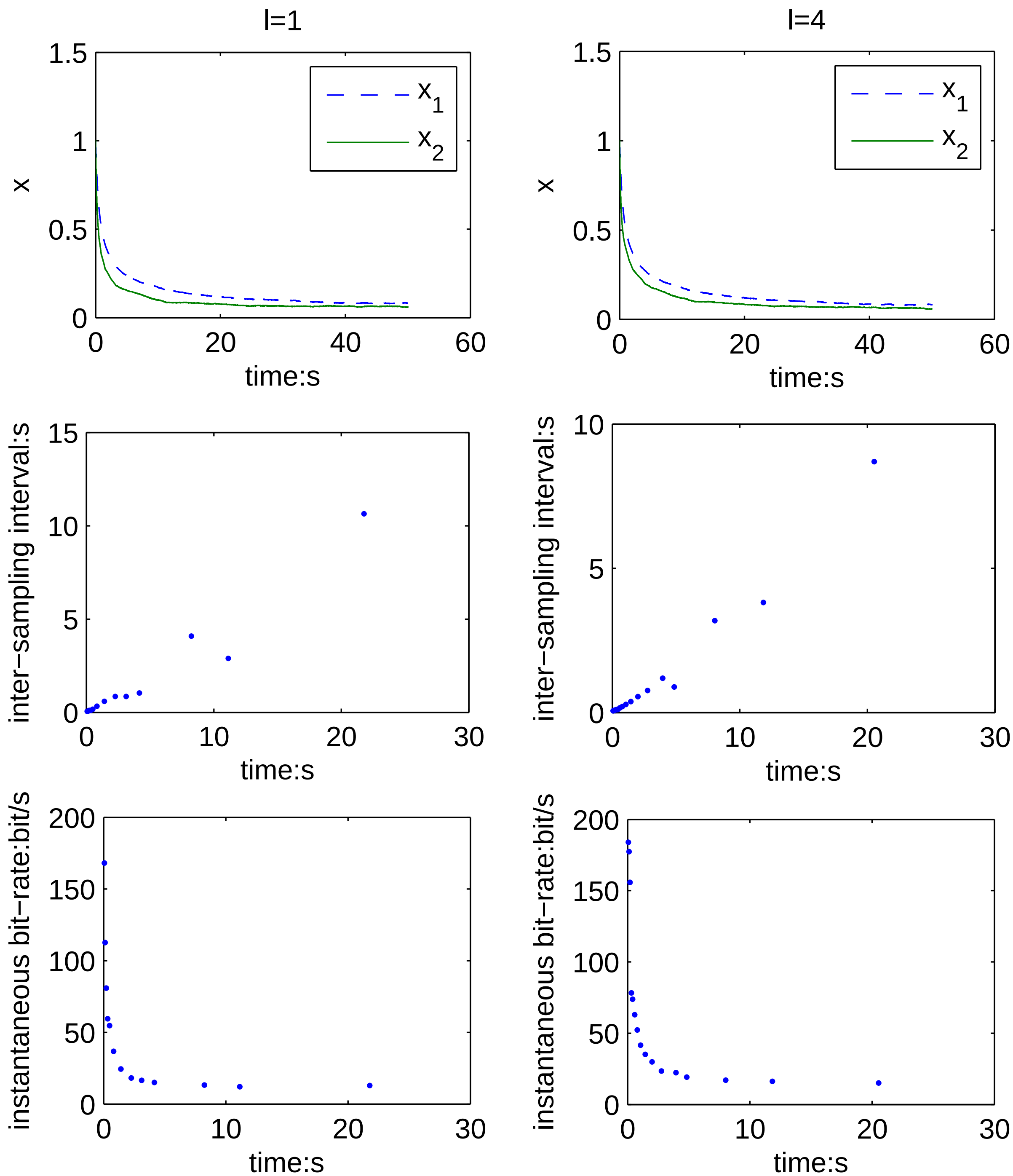

Fig. 5. Performance, inter-sampling interval and instantaneous bit-rate with $\rho_\theta = \rho_\Delta = 0.6$, and $l = 1, 4$ for noisy case.

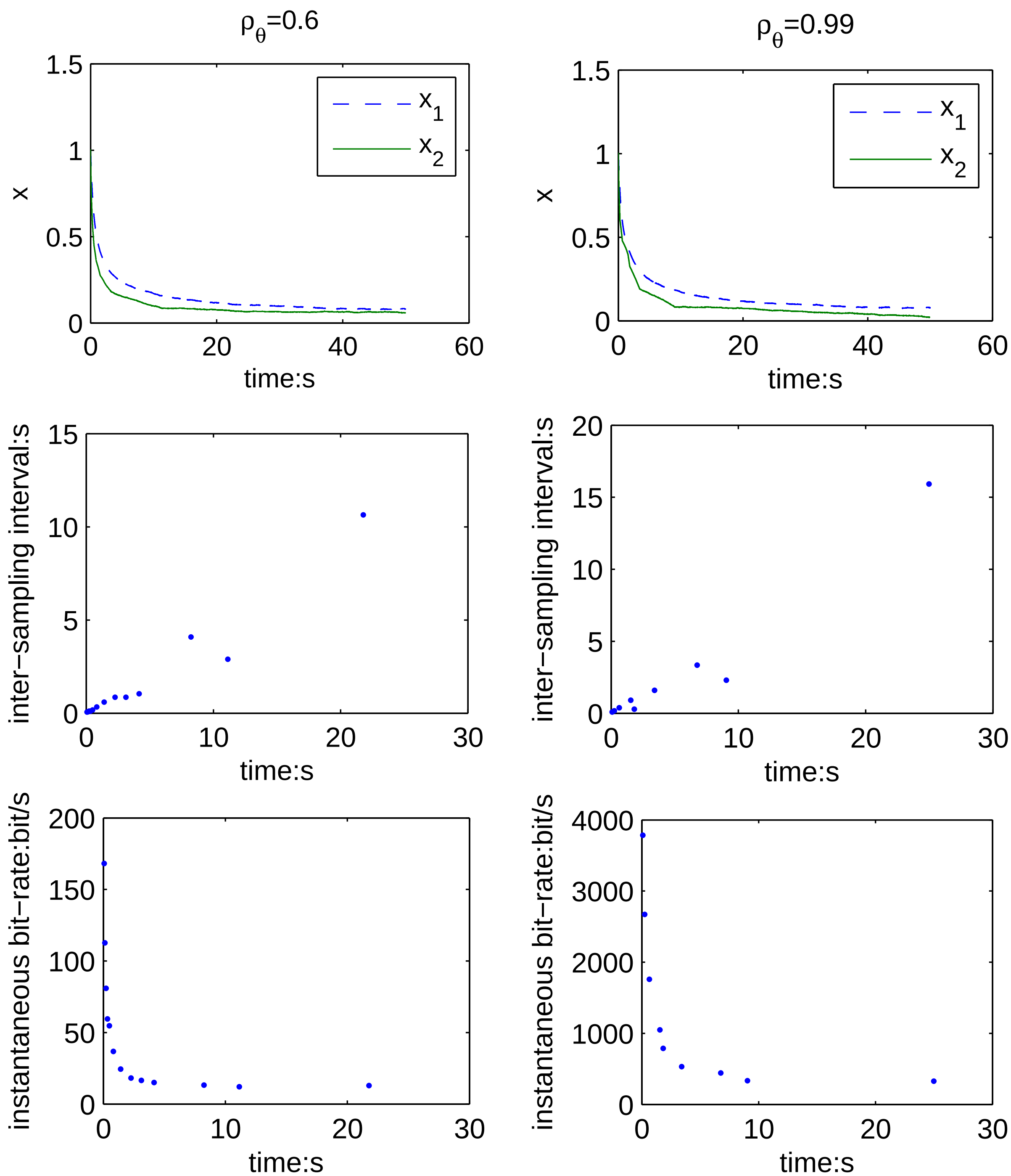

Fig. 6. Performance, inter-sampling interval and instantaneous bit-rate with $\rho_\theta = 0.6, 0.99$, $\rho_\Delta = 0.6$ and $l = 1$ for noisy case.

When $\bar{w} = 0.1$, the system performance, inter-sampling interval, and instantaneous bit-rate are shown in Figure 5. Comparing the left plots ($l = 1$) with the right plots($l = 4$), we find that for both cases, the systems had similar system performances, inter-sampling intervals, and instantaneous bit-rates, and both are ISS and efficiently attentive. This is because when $\bar{w} \neq 0$, and x is small, for both cases, the term associated with $\bar{w}$ dominated the threshold and the quantization error, and hence for both cases, the systems had similar performances, inter-sampling intervals, and instantaneous bit-rates.

C. The inter-sampling interval and the instantaneous bit-rate

In this section, we will fix $l = 1$ and $\rho_\Delta = 0.6$. By varying $\rho_\theta = 0.6, 0.99$, we want to explore the tradeoff between the inter-sampling interval and the instantaneous bit-rate. The simulation results are shown in Figure 6. The left plots are the results for $\rho_\theta = 0.6$, and the right plots are the results for $\rho_\theta = 0.99$. The top plots give the system performance. we see that both of them are ISS. The middle plots show the inter-sampling intervals. We see that the inter-sampling interval of the system with $\rho_\theta = 0.99$ is about 2 times of the inter-sampling interval of the system with $\rho_\theta = 0.6$. The bottom plots show the instantaneous bit-rates. we find that the instantaneous bit-rate of the system with $\rho_\theta = 0.99$ is about 20 times of the instantaneous bit-rate of the system with $\rho_\theta = 0.6$. This experiment showed the tradeoff between inter-sampling interval and instantaneous bit-rate. When we increased the threshold, while the inter-sampling interval increased, the required instantaneous bit-rate also increased. Therefore, for bandwidth limited systems, we should not only focus on lengthening the inter-sampling interval, but also need to guarantee the channel bandwidth satisfies the required instantaneous bit-rate.

VIII. CONCLUSION

This paper studies the input-to-state stability and efficient attentiveness for bandwidth limited event-triggered control systems. A system is said to be efficiently attentive, if the inter-sampling interval gets longer and the required instantaneous bit-rate gets smaller when the system state goes to the origin. We first talk about how to design the event-trigger and the quantization map, then provide the acceptable delay preserving ISS, and finally gives a sufficient condition of efficient attentiveness. Our simulation results demonstrated these main results, and indicates that for bandwidth limited systems, we should not only focus on lengthening the inter-sampling interval, but also need to guarantee the channel bandwidth satisfies the required instantaneous bit-rate.

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