Math 10250 Review for Exam 3

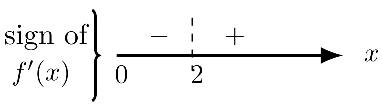
1. Find all critical points of the function $f(x) = x^3e^{-2x}$. Determine if each is a local maximum or local minimum and whether or not it is a global maximum or minimum Ans: Global max at $x = 1.5$. No global min

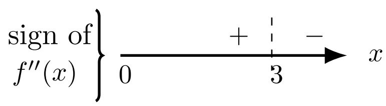
2. Find all critical points of the function $f(x) = 3x \ln x$, $x > 0$. Where is the function increasing and where is it decreasing? Use the second derivative to describe the concavity of the function. Ans: Decreasing on $(0, 1/e)$, increasing on $(1/e, \infty)$, concave up.

3. Find where the **slope** of the function $f(x) = x^3 + 3x^2 - 9x + 17$ is (a) increasing and (b) decreasing. Also, sketch its graph. Ans: (a) $(-1, \infty)$; concave up and (b) $(-\infty, -1)$; concave down

4. Sketch a graph of the function $f(x) = \dfrac{2x}{x-3}$. List all vertical and horizontal asymptotes, and describe where the graph is increasing, decreasing, concave up, and concave down. Ans: Vertical asym. $x = 3$; Horizontal asym. at $y = 2$ and the function is decreasing everywhere.

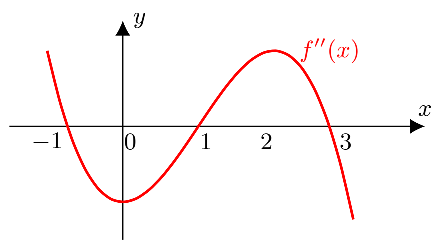
5. Sketch the graph of a function $f(x)$ defined for $x > 0$ and such that: $\lim_{x\to 0^+} f(x) = \infty$, $\lim_{x\to\infty} f(x) = 4$, and

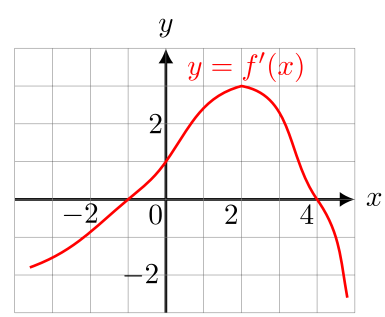




Make sure that increasing/decreasing, concavity, inflection points, and max/min are clearly shown!

6. The graph of the **second derivative** of $f(x)$ is shown in next figure.
(a) Find all inflection points of $f(x)$. (Ans: $x = -1, 1, 3$)
(b) Where is $f(x)$ concave up? (Ans: $(-\infty, -1)$ and ?)
(c) Where is $f(x)$ concave down? (Ans: $(-1, 1)$ and ?)





7. The graph of the **derivative** of $f(x)$ is shown in next figure. (a) Find all critical points of $f(x)$, (b) Where is $f(x)$ increasing? decreasing? (c) Where is $f(x)$ concave up? concave down? (d) Give all inflection points. (a) $x = -1, 4$ (b) increasing $(-1, 4)$; decreasing $(-\infty, -1)$ and $(4, \infty)$ (c) concave up $(-\infty, 2)$; concave down $(2, \infty)$ (d) $x = 2$

8. If $f(x)$ satisfies the equation $f'(x) = \ln x$, $x > 0$, then find the concavity of its graph. Ans: Concave up

9. Let $f(x)$ be a differentiable function defined for all numbers x. If you are told that its derivative is:
$$f'(x) = 8e^{x-2}(x^3 - x),$$
then, using this fact, find all the local extrema of $f(x)$. (Ans: local min. at $-1, 1$ and local max. at $x = 0$)

10. If the second derivative of a function $f(x)$ is given by $f''(x) = 8e^x x(x^3 - 9x)$, then find its inflection points and determine its concavity. (Ans: inflection points are at $x = -3$ and 3, concave down in $(-3, 3)$ and concave up elsewhere)

11. Suppose the demand function for a certain product is given by the equation $q = 10{,}125 - 150p^{\frac{3}{2}}$, $0 \le p \le 25$, where p is the price per unit and q is the number of items sold. What price maximizes revenue? Ans: $p = 9$

12. Find the global maximum and minimum value, if any, of the function $f(x) = 3x^4 - 8x^3 + 5$ on $[1, \infty)$ Ans: Global min $= -11$ occurring at $x = 2$ and global max none.

13. A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can she do this so as to minimize the cost of the fence? Ans: 1.5 by 1

14. A circular cylindrical can, open at one end, has external surface area 27π cm^2. Find the maximum volume of such a can. Ans: 27π cm^3

15. A box with a square base and a top is to be built with a volume of 20 cubic feet. The material for the base has density 3 lb per square foot, the material for the top has density 2 lb per square foot, and the material for the side has density 1 lb per square foot. What should the dimension of the box be so that its weight is minimum? Ans: 2 by 2 by 5

16. Find the area of the largest rectangle that can be inscribed in a semicircle of radius 1 with the base of the rectangle along a diameter of the semicircle.

17. Find all functions whose derivative is equal to $\sqrt{x}$.

18. Find the function whose instantaneous rate of change at each t is $e^{0.2t}$ and whose value at $t = 0$ is 8.

19. Find the function whose slope at each x is $8x^3 - \dfrac{1}{x}$ and whose graph passes through $(1, 4)$.

20. The marginal profit function of a company producing and selling x pairs of wool socks per week is $MP(x) = -0.1x + 150$. If the profit from selling 1000 pairs of socks is \$61,000, what is the profit function? What is the profit from selling 1500 pairs of socks? Ans: $P(x) = -0.05x^2 + 150x - 39{,}000$; $P(1500) = 73{,}500$

21. The rate of production of a certain item, in millions of units per month, is given by $P'(t) = 8te^{-t^2}$. Find the production function $P(t)$ if $P(0) = 5$. Ans: $P(t) = -4e^{-t^2} + 9$

22. Find the indefinite integral using substitution. Check your answer by differentiating.

    i) $\displaystyle\int x^2 e^{x^3}\,dx$

    ii) $\displaystyle\int \frac{4x^3 - 2x}{x^4 - x^2 - 9}\,dx$

    iii) $\displaystyle\int \frac{\ln(x^4 + x^2 + 1)}{x^4 + x^2 + 1}(4x^3 + 2x)\,dx$

    iv) $\displaystyle\int (x^3 + x - 1)^{99}(3x^2 + 1)\,dx$

    v) $\displaystyle\int \frac{e^x}{(10 + e^x)^4}\,dx$

    vi) $\displaystyle\int \frac{2x^3 + x}{(x^4 + x^2 + 1)^{3/2}}\,dx$

23. Solve the differential equation: $\dfrac{dy}{dx} = x^3 e^{x^4} + \dfrac{4x^3 + 2x}{x^4 + x^2 + 1} + x^{3/5} + 8$.

24. If $F(t) = e^{-t^2} + t$ is an antiderivative of a function $f(t)$, then solve the initial value problem: $\dfrac{dy}{dt} = f(t)$, $y(0) = 8$.

25. A Mustang GT is traveling along a straight road at 150 feet per second when the driver steps on the break. At that point the car starts decelerating at a constant rate of 30 feet per second squared until it stops. How long does it take to stop? What is the concavity of the position function? Ans: 5 sec, concave down.

26. What information about a function $f(x)$ does the sign of the derivative $f'(x)$ provide?

27. What information about a function $f(x)$ does the sign of the second derivative $f''(x)$ provide?

28. Is it possible to cover the **whole** xy-plane with graphs of antiderivatives of the function $f(x) = x$? Explain!

29. What is Calculus good for?