Name ________________________________ Date ____________________

Math 10250 Activity 14: The Derivative as a Rate (Section 3.3)

GOAL: To focus our attention on the interpretation of the derivatives as a rate of change and learn what it represents in different physical context. For example, velocity is derivative of the position function, and acceleration is the derivative of the velocity function.

▶ Estimating the derivative (based on its definition!)

- Forward difference formula:

$\boxed{f'(a)} \approx$ Slope of chord PQ $= \boxed{\dfrac{f(a+h)-f(a)}{h}}$ h small!

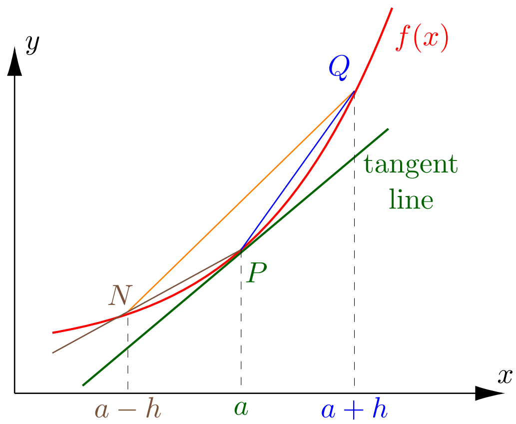
- Backward difference formula:

$\boxed{f'(a)} \approx$ Slope of chord NP $= \boxed{\dfrac{f(a)-f(a-h)}{h}}$ h small!

- Central difference formula:

$\boxed{f'(a)} \approx$ Slope of chord NQ $= \boxed{\dfrac{f(a+h)-f(a-h)}{2h}}$ h small!

Note: In many applications the derivative is computed numerically using one of these formulas.



Example 1

x	2.98	2.99	3	3.01	3.02
$f(x)$	7.87	7.95	8	8.06	8.09

Give as many estimates as possible for each of the following derivatives of $f(x)$ with the table above:

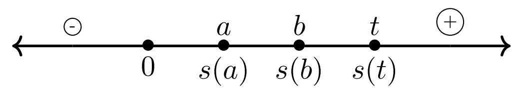
(a) $f'(3) \approx \dfrac{f(3+0.01)-f(3-0.01)}{2(0.01)} = \dfrac{8.06-7.95}{0.02} = \dfrac{0.11}{0.02} = \dfrac{11}{2} = \boxed{5.5}$

(b) $f'(2.98) \approx \dfrac{f(2.98+0.01)-f(2.98)}{0.01} = \dfrac{7.95-7.87}{0.01} = \dfrac{0.08}{0.01} = \boxed{8}$

(c) $f'(3.02) \approx \dfrac{f(3.02)-f(3.02-0.01)}{0.01} = \dfrac{8.09-8.06}{0.01} = \dfrac{0.03}{0.01} = \boxed{3}$

▶ Average and instantaneous velocity

- $s(t)$ = Position of object at time t from some fixed point O.



- Average velocity over the time interval $a \le t \le b = \dfrac{\text{change in position}}{\text{change in time}} = \dfrac{s(b)-s(a)}{b-a}$

Example 2 A puppy on Douglas Road is 60 meters west of the 7-Eleven at 12:00PM. If the position (in meters) of the puppy measured from 7-Eleven (origin O) t minutes after 12:00PM is given in Figure 1, answer the following questions about the puppy:

(a) What is its position and distance traveled when $t = 10$?
$s(10) = 0$, *distance traveled* $= \boxed{60}$.

(b) What is its position and the distance traveled when $t = 80$?
$s(80) = -60$, *dist.* $= 60 + 40 + 40 + 60 = \boxed{200}$.

(c) Did the puppy stop for a break? If yes, when and how long?
Yes, puppy stoped for 20 *minutes when* $40 \le t \le 60$.

(d) What is its average velocity for $0 \le t \le 20$?
$\dfrac{s(20)-s(0)}{20} = \dfrac{20-(-60)}{20} = \dfrac{80}{20} = \boxed{4}$.

(e) What is its average velocity between 12:20PM and 1:10PM?
$\dfrac{s(70)-s(20)}{70-20} = \dfrac{20-20}{50} = \boxed{0}$.

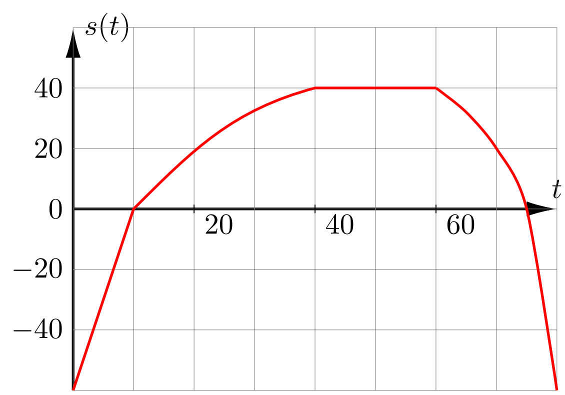


Figure 1

(f) What is its average velocity for $0 \le t \le 80$? What about average speed?

$$\textit{average velosity} = \frac{s(80)-s(0)}{80-0} = \frac{(-60)-(-60)}{80} = \boxed{0}.$$

Remarks:

- If average velocity is **positive** then object has moved in the *positive direction*.
- If average velocity is **negative** then object has moved in the *negative direction*.
- Average speed between $a \le t \le b = \dfrac{\textit{distance}}{\textit{time}}$

ave. speed for $0 \le t \le 80$
$$= \frac{60+40+40+60}{80} = \frac{200}{80} = \frac{5}{2} = 2.5$$

▶ Instantaneous Velocity, Speed, and Acceleration

If $s(t)$ is the position of an object from a fixed point O. Then we define its (instantaneous) velocity, speed and acceleration as follows:

- Instantaneous velocity $v(t)$ = Rate of change of position $\stackrel{?}{=} \dfrac{ds}{dt} = s'(t)$

  If $v(t) > 0$ then the object is moving to right.

  If $v(t) < 0$ then the object is moving to left.

- Instantaneous speed = Magnitude of velocity $\stackrel{?}{=} \left|\dfrac{ds}{dt}\right|$

- Instantaneous acceleration $a(t)$ = Rate of change of velocity $\stackrel{?}{=} \dfrac{dv}{dt} = \dfrac{d^2s}{dt^2}$

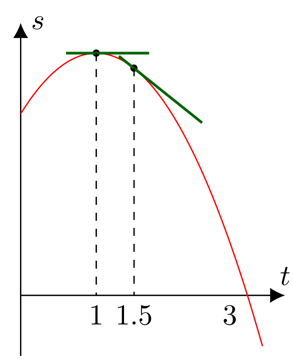
  If $a(t) > 0$ then velocity of object is *accelerating*.

  If $a(t) < 0$ then velocity of object is *decelerating*.

Example 3 A ball is thrown into the air and its height in feet after t seconds is given by $s = -16t^2 + 32t + 48$ until it hits the ground.

Note: The ball hits the ground when $s(t) = 0$ or

$$-16t^2 + 32t + 48 = 0 \Rightarrow t^2 - 2t - 3 = 0 \Rightarrow (t-3)(t+1) = 0 \Rightarrow \boxed{t=3}$$



(a)Write a formula for the ball's velocity until it hits the ground.

$$v(t) = \frac{ds}{dt} = \boxed{-32t+32}$$

(b) What is its velocity at the end of 1 second? In what direction (up or down) is it moving at the end of 1 second? What about its speed?

$v(1) = -32 + 32 = 0 = \textit{speed.}$ *Not up not down*

(c)What is its velocity at the end of 1.5 seconds? In what direction (up or down) is it moving at the end of 1.5 seconds? What about its speed?

$v(1.5) = -32(1.5) + 32 = -\frac{32}{2} = \boxed{-16}$ *ft/sec, down.* *Speed* $|v(1.5)| = \boxed{16}$ *ft/sec*

(d) What is the ball's acceleration at the end of 0.5 seconds? What is the ball's acceleration after 1 second?

$a(t) = \dfrac{dv}{dt} = -\boxed{32}$. *Ball's acceleration is same at all times including* $t = 1$ *second.*