

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

## Practice A – Exam 1

This exam is in 2 parts on 10 pages and contains 14 problems worth a total of 100 points. You have to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. Good luck!

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**You must record here your answers to the multiple choice problems.**

Place an  $\times$  through your answer to each problem.

- |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|
| 1.  | (a) | (b) | (c) | (d) | (e) |
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MC. \_\_\_\_\_

11. \_\_\_\_\_

12. \_\_\_\_\_

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14. \_\_\_\_\_

Tot. \_\_\_\_\_

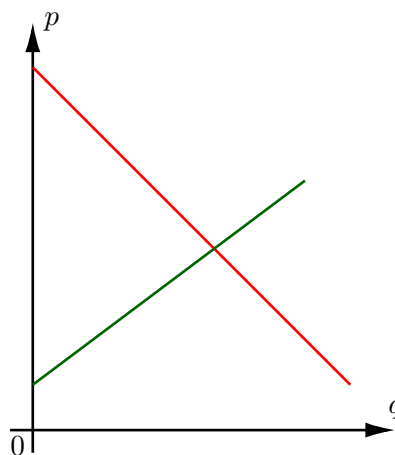
### Multiple Choice

1. (5 pts.) In a region, in 2010 there were about 20,000 units (Peak Kilowatts) of electricity produced from sunlight and in 2015 there were 140,000 units produced. Assuming that the electricity  $E$  produced from sunlight is a linear function of time, find a formula for it. (Let  $t = 0$  be the year 2010.)

- (a)  $E = 24,000t - 20,000$
- (b)  $E = 20,000t + 24,000$
- (c)  $E = -24,000t + 20,000$
- (d)  $E = 20,000t + 140,000$
- (e)  $E = 24,000t + 20,000$

2. (5 pts.) In a certain region, the demand curve for iPhone 8 is modeled by  $q = -\frac{1}{2}p + 8$  and the supply curve is modeled by  $q = \frac{2}{3}p - \frac{4}{3}$ , where the price  $p$  is measured in hundreds of dollars and the quantity in millions of units. Which of the following statements is **FALSE**?

- (a) The supply function is increasing.
- (b) The equilibrium quantity is 5.
- (c) The demand function is decreasing.
- (d) The equilibrium quantity is 4.
- (e) The equilibrium price is 8.



3. (5 pts.) A company determines that its profit  $P$  (in millions) when it sells its product at the price of  $x$  dollars per unit is given by the function

$$P(x) = -10(x - 12)^2 + 250.$$

Determine the range of prices for which the company makes a profit (i.e.  $P(x) > 0$ ).

- (a)  $10 < x < 25$
- (b)  $x < 7$
- (c)  $7 < x < 17$
- (d)  $x > 17$
- (e)  $10 < x < 12$

4. (5 pts.) Compute the following limit:  $\lim_{h \rightarrow 0} \frac{5(3 + h)^2 - 45}{h}$ .

- (a) 30
- (b) 45
- (c) It does not exist.
- (d) 40
- (e) 15

5. (5 pts.) A company estimates that when it spends  $x$  million dollars to advertise its product, then its annual revenue  $R$ , in millions of dollars, is modeled by the function

$$R(x) = \frac{400x + 1200}{2x + 5}.$$

What is the limiting value of the revenue  $R$  if the company keeps spending more and more money in advertising?

- (a) 800
- (b) 1600
- (c) 200
- (d) 240
- (e) 400

6. (5 pts.) Let  $f(x) = \frac{x - 5}{x^2 - 8x + 15}$ . Which of the following statements is **FALSE**?

- (a) The line  $x = 3$  is a vertical asymptote.
- (b) The limit of  $f(x)$  as  $x \rightarrow \infty$  is equal to zero.
- (c) The  $x$ -axis (i.e.  $y = 0$ ) is a horizontal asymptote.
- (d) The lines  $x = 3$  and  $x = 5$  are vertical asymptotes.
- (e) The natural domain of  $f(x)$  is the set of all real numbers except  $x = 3$  and  $x = 5$ .

7. (5 pts.) The following table lists the temperature  $H(t)$  at Notre Dame during a day in September at certain times from 1:00 AM till 11:00 AM.

$t$	1	3	5	7	9	11
$H(t)$	64	55	52	61	70	78

Assuming that the temperature  $H(t)$  is a **continuous** function of  $t$ , in which of the following time intervals can you be sure that the temperature  $H(t)$  attained the value of 63?

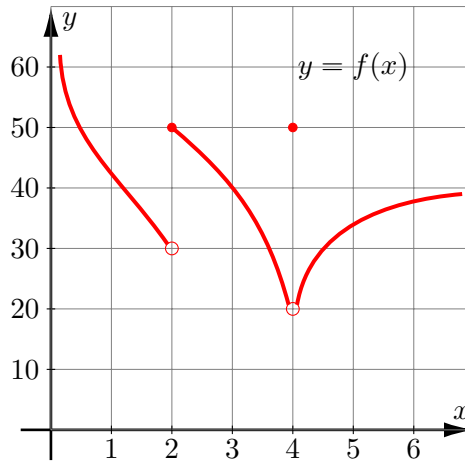
- (a)  $[1, 3]$  and  $[5, 7]$  only.
- (b)  $[1, 3]$  only.
- (c)  $[5, 7]$  and  $[7, 9]$  only.
- (d)  $[1, 3]$  and  $[7, 9]$  only.
- (e)  $[1, 3]$  and  $[9, 11]$  only.

8. (5 pts.) Determine the constant  $c$  so that the following function is continuous:

$$f(x) = \begin{cases} \frac{x^2 + 6x - 7}{x - 1}, & x \neq 1 \\ c, & x = 1. \end{cases}$$

- (a) 6
- (b) 7
- (c) 8
- (d) 0
- (e) 1

9. (5 pts.) Let  $f(x)$  be the function whose graph is shown below. Which of the following statements is **FALSE**?



- (a)  $\lim_{x \rightarrow 2^-} f(x) = 30$
- (b)  $\lim_{x \rightarrow \infty} f(x) = 40$
- (c)  $\lim_{x \rightarrow 0^+} f(x) = \infty$
- (d)  $\lim_{x \rightarrow 2^+} f(x) = 50$
- (e)  $f(x)$  is continuous at  $x = 4$ .

10. (5 pts.) Let  $f(x)$  be the function whose graph is shown above. Compute  $\lim_{x \rightarrow 4} \frac{\sqrt{xf(x) + 20}}{x^2 - 3x + 1}$ .

- (a) 3
- (b) 2
- (c) 1
- (d) 4
- (e)  $\infty$

**Partial Credit**

You must **show your work** on the partial credit problems to receive credit!

**11.** (12 pts.) [**Show your work**]

(i) (6 points) Determine the natural domain of the function

$$f(x) = \frac{x + 2}{x + 1}$$

and find its inverse  $g(x)$ .

**Answer:** \_\_\_\_\_

(ii) (6 pts.) When the price per unit of a certain Flash Drive is \$20 then 5000 units are sold. However, when the price is **increased** by \$5, its demand is **decreased** by 1000 units. Find its demand assuming that it is modeled by a linear function.

**Answer:** \_\_\_\_\_

**12.** (12 pts.) [**Show your work**]

(A) The demand for a certain item is  $q = -0.5x + 95$ , where  $q$  denotes the units of quantity sold and  $x$  the unit price in dollars. Also, the cost function is given by  $C(q) = 10q + 5000$ .

(i) (4 pts.) Find the revenue  $R$  as a function of the price  $x$ , i.e.  $R = R(x)$ .

**Answer:** \_\_\_\_\_

(ii) (4 pts.) Find the profit  $P$  as a function of the price  $x$ , i.e.  $P = P(x)$ .

**Answer:** \_\_\_\_\_

(B) (4 pts.) (Independent of A.) Suppose that today you invest \$10,000 at an annual rate of 5%. Find the balance at the end of 10 years if the interest is compounded **daily**.

**Answer:** \_\_\_\_\_



**13.** (12 pts.) [**Show your work**]

Polyphemus (the gigantic one-eyed son of Poseidon) threw a rock at the escaping Odysseus and his companions. Assume that the height of the rock at time  $t$  is modeled by the quadratic function

$$H(t) = -16t^2 + 1600t,$$

where the time  $t$  is measured in seconds and the height  $H$  in feet.

(i) (6 pts.) By completing the square, write  $H(t)$  in the form  $H(t) = a(t - h)^2 + k$ . **Show clearly all your steps.**

**Answer:** \_\_\_\_\_

(ii) (3 pts.) Find the maximum height that the rock reached and the time  $t$  when this maximum was attained.

Maximum height  $\stackrel{?}{=}$  \_\_\_\_\_ when  $t \stackrel{?}{=}$  \_\_\_\_\_.

(iii) (3 pts.) Find the time it took for the rock to hit the ground (luckily missing its target!).

**Answer:** \_\_\_\_\_

14. (14 pts.) [Show your work]

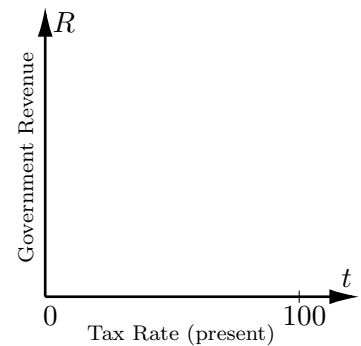
(A) (6 pts.) Find the limit:  $\lim_{h \rightarrow 0} \frac{\sqrt{25+h} - 5}{h}$ . (Show your work!)

Answer: \_\_\_\_\_

(B) (Independent of A.) (6 pts.) The federal debt now is about \$21.5 trillion (above the about 20.1 trillion GDP). How much it will become (future value) in 10 years if we assume that no additional debt is added during this period but this amount has been borrowed at an annual interest rate of 4% compounded **continuously**?

Answer: \_\_\_\_\_

(C) (Independent of A and B.) (2 pts.) The **Laffer curve** illustrates the idea that government can maximize tax revenue by setting tax rates at an optimal value  $t_{max}$  and that both 0% tax rate and 100% tax rate will generate zero government revenue. Draw this curve in the figure at right, assuming that it is modeled by a quadratic function.



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