Department of Mathematics
University of Notre Dame
Math 10250 – Elem. of Calc. I

Name:________________________________

Instructor:____________________________

Practice A – Exam 3

This exam is in 2 parts on 10 pages and contains 14 problems worth a total of 100 points. You have to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. Good luck!

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You must record here your answers to the multiple choice problems.
Place an × through your answer to each problem.

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MC.	______
11.	______
12.	______
13.	______
14.	______
Tot.	______

Multiple Choice

1. (5 pts.) Consider the curve $x^2 + y^2 = xe^y$. Find $\dfrac{dy}{dx}$.

(a) $\dfrac{dy}{dx} = 2x + 2y - xe^y - e^y$

(b) $\dfrac{dy}{dx} = \dfrac{xe^y + e^y - 2x}{2y}$

(c) $\dfrac{dy}{dx} = \dfrac{2x}{xe^y - 2y}$

(d) $\dfrac{dy}{dx} = \dfrac{e^y - 2x}{2y - xe^y}$

(e) $\dfrac{dy}{dx} = \dfrac{2x}{e^y - 2y}$

2. (5 pts.) Let

$$f(x) = e^{x-2}(x^3 - 3x^2 + 5x - 5).$$

You do not have to verify the fact that

$$f'(x) = e^{x-2}(x^3 - x).$$

Using this fact, find all the relative extrema of $f(x)$.

(a) Local maximum at $x = 0$; local minimum at $x = -1$ and at $x = 1$.

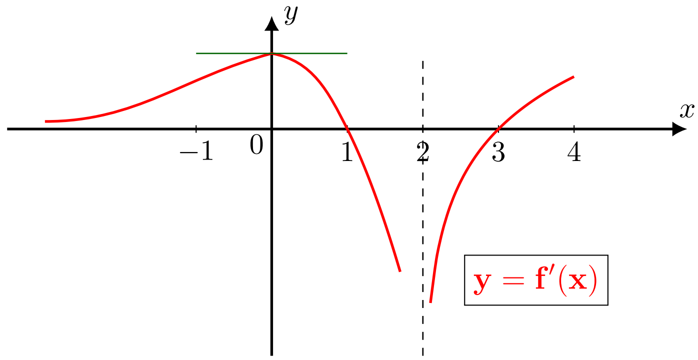
(b) Local maximum at $x = -1$ and at $x = 1$; local minimum at $x = 0$.

(c) Local maximum at $x = 0$; no local minima.

(d) Local maximum at $x = -1$ and at $x = 1$; local minimum at $x = 0$ and at $x = 2$.

(e) Local maximum at $x = 0$ and at $x = 2$; local minimum at $x = -1$ and at $x = 1$.

The following graph will be used for problems 3 and 4. It is the graph of **the derivative** $f'(x)$ of some function $f(x)$. In both parts, assume that $f(x)$ is defined for **all** x in $(-\infty, \infty)$.



3. (5 pts.) Referring to the graph of $f'(x)$ above, what are the critical points of $f(x)$? (Note we are asking about $f(x)$, not $f'(x)$.)

(a) $x = 0$.

(b) $x = 1, 2, 3$.

(c) $x = 1, 3$.

(d) $x = 0, 2$.

(e) $x = 0, 1, 2, 3$.

4. (5 pts.) Referring to the graph of $f'(x)$ above, where is the graph of $f(x)$ **concave up**? (Note we are asking about $f(x)$, **not** $f'(x)$.)

(a) $(-1, 2)$ and $(2, \infty)$.

(b) Nowhere.

(c) $(-\infty, -1)$.

(d) $(-\infty, 0)$ and $(2, \infty)$.

(e) $(0, 2)$.

5. (5 pts.) Let $f(x)$ be a **differentiable** function defined for all x in $(-\infty, \infty)$ and satisfying:

- $f(1) < f(3)$;
- $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = -\infty$.
- $f'(1) = f'(2) = f'(3) = 0$ and the sign of $f'(x)$ for all other x is described by the following chart:

f'	$+$		$-$		$+$		$-$
		1		2		3	

Which of the following is **FALSE**? [Hint: sketch the graph.]

(a) $f(x)$ has a local minimum at $x = 2$;

(b) $f(x)$ has a global maximum at $x = 3$;

(c) $f(1) > f(2)$;

(d) $f(x)$ does not have any vertical asymptotes.

(e) $f(x)$ has a global minimum at $x = 2$;

6. (5 pts.) Let $f(x)$ be a **differentiable** function defined for all x on $(-\infty, \infty)$ and satisfying:

- $f(1) = 2$, $f(3) = 6$;
- $f'(1) = f'(3) = 0$;
- $f''(2) = 0$.
- We know the following about the sign of $f'(x)$ and $f''(x)$:

f'	$-$		$+$		$+$		$-$
		1		2		3	

f''	$+$		$-$
		2	

Which of the following is **FALSE** given this information? [Hint: sketch the graph.]

(a) $f(x)$ has a local minimum at $x = 1$.

(b) $f(x)$ has a local maximum at $x = 3$.

(c) $f(2) < 2$.

(d) $f(x)$ has an inflection point at $x = 2$.

(e) $f(x)$ is increasing on $(1, 3)$.

7. (5 pts.) Paraná is a company that ships boxes all over the world. To save money, they want to minimize the cardboard they use for their boxes (i.e. minimize the surface area of the box, including bottom and top). For one product that they want to ship, they want to use a rectangular box whose base is a **square** and whose volume is 100 cm^3. Which of the following functions will they have to minimize in order to decide on the optimal dimensions of the box? In the following, x represents one side of the base and $A(x)$ represents the total surface area. [Hint: draw the box.]

(a) $A(x) = 2x^2 + \dfrac{400}{x^2}$

(b) $A(x) = 2x^2 + \dfrac{100}{x}$

(c) $A(x) = 2x^2 + \dfrac{100}{x^2}$

(d) $A(x) = 2x^2 + \dfrac{400}{x}$

(e) $A(x) = x^2 + \dfrac{4}{x}$

8. (5 pts.) Find the general solution of the following differential equation:

$$\frac{dy}{dx} = x^{1/2} + \frac{1}{x} + e^{4x} \quad (x > 0)$$

(a) $y(x) = \frac{2}{3}x^{3/2} + \ln x + \frac{1}{4}e^{4x} + C$

(b) $y(x) = \frac{1}{2}x^{-1/2} - \frac{1}{x^2} + 4e^{4x} + C$

(c) $y(x) = \frac{2}{3}x^{3/2} - \ln x + \frac{1}{4}e^{4x} + C$

(d) $y(x) = \frac{2}{3}x^{3/2} + \ln x + 4e^{4x} + C$

(e) $y(x) = \frac{2}{3}x^{3/2} - \ln x + 4e^{4x} + C$

9. (5 pts.) Suppose you use the substitution $u = x^2 + 1$ to evaluate the integral

$$\int x^3 e^{x^2+1} dx.$$

Which of the following integrals is the result of this substitution? [Hint: $x^3 = x^2 \cdot x$]

(a) $\displaystyle\int (u-1)e^u du$

(b) $\displaystyle\int ue^u du$

(c) $\displaystyle\frac{1}{2}\int ue^u du$

(d) $\displaystyle\frac{1}{2}\int ue^{u-1} du$

(e) $\displaystyle\frac{1}{2}\int (u-1)e^u du$

10. (5 pts.) Find the global maximum value of $f(x) = x^3 - 12x + 4$ **on the interval** $[-1, 3]$.

(a) -12

(b) 15

(c) -5

(d) 20

(e) There is no global maximum.

Partial Credit

You must **show your work** on the partial credit problems to receive credit!

11. (12 pts.) [**Show your work**]

For the Rose Bowl parade, Sofia displays a perfectly spherical balloon, which she can pump air into or out of.

(a) (5 points) If the formula for the volume of a sphere is

$$V = \frac{4}{3}\pi r^3,$$

how are $\dfrac{dV}{dt}$ and $\dfrac{dr}{dt}$ related?

Answer: ____________________

(b) (1 point) After the Rose Bowl parade, Sofia wants to **deflate** her balloon. Are the volume and radius increasing or decreasing as she does this?

Answer: ____________________

(c) (6 points) Her pump **releases** air from the balloon at a steady 48π ft^3/minute. At a certain moment she notices that the radius is decreasing at a rate of one inch per minute ($\frac{1}{12}$ ft/minute). What is the radius of the balloon at that moment? [Pay attention to part (b).]

Answer: ____________________

12. (12 pts.) [**Show your work**]

Let $f(x) = \dfrac{x}{x^2+1}$. It is a fact, which you do not have to verify, that

$$f'(x) = \frac{1-x^2}{(x^2+1)^2} \quad \text{and} \quad f''(x) = \frac{2x(x^2-3)}{(x^2+1)^3}.$$

In the following parts of this problem, your explanation counts more than the actual answer so

carefully explain your answers!!!!!!!

(a) (2 points) What are all horizontal and vertical asymptotes of $f(x)$?

Answer: ______________________________

(b) (2 points) Find all the critical points and points where $f(x)$ is not defined, if any.

Answer: ______________________________

(c) (3 points) Find all intervals where $f(x)$ is increasing and all intervals where $f(x)$ is decreasing.

Answer: ______________________________

(d) (3 points) Find all intervals where $f(x)$ is concave up and all intervals where $f(x)$ is concave down.

Answer: ______________________________

(e) (2 points) Find all local minimum points and all local maximum points.

Answer: ______________________________

13. (12 pts.) [**Show your work**]

(a) (6 points) Solve the following initial value problem: $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{x}} - xe^{x^2}, \quad y(1) = 4.$

Answer: ________________

(b) (6 points) **(Independent of (a)).** Compute the following indefinite integral:

$$\int \frac{x \ln(x^2 + 1)}{x^2 + 1} dx.$$

Answer: ________________

14. (14 pts.) [**Show your work**]

Pete's Skateboards produces all the skateboards in the state. They have determined that the skateboard market is such that for a production level of q skateboards per month **(measured in the tens of thousands, e.g. 10,000 skateboards corresponds to** $q = 1$**)**, the market will allow them to set a price of $p = e^{8-2q^2}$. Furthermore, they can produce at most 20,000 skateboards a month.

(a) (2 points) What interval describes the possible values of q?

Answer: ______________

(b) (3 points) Write the revenue, R, as a function of q.

Answer: R = ________________

(c) (3 points) Find $\dfrac{dR}{dq}$.

Answer: $\dfrac{dR}{dq} =$ ________________

(d) (3 points) Find the critical point(s) of R.

Answer: ______________

(e) (3 points) What is the maximum revenue that they can produce in a month? [It's ok if your answer has an "e" in it.]

Answer: ______________

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