Practice B – Math 10250 Exam 1 Solutions

1.) We need to find the equation of the line passing from the point $(p = 630, q = 200,000)$ and has slope $m = \frac{\Delta q}{\Delta p} = \frac{20,000}{-50} = -400$. Therefore using the point-slope formula we get: $q - 20,000 = -400(p - 630)$, or $\boxed{q = -400p + 452,000}$.

2.) Since the minimum value of r is 4 and occurs when $t = 2$ (years), we conclude that $h = 2$ and $k = 4$. Thus $r(t) = a(t-2)^2 + 4$. Since $6.1 = r(0) = a(0-2)^2 + 4 = 4a + 4$ we get $a = (6.1 - 4)/4 = 0.525$. Thus $r(t) = 0.525(t-2)^2 + 4$. Letting $t = 1$ (years) gives $r(1) = 0.525(1-2)^2 + 4 = 0.525 + 4$, or $\boxed{r(1) = 4.525}$.

3.) Using the identity $A^2 - B^2 = (A - B)(A + B)$ we have

$$P(x) = -20(x-40)^2 + 2000 = -20[(x-40)^2 - (10)^2] = -20[(x-40-10)(x-40+10)] = -10(x-50)(x-30),$$

which shows that $P(x) > 0$ if $\boxed{30 < x < 50}$. (Or, find the roots of $P(x) = 0$ and then check the sign of $P(x)$ between them!)

4.) Using the identity $(A + B)^2 = A^2 + 2AB + B^2$ we have

$$\lim_{h\to 0} \frac{4(5+h)^2 - 100}{h} = \lim_{h\to 0} \frac{4(25 + 10h + h^2) - 100}{h} = \lim_{h\to 0} \frac{40h + 4h^2}{h} = \lim_{h\to 0}(40 + 4h) = 40 + 0 = \boxed{40}.$$

5.) The function $f(x)$ is **not** continuous at $x = 3$ since its value there is 80, which is different from its limit as $x \to 3$. Note that this limit is 100.

6.) Looking at the graph of $f(x)$ we see that $\lim_{x\to 3} f(x) = 100$. Therefore, applying the limit law's we have

$$\lim_{x\to 3} \frac{x^2 + 10x + 1}{\sqrt{f(x)}} = \frac{\lim_{x\to 3}[x^2 + 10x + 1]}{\lim_{x\to 3}\sqrt{f(x)}} = \frac{\lim_{x\to 3} x^2 + 10\lim_{x\to 3} x + 1}{\sqrt{\lim_{x\to 3} f(x)}} = \frac{9 + 30 + 1}{\sqrt{100}} = \frac{40}{10} = \boxed{4}.$$

7.) We have

$$\lim_{x\to\infty} R(x) = \lim_{x\to\infty} \frac{800x + 2100}{2x + 7} = \lim_{x\to\infty} \frac{800 + 2100/x}{2 + 7/x} = \frac{800 + \lim_{x\to\infty}[2100/x]}{2 + \lim_{x\to\infty}[7/x]} = \frac{800 + 0}{2 + 0} = 400.$$

Thus, if the company keeps spending more and more money in advertising then the revenue's limiting value is $\boxed{\$400}$ million.

8.) Writing $f(x) = \frac{x-8}{(x-8)(x-7)} \overset{x\neq 8}{=} \frac{1}{x-7}$, we see that $x = 7$ is a vertical asymptote since $\lim_{x\to 7^\pm} \frac{1}{x-7} = \pm\infty$. Also, we have that $y = 0$ is a horizontal asymptote, since $\lim_{x\to\pm\infty} \frac{1}{x-7} = 0$. The natural domain of the function $f(x)$ consists of all numbers except $x = 7$ and $x = 8$, which are the zeros of the denominator. Note, however, that $x = 8$ is **not** a vertical asymptote since $\lim_{x\to 8^\pm} \frac{1}{x-7} = 1$.

9.) Since temperature is a continuous function of time and the value 62 is between $H(2) = 63$ and $H(4) = 54$, and also between $H(8) = 61$ and $H(10) = 70$ by the intermediate value theorem the temperature assumes the value 62 in the time intervals $[2, 4]$ and $[8, 10]$ for certain.

10.) First we solve the equation $y = \frac{x+5}{x-3}$ for x. For this we multiply the equation by $x - 3$ and get $xy - 3y = x + 5$, or $xy - x = 3y + 5$, or $(y-1)x = 3y + 5$, or $x = \frac{3y+5}{y-1}$. Next, we interchange x and y and obtain $y = \frac{3x+5}{x-1}$. Thus the inverse of $f(x)$ is given by the function $\boxed{g(x) = \frac{3x+5}{x-1}}$. Observe the natural domain of $f(x)$ consists of all numbers $x \neq 3$ and of $f(x)$ consists of all numbers $x \neq 1$.

11.) (Ai) The revenue function is: $R = x \cdot q = x(-0.2x + 100)$, or $\boxed{R(x) = -0.2x^2 + 100x}$.

(Aii) The profit function is: $P = R - C = -0.2x^2 + 100x - 5q - 2500$. Taking the expression of q from demand and substituting it to the profit function we get: $P = -0.2x^2 + 100x - 5(-0.2x + 100) - 2500$, or $\boxed{P(x) = -0.2x^2 + 101x - 3000}$.

(B) We need to find the equation of the line passing through the points $(p = 10,\ q = 5000)$ and $(p = 15,\ q = 3500)$. Since the slope is $m = \frac{\Delta q}{\Delta p} = \frac{3500-5000}{15-10} = -300$. Therefore using the point-slope formula we get: $q - 5000 = -300(p - 10)$, or $\boxed{q = -300p + 8000}$.

12.) (A) For $h \neq 0$ we have

$$\frac{\sqrt{16+h}-4}{h} = \frac{(\sqrt{16+h}-4)(\sqrt{16+h}+4)}{h(\sqrt{16+h}+4)} = \frac{(\sqrt{16+h})^2 - 4^2}{h(\sqrt{16+h}+4)} = \frac{\cancel{h}}{\cancel{h}(\sqrt{16+h}+4)} = \frac{1}{\sqrt{16+h}+4}.$$

Therefore, the given limit is equal to

$$\lim_{h\to 0} \frac{1}{\sqrt{16+h}+4} = \frac{1}{\sqrt{16+0}+4} = \boxed{\frac{1}{8}}.$$

(B) Using the formula $FV = PVe^{rt}$ with $PV = 17.76$, $r = 0.04$ and $t = 10$, we find that in 10 years the federal debt will be $FV = 17.76e^{0.04\cdot 10} = \boxed{17.76e^{0.4}}$ trillion dollars.

13.) (Ai) We have

$$\begin{aligned} H(t) &= -16t^2 + 1600t \\ &= -16[t^2 - 100t] \\ &= -16[t^2 - 2\cdot t\cdot 50] \\ &= -16[t^2 - 2\cdot t\cdot 50 + 50^2 - 50^2] \\ &= -16[(t-50)^2 - 50^2] \\ &= -16(t-50)^2 + 16\cdot 50^2 \\ &= \boxed{-16(t-50)^2 + 40,000} \end{aligned}$$

(Note that there are other ways for completing square.)

(Aii) Since $H(t) = -16(t-50)^2 + 40,000$ it takes its maximum value 40,000 feet at $t = 50$ seconds.

(B) Since $P(0) = 10$ we have $P_0 b^0 = 10$, which gives $P_0 = 10$. Furthermore, since $P(5) = 320$ we have $10b^5 = 320$, which gives $b = 2$. Thus, the animal population is given by the formula $\boxed{P(t) = 10 \cdot 2^t}$.

14.) (A) The function $f(x)$ is continuous everywhere except possibly at $x = 4$, which is the zero of the denominator. Since $\lim_{x\to 4} \frac{x^2+x-20}{x-4} = \lim_{x\to 4} \frac{(x-4)(x+5)}{x-4} = \lim_{x\to 4}(x+5) = 9$, we see that $f(x)$ is continuous at $x = 4$ too if we define $f(4) = 9$, or choose $\boxed{c = 9}$.

(B) The graph of such a function is shown in the next graph. (There are infinitely-many such functions. Choose yours!)