## Mathematics 10350 - Calculus A for the Life \& Social Sciences

Fall Semester 2023
Text: Calculus (4th Ed) - Early Transcendentals by Rogawski, Adams \& Franzosa Publisher: W. H. Freeman \& Co.

| Section | Instructor | Class Schedule | Office HR | email@nd.edu |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Arthur Lim | MWF 8:20-9:10 HAYE 117 | TBA | arthurlim |
| 2 | Matthew Weaver | MWF 9:25-10:15 HAGGAR 117 | TBA | mweaver6 |
| 3 | Anthony Gomez Fonseca | MWF 10:30-11:20 BOND 104 | TBA | agomezfo |
| 4 | Matthew Weaver | MWF 11:30-12:20 NIEU 127 | TBA | mweaver6 |
| 5 | Sudipta Ghosh | MWF 12:50-1:40 DBRT 136 | TBA | sghosh7 |
| 6 | Xiyan Zhong | MWF 2:00-2:50 BOND 104 | TBA | xzhong4 |


| Section | Teaching Assistant | Class Schedule | Office HR | email@nd.edu |
| :---: | :---: | :---: | :---: | :---: |
| 11 | Daniel Villanueva Vega | R 12:30-1:20 HAYE 231 | TBA | dvillanu |
| 12 | Brandon Fu | R 3:30-4:20 HAYE 231 | TBA | qfu3 |
| 21 | Daniel Villanueva Vega | R 11:00-11:50 FITZ 356A | TBA | dvillanu |
| 22 | Jui-Yun Hung | R 2:00 - 2:50 HAYE 231 | TBA | jhung2 |
| 31 | Atticus Stonestrom | R 11:00-11:50 HAYE 231 | TBA | astones3 |
| 32 | Annie Holden | R 12:30-1:20 PCTR 112 | TBA | aholden2 |
| 41 | Atticus Stonestrom | R 9:30-10:20 HAYE 231 | TBA | astones3 |
| 42 | Yuyan He | R 2:00-2:50 PCTR 112 | TBA | yhe27 |
| 51 | Luis Atzin Franco Reyna | R 9:30 - 10:20 COMO 330 | TBA | lfrancor |
| 52 | Luis Atzin Franco Reyna | R 11:00 - 11:50 DBRT 119 | TBA | lfrancor |
| 61 | Yuyan He | R 12:30 - 1:20 HAGGAR 117 | TBA | yhe27 |
| 62 | Brandon Fu | R 5:05-5:55 HAYE 231 | TBA | qfu3 |

Course Website:
https://www3.nd.edu/~m10350/
Most information for this course is posted on its website. These include instructors and TAs office hours and contact information, daily homework information, exam dates and venues, practice exams, and etc.

Calculator Policy: Calculators are NOT allowed on any of the quizzes or exams. You may use your calculators for homework and assignments, but it is strongly recommended that you do not rely on any of the graphing functions on the calculator.
Course Grade \& Breakdown (SUBJECTED TO CHANGE DUE TO HEALTH PROTOCOL):

|  | Date | Time | Room | Points |
| :---: | :---: | :---: | :---: | :---: |
| Gateway Quiz | During tutori | week 02 (20\%), w | , \& week 04 (50\%) | 50 |
| Midterm Test 01 | Tues Sep 19 | 8:00am - 9:15am | TBA | 100 |
| Midterm Test 02 | Thurs Oct 12 | 8:00am - 9:15am | TBA | 100 |
| Midterm Test 03 | Tues Nov 14 | 8:00am - 9:15am | TBA | 100 |
| Final Exam | Wed Dec 13 | $1: 45 \mathrm{pm}-3: 45 \mathrm{pm}$ | TBA | 150 |
| Online Hwk \& Assignments | Submit online or collected in class as scheduled on website |  |  | 75 |
| Participation | participation, attendance, activities \& quizzes |  |  | 25 |
| Total points: 600 |  |  |  |  |

Your final grade will be based on your total score out of 600 . All course activities are in-person except otherwise stated. Please note that course format and policies are subjected to change as we navigate the current pandemic situation.

Drop Policies: Each homework assignment will be graded out of 25 points. At the end of the semester THREE sets of homework of the lowest scores will be dropped. Homework would still count toward $12.5 \%$ of your grade.

Weighted Midterms: There are three midterm tests each worth 100 points. At the end of the semester, we will re-weight the three midterm scores so that lowest scores earns up to $25 \%$ of the 300 points, the middle scores earns up to $35 \%$ of the 300 points and the highest scores earns up to $40 \%$ of the 300 points. The sum of the weighted scores give you your midterm total out of 300 points.

There are three midterm tests each worth 100 points and one Gateway quiz worth 50 points.
Missed exams or quizzes: A student who misses an examination or quiz will receive zero points for that exam or quiz. If you have a valid excuse (illness, excused athletic absence, etc.) for missing an exam, please contact your section professor ASAP (preferably before the exam) to schedule a makeup exam. If you missed any tutorial work or quizzes, you must contact your TA immediately too.

Honor Code: Examinations, homework, assignment and quizzes are conducted under the honor code. While collaboration in small groups in doing homework is permitted (and strongly encouraged) in this course, copying is not. In particular, copying from the Student Solutions Manual is a violation of the Honor Code. Exams are closed book and are to be done completely by yourself with no help from others.

Homework \& Assignments: Online Homework and assignment problems are assigned daily. Their schedule is listed on the course website. We do not accept any late homework or assignment unless you have a documented medical or university excuse. Exceptions are handled case by case. You are encouraged to work on these problems in groups, but all online homework and assignments must be turned in individually. Remember that you will not learn anything by simply copying another student's work or the Student Solutions Manual. The main purpose of homework and assignments is to help you learn the material and assess yourself. Experience shows that students who take their homework seriously do very well in the course because they have a better understanding of the material. For detailed homework and assignment instructions, please see attached information.

Class Attendance: A student who accumulates more than 3 unexcused absences may be given an F grade.
Classroom Policies: Please do your best to show up on time and quietly enter the room if you are late. Please remember to respect your peers who are here to learn. Indeed, class disruptions will not be tolerated and offending parties will be asked to leave. During lectures you are encouraged to actively participate by answering and asking questions.

Study Tips are attached and also posted on the on the course website (http://www.nd.edu/~m10350). Please review it. The key point is to start early and be consistent.

Getting Help: You can get help for mastering the course material from the avenues below. More information can be obtained from the 10350 course website; click on "TUTORING \& HELP" or go directly to
https://www3.nd.edu/~m10350/tutor_help.html.

It is important that you get help soon when you have difficulty with the course. The earlier you meet with your instructor and TA, the more we can do to help and advise.

- Instructor \& TA's Office Hours - We highly encourage students to visit their professors/TAs when they have questions. Students can attend any office hours available, not just those of their section professors. See office hour schedule here: https://www3.nd.edu/~m10350/instructors.html
- Calculus A Tutoring Sessions: These are walk-in tutoring sessions that runs Sunday through Wednesday. See schedule posted here:
https://www3.nd.edu/~m10350/instructors.html
- Mathematics Help Room: The help room information is posted here:
https://math.nd.edu/undergraduate/student-resources/math-help-rooms/


## - Learning Resources Center (LRC) Help

Please note that instructors and tutors are NOT there to do your homework. In fact, tutors are instructed to guide you to the answer and not do your homework. Please do not ask the tutors to grade your homework, and be specific about what you want to discuss. You can find more information here:

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## MATH 10350 Course Work Policy

There are both online homework and paper-pencil assignments for this course.
Written Assignments are due in class according to the schedule posted on the Math 10350 website. The questions and problems to be turned in are posted on the course website. You are expected to submit your written assignment in the following manner:

- Write your name clearly on your homework.
- Your work has to be clearly and logically written, showing the method of solution, not just a final answer.
- Any work falling short of the above expectations may not be graded.

We do not accept any late homework or assignment unless you have a documented medical or university excuse. Exceptions are handled case by case. If you need to attend a school related event, you may turn in your assignment early or arrange to have your peer turn it in on the day it is due.

Online Homework is assigned daily and is due as scheduled on the course calendar:
https://www3.nd.edu/~m10350/homework.html

The online systems we are using is Mobius.

- Mobius is accessed through the ND canvas system.
- The ND e-mail address will be used to make all course related announcements. You must check your e-mail regularly daily.

All online homework should be done using paper and pencil, and be treated the same manner as written assignments. We encourage you to keep a record of your work for material submitted online; these are helpful when you review for an exam. Usually, you are expected to complete about 5 to 8 problems of your online homework assigned at the end of each class day. If you have difficulty solving the homework questions please see your $\mathrm{TA} /$ professor or visit the listed the math help resources above.

Absolutely no late homework will be accepted. You only need the course textbook and Mobius for this course.
Access Mobius at:
http://canvas.nd.edu/

All homework and assignment will be weighted the same. The three lowest of homework/assignment scores will be dropped. You should bookmark these pages.

Online Homework Submission Policies. All submission due dates for online homework on Mobius are fixed. You are highly encouraged to SUBMIT your homework well ahead of deadlines. We DO NOT accept excuses like: My computer/Webservers shut down just before I could submit my work on time. Save your answers as you enter them online. This ensures that no work is lost BEFORE the submission deadline. Enough buffer time is given to ensure timely submission of your work. All online homework are due at 11:59pm Tuesdays, Thursdays or Saturdays unless otherwise stated. In addition, after the deadline of a homework, you have 48 hours to complete a late homework to obtain up to $80 \%$ of the full score.

# Math 10350 (Calculus A) Syllabus 

Text: Calculus (Early Transcendentals) $4^{\mathrm{TH}}$ Edition
J. Rogawski, C. Adams \& R. Franzosa
Notes Functions and their Applications

- Applications include cost, revenue, and profit functions, velocity and position.
1.6 Exponential and Logarithmic Functions
- including application to Compound Interest
2.1 The Limits Idea: Instantaneous Velocity and Tangent Lines
2.2 Investigating Limits
2.3 Basic Limit Laws
2.4 Limits and Continuity
2.5 Indeterminate Forms
2.6 The Squeeze Theorem and Trigonometric Limits
2.7 Limits at Infinity
3.1 Definition of the Derivative
3.2 The Derivative as a Function
3.3 Product and Quotient Rules
3.4 Rates of Change
3.5 Higher Derivatives
3.6 Trigonometric Functions
3.7 The Chain Rule
3.8 Implicit Differentiation
3.9 Derivatives of General Exponential and Logarithmic Functions
3.10 Related Rates


### 4.1 Linear Approximation and Applications

4.2 Extreme Values
4.3 The Mean Value Theorem and Monotonicity
4.4 The Second Derivative and Concavity
4.5 L'Hopital's Rule
4.6 Analyzing and Sketching Graphs of Functions
4.7 Applied Optimization
4.8 Newton's Method
5.1 Approximating and Computing Area
5.2 The Definite Integral
5.3 The Indefinite Integral
5.4 The Fundamental Theorem of Calculus, Part I
5.5 The Fundamental Theorem of Calculus, Part II
5.6 Net (or Total) Change as the Integral of a Rate of Change
5.7 Substitution Method

### 11.1 Parametric Equations

## Basic Algebra Rules

## Exponential Rules:

$$
\begin{array}{ll}
a^{m} \cdot a^{n}=a^{m+n} & (a b)^{m}=a^{m} b^{m} \\
a^{0}=1 ; \quad a \neq 0 & \frac{a^{m}}{a^{n}}=a^{m-n} ; a \neq 0 \\
\left(a^{m}\right)^{n}=a^{m n} & \left(\frac{a}{b}\right)^{m}=\frac{a^{m}}{b^{m}} ; \quad b \neq 0
\end{array}
$$

## Distribution Law:

$$
a(b+c)=a b+a c \quad \frac{a+b}{c}=\frac{a}{c}+\frac{b}{c} \quad \frac{a-b}{c}=\frac{a}{c}-\frac{b}{c}
$$

## Quadratic Factoring:

$$
\begin{array}{ll}
(a+b)^{2}=a^{2}+2 a b+b^{2} & (a-b)^{2}=a^{2}-2 a b+b^{2} \\
a^{2}-b^{2}=(a-b)(a+b) &
\end{array}
$$

## Properties of Logarithm:

$\begin{array}{lll}\log _{a}(M N)=\log _{a} M+\log _{a} N & \log _{a}\left(\frac{M}{N}\right)=\log _{a} M-\log _{a} N & \log _{a}(M)^{t}=t \log _{a} M \\ \log _{a} a=1 & \log _{a} 1=0 & \\ \log _{a} a^{x}=x & a^{\log _{a} x}=x & \\ \text { Change of Base: } \quad \log _{a} M=\frac{\log _{b} M}{\log _{b} a} & \ln (M)^{t}=t \ln M \\ \ln (M N)=\ln M+\ln N & \ln \left(\frac{M}{N}\right)=\ln M-\ln N & \\ \ln e=1 & \ln 1=0 & \\ \ln e^{x}=x & e^{\ln x}=x & \end{array}$

Algebra Review - Arithmetic Rules \& Laws of Exponent

Complete the Arithmetic Operations below:
$a(b+c)=$
$\frac{a}{b}+\frac{c}{d}=$
$a \times b=$
$a(b c)=$
$\frac{a+b}{c}=$
$\frac{a}{b} \times \frac{c}{d}=$
$a \div \frac{b}{c}=$

$$
\frac{\frac{a}{b}}{\frac{c}{d}}=
$$

$(a+b)(c+d)=$
$(a+b)(a-b)=$
$(a+b)^{2}=$
$(a-b)^{2}=$

Complete the Laws of Exponents below:

$$
\begin{array}{lll}
a^{m} \cdot a^{n}= & (a b)^{m}= & \frac{a^{m}}{a^{n}}= \\
a^{0}= & ; a \neq 0 & a^{1 / m}= \\
\frac{1}{b^{m}}= & \left(a^{m}\right)^{n}= & \left(\frac{a}{b}\right)^{m}= \\
& &
\end{array}
$$

1. A lab technician has 700 kg of salt and wishes to make a solution with concentration $5.6 \mathrm{~g} / \mathrm{cm}^{3}$. How much water in $\mathrm{m}^{3}$ must he use to make the solution if all 700 kg of salt is used?
2. Simplify $\sqrt{25 a^{4} b^{3}} \times \frac{2}{b^{2}} \div \frac{5 a^{3}}{b^{2}}$ giving your answer in the form $k a^{m} b^{n}$.
3. Give the lowest common denominator of fractions or rational functions in the sums below then evaluate the sum giving your answer as a single rational number or function with no common factors between its numerator and denominator.
a. $\frac{3}{5}-\frac{7}{15}+\frac{2}{9}$
b. $\frac{3}{x^{2}}-\frac{x}{x^{2}-4}-\frac{2}{x^{2}+2 x}$
4. Simplify the following expression giving your answer in the form $\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ has no common factors.
$\frac{\left(x^{2}+2\right)^{3} \cdot 4-(4 x+1) \cdot 3\left(x^{2}+2\right)^{2} \cdot 2 x}{\left(x^{2}+2\right)^{6}}$

Math 10350 - Example Set 01B
Functions Review: Sections 1.1, 1.2, \& 1.3
(Basic Functions) Give an example for each type of basic functions below and give their general form:

## A. Power Function:

An example: $\qquad$

General form: $\qquad$

## B. Polynomial Function:

An example: $\qquad$

General form: $\qquad$
$\underline{\text { Special Cases }}$

Linear functions: $\qquad$

Quadratic functions: $\qquad$

## C. Rational Function:

An example: $\qquad$

General form: $\qquad$
D. Exponential Function:

An example: $\qquad$

General form: $\qquad$
E. Logarithmic Function:

An example: $\qquad$

General form: $\qquad$

## F. Trigonometric Function:

Examples:

1. The quantity $y$ relates to $x$ in each of the following graphs. For each graph determine whether $y$ is a function of $x$.


Price, Revenue, Cost \& Profit. Write an equation that connects the revenue from the sale of a certain product, the number of the product sold (or demand), and selling price of one unit of the product. How does revenue differ from the profit from the sale of the product?
2. (An application of Functions) A electronic company decides to set the sale price of a sound card at $\$ 60$ a piece for a monthly demand of 100 pieces. The sale price drops to $\$ 50$ a piece for a monthly demand of 200 pieces.

2a. Assuming that the sale price for one sound card is a linear function of the size of the monthly demand, find a formula for the sale price $s$ dollars per sound card in terms of the size $x$ of the monthly demand. What is the revenue function from the sales of the sound card?

$$
\left(-\frac{x}{10}+70 ;-\frac{x^{2}}{10}+70 x\right)
$$

2b. Suppose further that the company has a monthly overhead cost of $\$ 5000$ for producing the sound cards and a cost of $\$ 10$ for producing each piece of the sound card. What is the monthly profit from the sales of the sound card in terms of month production assuming that all items produced are sold?

$$
\left(-\frac{x^{2}}{10}+60 x-5000\right)
$$

3. (Composition of Functions) Let $f(x)=\frac{x+1}{x-2}$ and $g(x)=\frac{2}{x+1}$. Evaluate the following (a) $\frac{f(a+h)-f(a)}{h}$ and (b) $f(g(a))$ simplify your answer assuming that $a \neq-1$.

## Completing the Square Notes

Completing the square is an algebraic process applied to quadratic expressions of the form $x^{2}+a x$ to obtain a perfect square. Specifically we want to find a positive number $b$ such that

$$
x^{2}+a x+\ldots
$$

where both $b$ and $c$ are to be determined.

## Geometric interpretation of completing the square.

Interpret $x^{2}$ as the area of a and

Interpret $a x$ as the area of $\qquad$
$\qquad$

Piece together the square and rectangles together to see the method of completing the square.

Examples. Fill in the blanks for each quadratic expressions of the form $x^{2}+a x$ below to obtain a perfect square.
(a) $x^{2}+6 x+\quad=$ $\qquad$
(b) $x^{2}-4 x+=$ $\qquad$
(c) $x^{2}+5 x+\square=$ $\qquad$
3. (Completing the Square Practice) Rewrite each of the following quadratic functions in the form $A(x+B)^{2}+C$ : (i) $x^{2}-6 x-5$, (ii) $-2 x^{2}-8 x+1$. (iii) Graph the quadratic function in (ii). State the coordinates of the vertex and the equation of the axis of symmetry.
4. Rewrite the monthly profit function in Q1 in the form $A(x+B)^{2}+C$.
a. By scaling and translating $x^{2}$, graph the monthly profit function labelling the axis of symmetry, vertical intercept and vertex.
b. What is the maximum profit the company can make and when does that happen?

## Suggestions and Tips for Success in Calculus A

$*$ By failing to prepare,
you are preparing to fail.*
$\sim$ Benjamin Franklin

Learning mathematics takes consistent practice. Many of the opportunities for practice are furnished by the homework and assignment problems that you need to turn in regularly. In addition to homework problems, you should also honestly "test" yourself with extra problems in the textbook, and exam reviews. Your performance in these "self-tests" could give you a good gauge of your understanding of the material taught in class.

Aim to achieve ownership of mathematical knowledge.
By that we mean you could:
(1) Identify the tools needed to solve a given problem.
(2) Carry out the process of solution in a reasonable time frame without help from a tutor or peer.
(3) Give a reason for each step in the solution of the problem.
(4) Obtain insights from your computation besides giving a number.

Although we recognize that some individuals are more gifted in mathematics, consistency in your learning is still required to improve your chance of success. Mathematics is a hard subject; for many, much has to be done just to get by. However, the skills you gain and the opportunities opened to you from your training in the subject will make the challenges you face worthwhile. These skills are required in your future employment. For instance, see:

BusinessWeek: Math Will Rock Your World (Jan 23, 2006)
http://news.bbc.co.uk/2/hi/uk news/education/6954666.stm
Meet with your instructors (professor and TA) whenever you feel that you are falling behind or have trouble understanding the material to the level listed above. In addition to your instructors, you could also get help from Math department tutoring, and the LRC. For details look up:
http://www.nd.edu/~m10350/tutor_help.html
Ultimately, students need to be able to solve the problems by themselves. But you will be helped by the guided experience provided by the course. Use the resources available to you to support your learning. These resources include your instructor, tutoring services, and practice problems.

Many students in the past have been successful in Calculus.
With that said, we wish to give advice on how you should manage your time, and tips for handling various learning activities of the course, including the examinations.

Getting the Most of Classroom Instruction
Be in class and be prepared to listen attentively and take notes.
Work out all examples discussed in class. You should be able to work these out on your own in a reasonable time frame. Most problems should take no more than 20 minutes if you have grasped the material. You should be able to reason out and explain each step of your work.

Identify the material that you do not understand, and examples that you have a hard time working out or take a long time to complete.

Ask for help immediately. We highly encourage that you clarify any confusion with your instructors before the next class or at least on the same day as the next class. The point is to promptly resolve any confusion as soon as possible. You may still need to "test" yourself with a few similar problems after meeting with your instructor.

## Completing Your Homework/Assignments

Almost all homework is online. You may have occasional written assignments. Here are some good practices for completing your homework.

Start early to give a good margin of time for completing your assignment. Aim to complete 5 to 8 problems of the assigned homework at the end of each class day. Homework is posted online on Mobius which can be accessed through Canvas. Written homework will also be posted on Canvas.

Mark out those you have confidence doing and those you do not know how to start. Work on at least a couple of them to know how much help you need. Usually, you are expected to complete or at least seriously attempt all problems of your written assignment assigned at the end of each class day.

Attempt those problems that you are confident in completing first. Work on those that you are unsure of. Give yourself a full 20 minutes to seriously attempt these problems.

Ask for help promptly if you still cannot do your assignments. Bring along your scratch work.
File your assignment immediately when it is returned. You will need it for reviewing.
You should have a notebook that organizes your work for each online homework. Work out the questions just like you would a paper and pencil homework and keep your work for review later as part of your exam preparation.

## Do not wait till the last minute to complete and submit your work online.

## Preparing for Exams

Give yourself a week to prepare for each midterm (or final) exam. Get all the material you need to go through: (1) Sample exams, (2) Class notes, (3) Homework, and (4) Tutorial material.

There is obviously a lot of material to be covered. However, learning the course material should be a consistently on-going process and should NOT take place merely the week before the exam.

Start with the sample exam. Mark out all the problems that you are confident of solving, those for which you need to refresh your memory, and those you totally do not know how to attack.

Attempt as many problems as possible before review sessions.
Ask for help promptly from tutors and instructors. Bring along your attempts and scratch work; they help us see how to best facilitate your learning.

Attend review sessions. Please note that going to the review session alone does not mean that you are ready for the test.

Test yourself with an exam from a previous year. You need to be able to solve a problem by reasoning out each step of the solution.

Look for more practice problems in topics that you are still unsure of.

## Access for Calculus A online Homework system Mobius Assessments

The online homework system Mobius is accessed through Canvas. Follow the steps below to logged into Mobius.
(a) Access Canvas through InsideND by searching for the Canvas App or using the link:

## https://canvas.nd.edu/

(b) Log into Canvas using your NetID and password.
(c) Find your Canvas Calc A (10350) section for the semester in "Dashboard" on the left.
(d) In your Calc A section, click on "Module". You will see the link to 10350 Homework in the Mobius Assessment module. See figure below.

(e) Click on the "10350 Mobius Homework" link and launch Mobius.
(f) Mobius will launch in a new tap.
(g) There are six units of homework available. You can toggle between the units to see the sets of homework available. See figure below. You have eight attempts for each set of exercise.


Timed Exercises - 25 points each. There are four sets each consisting of five components of short timed exercises. You should Highest scores of all components will collected to make up 25 points. The timed feature is to check your speed and accuracy for basic computations. You can attempt each component eight times. You should aim to get 100\% for each component to count yourself proficient. Answers of previous attempts will NOT be saved for timed exercises.

## Fraction Ex

Simplify 01 Ex
Simplify 02 Ex

Gateway Prep - 25 points each. There will be a Gateway prep for each gateway 1, 2, and 3 . These are preparation exercises for the Gateway Quizzes in Week02, Week03, and Week04 during tutorial. Complete them before the due dates so you have an idea where you stand for the material tested in each quiz. You have five attempts for each set of homework but no time limit for each attempt Correct answers of previous attempts are saved for exercises with no time limit for each attempt.

The due dates are as stated for each homework set (highlighted in red). If the first due date is Aug 30 midnight. This means the work should be done by 11:59pm Aug 30 and the homework will close sometime after 11:59pm Aug 30. We are using the due date to pace your learning so please schedule time to keep the due dates.

After you submitted a set of homework, you can always see the correct answer. In some homework, solution of the graded work by clicking on the score or see details in the gradebook in Mobius. See an illustrative figure below. Please explore the links.

Learn from your mistakes in the provided solution. Understand the rules applied and process of solution but DO NOT just follow a "cook book" process or mere pattern recognition. Every step of the solution has a reason.



[^0]:    https://firstyear.nd.edu/resources/academic-support/learning-resource-center/

