Math 10360 Review for Exam 2

1. Find the function $y(t)$ if its slope is given by $21\sin(2t)\sin(5t)$ and that $y(0) = 3$. (Ans: $y = -\frac{3}{2}\sin(7t) + \frac{7}{2}\sin(3t) + 3$)

2. Find the total change for the function of $f(x)$ if $f'(x) = \cos^4(3x)$ over the interval $[0, \pi]$. (Ans: $\frac{3\pi}{8}$)

3. Explain why the integrals are improper and evaluate them using **limits**:

(i) $\displaystyle\int_0^1 \frac{xdx}{\sqrt[3]{1-x^2}}$ (Ans: 3/4) (ii) $\displaystyle\int_{-\infty}^0 xe^{2x}dx$ (Ans: $-1/4$) (ii) $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2}\,dx$ (Ans: 0)

4. Evaluate the following integrals:

(i) $\displaystyle\int_0^{\pi/4} \cos^5(2x)\,dx$ (ii) $\displaystyle\int \tan^3 x \sec x\,dx$ (iii) $\displaystyle\int x\tan^{-1} x\,dx$ (iv) $\displaystyle\int_{-\infty}^0 \frac{e^x}{1+e^{2x}}dx$

(Ans: (i) 4/15; (ii) $\frac{\sec^3 x}{3} - \sec x + C$; (iii) $\frac{1}{2}(x^2\arctan(x) - x + \arctan(x)) + C$; (iv) $\frac{\pi}{4}$)

5. Find the area under the curve $y = \dfrac{8}{x^4-1}$ for $2 \le x \le 5$.

(Ans: $\int_2^5 \frac{8}{x^4-1}dx = \int_2^5 \frac{2}{x-1} - \frac{2}{x+1} - \frac{4}{x^2+1}dx = 2\ln 2 - 4\arctan(5) + 4\arctan(2)$)

6. To provide for new catch, the king crab was released into the Barents sea. However such crab has now reached waters near the north pole to the dismay of many environmental scientist. If the population $P(t)$ (in thousands) of the king crab a year after they have been discovered in northern waters is modeled by

$$\frac{dP}{dt} = \frac{100}{t(\ln t)^2},$$

estimate the total change in the numbers of such crab a long time after two years of their discovery?

(Ans: $\int_2^\infty \frac{100}{t(\ln t)^2}dt = \frac{100}{\ln 2}$)

7. Write down, but DO NOT solve for the constants, the partial fraction decomposition of each of the rational expressions. Be sure to long divide if the expression is improper.

a. $\dfrac{5}{2x^2-3x-2}$ **b.** $\dfrac{2}{x^4-x^3}$ **c.** $\dfrac{x^3+2x^2}{x^2-1}$ **d.** $\dfrac{x^2+1}{(x^2+x+1)^2(2x+5)}$

(Ans: (a) $\frac{A}{2x+1} + \frac{B}{x-2}$; (b) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1}$; (c) $x + 2 + \frac{A}{x-1} + \frac{B}{x+1}$; (d) $\frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{(x^2+x+1)^2} + \frac{E}{2x+5}$)

8. Perform the integration $\displaystyle\int \frac{x^3+2x^2}{x^2-1}\,dx$. (Ans: $\frac{x^2}{2} + 2x + \frac{3}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C$)

9. Parametrize in two ways using rectangle coordinates the region bounded by $y = 4 - x^2$ and the chord joining the points $(0, 4)$ and $(2, 0)$. If the mass density of the region is given by $\rho(x, y) = xe^y$ kg/m^2, find the total mass of the region.

Ans: 12. $R = \{(x, y) : 0 \le x \le 2, -2x + 4 \le y \le 4 - x^2\}$; $R = \{(x, y) : 0 \le y \le 4, \frac{4-y}{2} \le x \le \sqrt{4-y}\}$; Mass $= \int_0^2 \int_{-2x+4}^{4-x^2} xe^y\,dydx$

10. Parametrize as a single region the region R bounded by $y = 2x - 1$, $y = \sqrt{x}$, $x = 0$. Using your answer, evaluate the following double integral: $\displaystyle\iint_R (6x^2 + 4xy)\,dA$.

(Ans: 13. $\iint_R (6x^2+4xy)dA = \int_{x=0}^1 \int_{y=2x-1}^{\sqrt{x}} (6x^2+4xy)dydx = \int_0^1 (6x^{5/2} + 16x^2 - 20x^3 - 2x)dx = \frac{22}{21}$)

11. The population density of a certain kind of antelope leaving inside the circle $x^2 + y^2 = 100$ is given by the function $\rho(x, y) = x^2 + 3y^2$ antelope per sq. km Find the total population of the antelope.

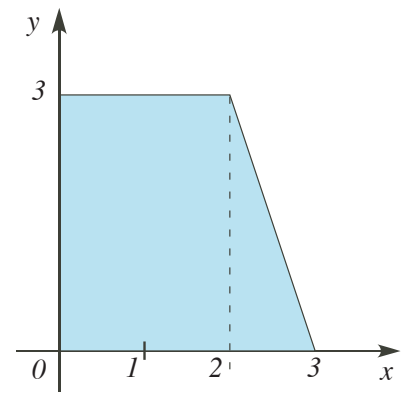
Ans: 13. $\iint_R \rho(x, y)dA = \int_0^{10} \int_0^{2\pi} (r^2 + 2r^2 \sin^2 \theta) r d\theta dr = 10000\pi$

12. R is a trapezoidal metal plate as shown with dimensions in meters. If the mass density of the plate is

$$\rho(x, y) = 6x^2 + 2xy. \quad \text{kg/m}^2$$

find the total mass of the plate.

Ans: 14. $\int_0^3 \int_0^{(9-y)/3} (6x^2 + 2xy)\, dxdy = \frac{489}{4}$ kg.



13. Estimate the value of $\int_1^4 \cos^8 x\, dx$ with (a) midpoint rule with 6 equal sub-interval; (b) trapezoidal rule with 6 equal sub-interval; and (c) Simpson's rule with 6 equal sub-interval. Leave your answer in terms of cosine.

14. Consider the region R bounded by the curves $y = \cos(x)$, $y = -1$, and $y-$axis. Find the volume of each of the following solids:

14a. The solid generated when the region is revolved around the line $y = -2$.

14b. The solid generated when the region is revolved around the line $x = -2$.

14c. The solid whose base is R and the slices perpendicular to the $x-$axis are triangles with height $\sin(x)$.

Ans: 14a. $\int_0^\pi \pi((\cos x + 2)^2 - 1)\, dx$; 14b. $\int_0^\pi 2\pi(x + 2)(\cos x + 1)\, dx$; 14c. $\int_0^\pi \frac{1}{2}(\cos x + 1) \sin x\, dx$

Math 10360: Calculus B
Exam II sample
October 15, 2035

Name: _____

Class Time: _____

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

_____

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	a	b	c	d	e
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6.	a	b	c	d	e
7.	a	b	c	d	e
8.	a	b	c	d	e
9.	a	b	c	d	e
10.	a	b	c	d	e

Please do NOT write in this box.

Multiple Choice	_____
11.	_____
12.	_____
13.	_____
14.	_____
Total	_____

Name: _____

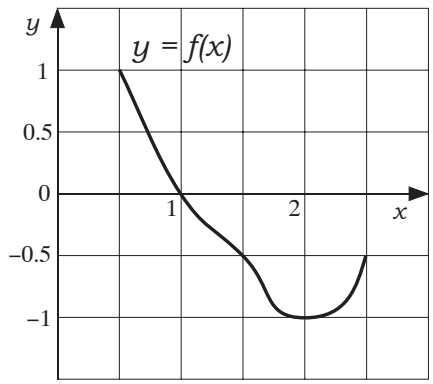
Class Time: _____

Multiple Choice

1.(5 pts.) The graph of $f(x)$ is given below. Estimate the value of

$$\int_{0.5}^{2.5} f(x)\,dx$$

using **Trapezoidal Rule** with **four** equal subintervals (that is T_4).



(a) $-3/4$

(b) $-1/2$

(c) $-5/8$

(d) $-1/4$

(e) $-9/8$

2.(5 pts.) Consider the region R enclosed between $y = \cos(x)$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ and the $x-$axis. Find the volume of the solid generated when R is rotated about the axis $y = -\pi$

(a) $2\pi^2$

(b) $2\pi^2(1-\pi)^2$

(c) $(\pi - 1)^2$

(d) $\frac{\pi^2}{2}$

(e) $\frac{9\pi^2}{2}$

Name: _____

Class Time: _____

3.(5 pts.) The rate of change of temperature Q (in ^{o}C) in a freezer is given by

$$\frac{dQ}{dt} = \sin^2\left(\frac{t}{2}\right).$$

What is the total change in the temperature over the time interval $\left[\frac{\pi}{2}, \pi\right]$?

(a) $\left(\frac{\pi}{4} + \frac{1}{2}\right)$ ^{o}C

(b) $\frac{1}{2}$ ^{o}C

(c) $\left(\frac{\pi}{4} - \frac{1}{2}\right)$ ^{o}C

(d) $-\frac{\pi}{4}$ ^{o}C

(e) $\frac{\pi}{4}$ ^{o}C

4.(5 pts.) Evaluate the integral $\displaystyle\int_0^1 \frac{1}{(x+1)(x+2)}\,dx$

(a) $\ln 3 - \ln 2$

(b) $\ln 4$

(c) $\ln 6 - \ln 2$

(d) $2\ln 2 - \ln 3$

(e) $\ln 2 - \ln 6$

Name: _____

Class Time: _____

5.(5 pts.) Find the area under the graph of $y = 2xe^{-x^2}$ over the interval $[2, \infty)$.

(a) $\frac{4}{3}e^{-3}$

(b) e^{-4}

(c) $8e^{-4}$

(d) $2e^{-4}$

(e) ∞

6.(5 pts.) Evaluate $\int_0^1 \frac{e^x}{e^x - 1} dx$.

(a) 1

(b) 0

(c) $\frac{1}{(e-1)^2}$

(d) $\ln(e-1)$

(e) Divergent

Name: _____

Class Time: _____

7.(5 pts.) Using Polar Coordinates, evaluate the following integral:

$$\int_0^2 \int_{-\sqrt{4-y^2}}^0 y \, dx \, dy.$$

(a) $\frac{4}{3}$

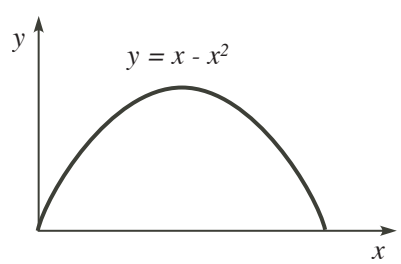
(b) $\frac{8}{3}$

(c) $\frac{1}{2}$

(d) 2

(e) 1

8.(5 pts.) A city is built in the region bounded by $y = x - x^2$ and the $x-$axis. If the population density of the city is given by $f(x, y) = 6x^2y$ thousand per sq. km, find the total population of the city.



(a) 1/30

(b) 16/15

(c) 71/35

(d) 1/25

(e) 1/35

Name: _____

Class Time: _____

9.(5 pts.) Water is flowing into a vessel at a rate of $\dfrac{5}{2t+3}$ cubic feet per minute. If the vessel is empty at time $t = 0$, find the volume (in ft^3) $V(t)$ of water in the tank at any t.

(a) $V(t) = \dfrac{5}{2}\ln|2t+3| - \dfrac{5}{2}\ln(3)$

(b) $V(t) = 5\ln|2t+3|$

(c) $V(t) = 5\ln|2t+3| - 5\ln(3)$

(d) $V(t) = \dfrac{5}{18} - \dfrac{5}{2(2t+3)^2}$

(e) $V(t) = \dfrac{5}{2}\ln|2t+3|$

10.(5 pts.) Let (x, y) denote the usual rectangular coordinates. Write the following function $f(x, y)$ in polar coordinates (r, θ).

$$f(x, y) = 2x^2 + 2y^2 + \frac{x}{y}.$$

(a) $f(r, \theta) = 2r^2 + \tan\theta.$

(b) $f(r, \theta) = r^2\cos(2\theta) + \cot\theta.$

(c) $f(r, \theta) = 2r^2 + \cot\theta.$

(d) $f(r, \theta) = 2r + \tan\theta.$

(e) $f(r, \theta) = 2\sqrt{r} + \cot\theta.$

Name: _____

Class Time: _____

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(12 pts.) The slope of the graph of $f(x)$ is given by

$$\arctan(2x).$$

Find the formula for $f(x)$ if its graph passes through the point $\left(\frac{1}{2}, 0\right)$

Name: _____

Class Time: _____

12.(12 pts.)

A1. Give the **limit** definition of the following improper integral:

$$\int_0^8 \frac{1}{x^{2/3}}\,dx \stackrel{?}{=} \underline{\qquad\qquad\qquad\qquad}$$

A2. Evaluate the improper integral below showing clearly how you use limits:

$$\int_0^8 \frac{1}{x^{2/3}}\,dx$$

B. (Not Related To Above) Write out the form of the partial fraction decomposition of the following rational functions. **You DO NOT need to solve for the coefficients.**

$$\frac{x+1}{(x+2)^2(x^2+1)} \stackrel{?}{=}$$

C. (Not Related To Above) Find the partial decomposition of the following rational functions. <u>Solve for all coefficients.</u>

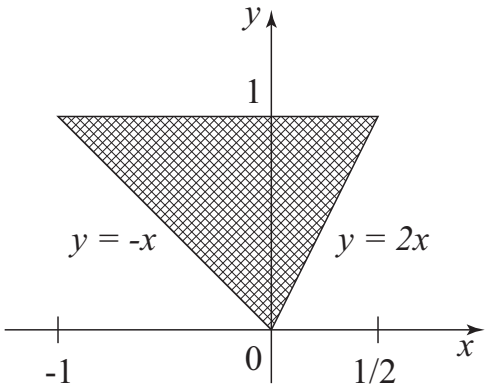
$$\frac{x^2+4}{x^2-4} \stackrel{?}{=}$$

Name: _____

Class Time: _____

13.(12 pts.) The Region Q in the $xy-$plane is given below. Evaluate the following double integral over Q

$$\iint_Q (8x + 8y)\, dA$$



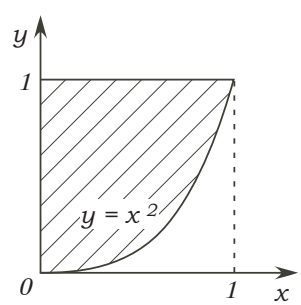
Name: _____

Class Time: _____

14.(12 pts.) **[Part A].** Find the total mass of the quarter disc sitting in the first quadrant with radius 2 and mass density is given by

$$\rho(x, y) = 4e^{2x^2+2y^2} + y \quad \text{kg/m}^2.$$

[Part b] Not Related to the Above. Find the total mass of the shaded region shown below if the mass density of the region is given by $\rho(x, y) = 4xe^{2y} \quad \text{kg/m}^2$.



Name: _____

Class Time: _____

You may find these identiies helpful in the test:

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A\sin B = -\frac{1}{2}\left[\cos(A+B) - \cos(A-B)\right]$$

Math 10360: Calculus B
Exam II sample
October 15, 2035

Name: _____

Class Time: ANSWERS

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- No calculators.
- The exam lasts for one hour and 15 minutes.
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_____

Good Luck!

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1.	a	b	•	d	e
2.	a	b	c	d	•
3.	•	b	c	d	e
4.	a	b	c	•	e
5.	a	•	c	d	e
6.	a	b	c	d	•
7.	a	•	c	d	e
8.	a	b	c	d	•
9.	•	b	c	d	e
10.	a	b	•	d	e

Please do NOT write in this box.

Multiple Choice _____

11. _____

12. _____

13. _____

14. _____

Total _____

Math 10360: Calculus B
Exam II
March 18, 2020

Name: _____

Class Time: _____

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4.	a	b	c	d	e
5.	a	b	c	d	e
6.	a	b	c	d	e
7.	a	b	c	d	e
8.	a	b	c	d	e
9.	a	b	c	d	e
10.	a	b	c	d	e

Please do NOT write in this box.

Multiple Choice	_____
11.	_____
12.	_____
13.	_____
14.	_____
Total	_____

Name: _____

Class Time: _____

Multiple Choice

1.(5 pts.) Find the total change for the function of $f(x)$ if $f'(x) = \cos(3x)\cos(x)$ over the interval $[0, \pi/4]$.

(a) 0

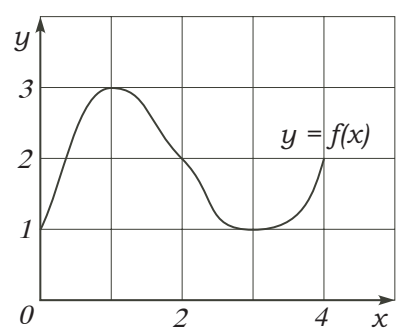
(b) $\frac{1}{4}$

(c) $\frac{1}{2}$

(d) $-\frac{1}{4}$

(e) $-\frac{1}{2}$

2.(5 pts.) Estimate the area under the graph of $y = f(x)$ over $[0, 4]$ using **Simpson's Rule** with **four** equal subintervals.



(a) 9

(b) $25/3$

(c) $25/2$

(d) $15/2$

(e) $23/3$

Name: _____

Class Time: _____

3.(5 pts.) Find y in terms of t if

$$\ln(1-y) - \ln(2-y) = t.$$

(a) $y = \dfrac{1-e^t}{2-e^t}$.

(b) $y = \dfrac{1+e^t}{1+2e^t}$.

(c) $y = \dfrac{2-e^t}{1-e^t}$.

(d) $y = \dfrac{1-2e^t}{1-e^t}$.

(e) $y = \dfrac{1+2e^t}{1+e^t}$.

4.(5 pts.) The population density of a sector of a city area is given by

$$\rho(r,\theta) = \frac{\sin\theta}{\sqrt{4-r^2}} \text{ thousand per sq. km}$$

where $0 \le r \le 1$ and $\dfrac{\pi}{4} \le \theta \le \dfrac{3\pi}{4}$. Find the total population in thousands.

(a) $\sqrt{2} \cdot \arctan(1/2)$

(b) $\dfrac{\pi\sqrt{2}}{12}$

(c) $\dfrac{\pi\sqrt{2}}{4}$

(d) $\dfrac{\pi\sqrt{2}}{6}$

(e) $\sqrt{2}(2-\sqrt{3})$

Name: _____

Class Time: _____

5.(5 pts.) Evaluate the integral $\int_0^{\pi/4} \sec^2(x)\tan^2(x)\,dx$

(a) $\sqrt{2}$

(b) $\frac{1}{3}$

(c) 1

(d) $\frac{\pi^3}{192}$

(e) $\frac{2\sqrt{2}}{3}$

6.(5 pts.) A vessel in a fountain is drained or filled with water such that its rate of change of volume V is given by

$$\frac{dV}{dt} = \sin^3(t) \quad \text{ft}^3/\text{min}.$$

What is the total change in volume over the time interval $[0, \pi/2]$ (in minutes)?

(a) 1 ft^3

(b) $\frac{1}{3}$ ft^3

(c) $\frac{2}{3}$ ft^3

(d) $\frac{1}{4}$ ft^3

(e) $-\frac{1}{4}$ ft^3

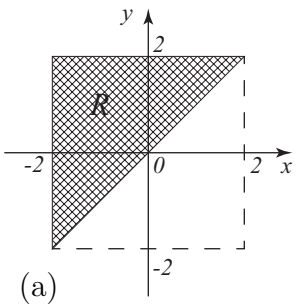
Name: _____

Class Time: _____

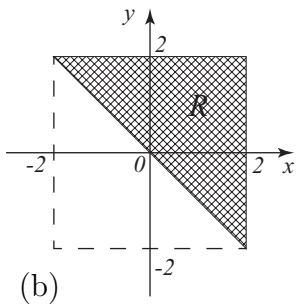
7.(5 pts.) Consider the region R given by the inequalities

$$-2 \leq y \leq 2$$
$$-2 \leq x \leq y$$

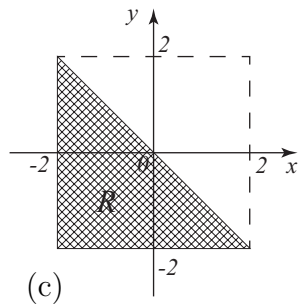
Which one of the figures below best depict the region R?



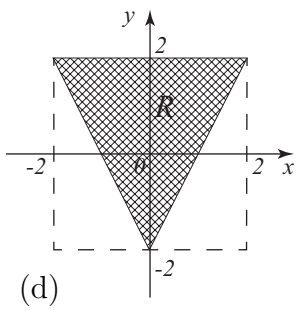
(a)



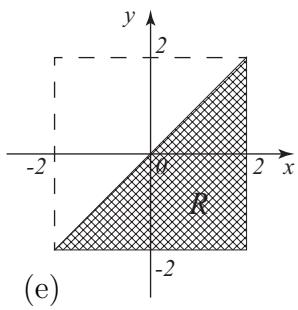
(b)



(c)



(d)



(e)

8.(5 pts.) Find the area under the graph of $y = \dfrac{4}{1+x^2}$ over the interval $[1, \infty)$.

(a) 2π

(b) 3π

(c) π

(d) $\pi/2$

(e) ∞

Name: _____

Class Time: _____

9.(5 pts.) Evaluate $\int_0^1 \frac{4}{x^2-4}dx$.

(a) $-\ln(3)$

(b) $2\ln(2) - \ln(3)$

(c) Divergent

(d) $\frac{\ln(4)}{\ln(3)}$

(e) $4\ln(3) - 4\ln(4)$

10.(5 pts.) Evaluate $\int_0^3 \frac{1}{x-1}dx$.

(a) 2

(b) $\ln(2)$

(c) 1

(d) 0

(e) Divergent

Name: _____

Class Time: _____

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(12 pts.) Consider a 20 meter long chain laying at the foot of a 50 meter building with non-uniform mass density $\rho(y) = e^{-2y}$ kg for $0 \leq y \leq 20$. Find the work done to lift the chain at its lighter end so that this end is at the top of the building and the rest of chain dangles freely.

Name: _____

Class Time: _____

12.(12 pts.) Perform the integral below.

$$\int \frac{x^4 + 2x^3 + 2}{x^3 + x^2}\, dx$$

Name: _____

Class Time: _____

13.(12 pts.) **Part A.** Solve the following initial value problem:

$$\frac{dy}{dx} = \frac{2x + \sin x}{e^y}; \qquad y(0) = 0.$$

Part B. (Not related to above). Give the limit definition of the following improper integral:

$$\int_1^\infty \frac{2}{x^3}\,dx \stackrel{?}{=} \underline{\hspace{6cm}}$$

Part C. Evaluate the improper integral below showing clearly how you use limits:

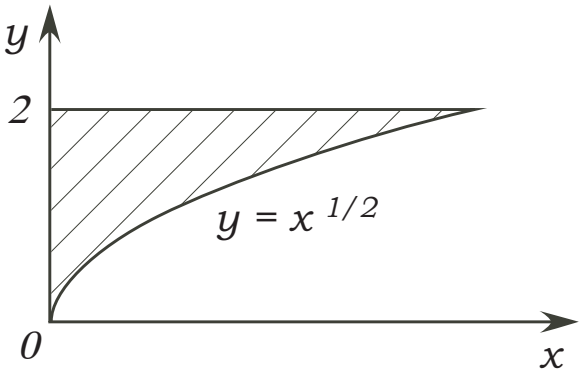
$$\int_1^\infty \frac{2}{x^3}\,dx$$

Name: _____

Class Time: _____

14.(12 pts.) **(Part A).** R is the shaded region as shown. Evaluate the following double integral showing all your steps.

$$\iint_R ye^x \, dA$$



(Part B) Not related to the above. Evaluate the following double integral. Hint: You may use any coordinate system.

$$\int_0^2 \int_0^{\sqrt{4-x^2}} (x^2 + x + y^2) \, dy \, dx =$$

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You may find these formulae helpful in the test:

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A\sin B = -\frac{1}{2}\left[\cos(A+B) - \cos(A-B)\right]$$

Math 10360: Calculus B
Exam II
March 18, 2020

Name: _____

Class Time: ANSWERS

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for one hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

_____

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	a	●	c	d	e
2.	a	b	c	d	●
3.	a	b	c	●	e
4.	a	b	c	d	●
5.	a	●	c	d	e
6.	a	b	●	d	e
7.	●	b	c	d	e
8.	a	b	●	d	e
9.	●	b	c	d	e
10.	a	b	c	d	●

Please do NOT write in this box.

Multiple Choice	_____
11.	_____
12.	_____
13.	_____
14.	_____
Total	_____

Name: _____

Class Time: _____

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(12 pts.) The slope of the graph of $f(x)$ is given by

$$\arctan(2x).$$

Find the formula for $f(x)$ if its graph passes through the point $\left(\frac{1}{2}, 0\right)$

$$f'(x) = \arctan(2x) \Rightarrow f(x) = \int \underbrace{\arctan(2x)}_{u} \underbrace{dx}_{dv}$$

$$u = \arctan(2x) \Rightarrow du = \frac{1}{1+(2x)^2} \cdot 2\,dx$$

$$dv = dx \Rightarrow v = x$$

$$f(x) = x\arctan(2x) - \int x \cdot \frac{2}{1+4x^2}\,dx$$

$$= x\arctan(2x) - \underbrace{\int \frac{2x}{1+4x^2}\,dx}$$

$$= x\arctan(2x) - \int \frac{2}{w} \cdot \frac{1}{8}\,dw$$

$$= x\arctan(2x) - \frac{1}{4}\ln|w| + C$$

$$= x\arctan(2x) - \frac{1}{4}\ln|1+4x^2| + C$$

$$w = 1+4x^2$$
$$dw = 8x\,dx$$
$$x\,dx = \frac{1}{8}dw$$

$$f\left(\frac{1}{2}\right) = 0 = \frac{1}{2}\arctan(1) - \frac{1}{4}\ln\left(1+\frac{4}{4}\right) + C$$

$$\Rightarrow C = -\frac{\pi}{8} + \frac{1}{4}\ln 2$$

$$f(x) = x\arctan(2x) - \frac{1}{4}\ln(1+4x^2) - \frac{\pi}{8} + \frac{1}{4}\ln(2)$$

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12.(12 pts.)

A1. Give the **limit** definition of the following improper integral:

$$\int_0^8 \frac{1}{x^{2/3}}\,dx \stackrel{?}{=} \lim_{a\to 0^+} \int_a^8 \frac{1}{x^{2/3}}\,dx \stackrel{\text{OR}}{=} \lim_{a\to 0^+} \int_a^8 x^{-2/3}\,dx$$

A2. Evaluate the improper integral below showing clearly how you use limits:

$$\int_0^8 \frac{1}{x^{2/3}}\,dx = \lim_{a\to 0^+} \left[3x^{1/3}\right]_a^8 = \lim_{a\to 0^+} \left[3(8)^{1/3} - 3a^{1/3}\right]$$

$$= 3(2) = 6$$

B. (Not Related To Above) Write out the form of the partial fraction decomposition of the following rational functions. **You DO NOT need to solve for the coefficients.**

$$\frac{x+1}{(x+2)^2(x^2+1)} \stackrel{?}{=} \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{Cx+D}{x^2+1}$$

C. (Not Related To Above) Find the partial decomposition of the following rational functions. Solve for all coefficients.

$$\frac{x^2+4}{x^2-4} \stackrel{?}{=} 1 + \frac{8}{x^2-4}$$

$$\begin{array}{r} 1 \\ x^2-4 \,\overline{)\, x^2+4} \\ -)\; x^2-4 \\ \hline 8 \end{array}$$

$$\frac{8}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$8 = A(x+2) + B(x-2)$$

$x=2$: $8 = 4A \Rightarrow A = 2$

$x=-2$: $8 = -4B \Rightarrow B = -2$

$$\frac{x^2+4}{x^2-4} = 1 + \frac{2}{x-2} - \frac{2}{x+2}.$$

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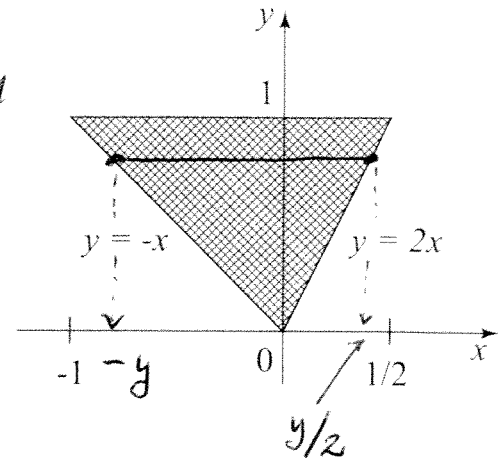
13. (12 pts.) The Region Q in the $xy-$plane is given below. Evaluate the following double integral over Q

$$\iint_Q (8x+8y)\,dA = \int_0^1 \int_{-y}^{\frac{1}{2}y} (8x+8y)\,dx\,dy$$



$$= \int_0^1 \left[4x^2 + 8xy\right]_{-y}^{\frac{1}{2}y} dy$$

$$= \int_0^1 \left[y^2 + \cancel{4y^2} - \cancel{4y^2} + 8y^2\right] dy$$

$$= \left[\frac{9y^3}{3}\right]_0^1 = \left[3y^3\right]_0^1 = 3$$

14 [Part A]. Find the total mass of the quarter disc sitting in the first quadrant with radius 2 and mass density is given by

$$\rho(x,y) = 4e^{2x^2+2y^2} + y \quad \text{kg/m}^2.$$

Convert to Polar coordinates first:

$$\text{Quarter disc } Q: \quad 0 \le r \le 2; \qquad 0 \le \theta \le \pi/2$$

$$\rho(x,y) = 4e^{2x^2+2y^2} + y = 4e^{2(x^2+y^2)} + y$$

$$\rho(r,\theta) = 4e^{2r^2} + r\sin\theta$$

$$\text{Mass} = \iint_Q \rho(x,y)\, dA = \iint_Q 4e^{2x^2+2y^2} + y\, dA = \int_{r=0}^{2}\int_{\theta=0}^{\pi/2} \left(4e^{2r^2} + r\sin\theta\right)\, r\, d\theta\, dr$$

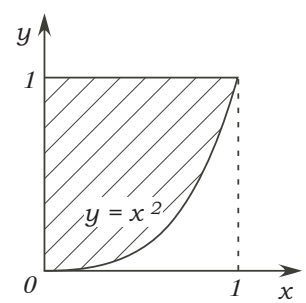
$$= \int_{r=0}^{2}\int_{\theta=0}^{\pi/2} \left(4e^{2r^2} + r\sin\theta\right)\, r\, d\theta\, dr = \int_{r=0}^{2}\int_{\theta=0}^{\pi/2} \left(4re^{2r^2} + r^2\sin\theta\right)\, d\theta\, dr$$

$$= \int_{r=0}^{2} \left[4re^{2r^2}\theta - r^2\cos\theta\right]_0^{\pi/2}\, d\theta\, dr = \int_0^2 \left[4re^{2r^2}\frac{\pi}{2} - r^2\cos\frac{\pi}{2} - (0 - r^2 cos(0))\right]\, dr$$

$$= \int_0^2 \left[2\pi re^{2r^2} + r^2\right]\, dr = \int_0^2 2\pi re^{2r^2}\, dr + \int_0^2 r^2\, dr = \int_0^8 \pi e^u\, \frac{1}{2}du + \left[\frac{r^3}{3}\right]_0^2$$

$$= \left[\frac{\pi e^u}{2}\right]_0^8 + \frac{8}{3} = \left(\frac{\pi e^8}{2} - \frac{\pi}{2} + \frac{8}{3}\right) \text{ kg}$$

[Part b] Not Related to the Above. Find the total mass of the shaded region shown below if the mass density of the region is given by $\rho(x,y) = 4xe^{2y} \quad \text{kg/m}^2$.



$$\text{Mass} = \iint_R \rho(x,y)\, dA = \int_0^1\int_{x^2}^1 4xe^{2y}\, dy\, dx = \int_0^1 \left[2xe^{2y}\right]_{x^2}^1\, dx$$

$$= \int_0^1 \left[2xe^2 - 2xe^{2x^2}\right]\, dx = [x^2e^2]_0^1 - \int_0^1 2xe^{2x^2}\, dx = e^2 - 0 - \int_0^2 \frac{1}{2}e^u\, du$$

$$= e^2 - \frac{1}{2}e^2 + \frac{1}{2} = \left(\frac{1}{2}e^2 + \frac{1}{2}\right) \text{ kg}$$

Name: _____

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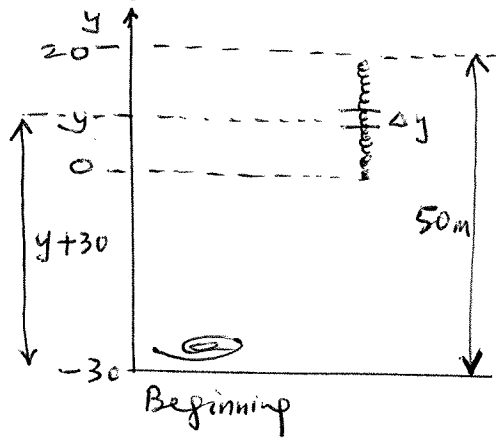
Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(12 pts.) Consider a 20 meter long chain laying at the foot of a 50 meter building with non-uniform mass density $\rho(y) = e^{-2y}$ kg for $0 \le y \le 20$. Find the work done to lift the chain at its lighter end so that this end is at the top of the building and the rest of chain dangles freely.



work done to lift the segment at level y from the ground

$\Delta W \approx$ Force $\times$ Displ.

$=$ (weight of the segment) $\times$ Displ.

$= \left(\begin{matrix}\text{weight density}\\ \text{at } y\end{matrix}\right)\left(\begin{matrix}\text{length}\\ \text{of segment}\end{matrix}\right) \times$ Displ.

$= (\rho(y) \cdot g)(\Delta y)(y+30)$

$= g(y+30)\rho(y)\Delta y = g(y+30)e^{-2y}\Delta y.$

$$W = \int_0^{20} g(y+30)e^{-2y}\,dy = g\int_0^{20} \underbrace{(y+30)}_{u}\,\underbrace{e^{-2y}\,dy}_{dv}$$

$u = y+30 \Rightarrow du = dy$

$dv = e^{-2y}dy \Rightarrow v = \frac{e^{-2y}}{-2}$

$$= g\left[\left[\frac{(y+30)}{-2}e^{-2y}\right]_0^{20} - \int_0^{20} \frac{-e^{-2y}}{2}\,dy\right]$$

$$= g\left[\frac{-50}{2}e^{-40} + \frac{30}{2}e^{0} + \left[\frac{e^{-2y}}{-4}\right]_0^{20}\right]$$

$$= g\left[-25e^{-40} + 15 - \frac{1}{4}e^{-40} + \frac{1}{4}\right]$$

$$= g\left[\frac{61}{4} - \frac{101}{4}e^{-40}\right] = \frac{1}{4}g\left(61 - 101e^{-40}\right) \text{ J.}$$

$10\ m/s^2$

$$= \frac{5}{2}\left(61 - 101e^{-40}\right) \text{ J.}$$

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12.(12 pts.) Perform the integral below.

$$\int \frac{x^4+2x^3+2}{x^3+x^2}\,dx$$

$$\begin{array}{r|l} & x+1 \\ \hline x^3+x^2 & x^4+2x^3+2 \\ & -)\ x^4+x^3 \\ \hline & x^3+2 \\ & -)\ x^3+x^2 \\ \hline & -x^2+2 \end{array}$$

$$\frac{x^4+2x^3+2}{x^3+x^2} = x+1 + \underbrace{\frac{2-x^2}{x^3+x^2}}_{\text{proper}}$$

$$\frac{2-x^2}{x^3+x^2} = \frac{2-x^2}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$2-x^2 = Ax(x+1) + B(x+1) + Cx^2$$

$x=0$: $2 = B$; $x=-1$: $2-(+1) = C \Rightarrow C = 1$

$$2-x^2 = Ax(x+1) + 2(x+1) + x^2$$
$$= Ax^2 + Ax + 2x + 2 + x^2$$
$$= (A+1)x^2 + (A+2)x + 2$$

Comparing the coeff. of x^2:

$-1 = A+1 \Rightarrow A = -2$

$$\int \frac{x^4+2x^3+2}{x^3+x^2}\,dx = \int \left(x+1 - \frac{2}{x} + \frac{2}{x^2} + \frac{1}{x+1}\right)dx$$

$$= \frac{1}{2}x^2 + x - 2\ln|x| - \frac{2}{x} + \ln|x+1| + C$$

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13.(12 pts.) **Part A.** Solve the following initial value problem:

$$\frac{dy}{dx} = \frac{2x + \sin x}{e^y}; \qquad y(0) = 0.$$

$$\int e^y dy = \int (2x + \sin x)\, dx$$

$$e^y = x^2 - \cos x + C$$

$$y(0) = 0 \Rightarrow e^0 = 0 - \cos(0) + C \Rightarrow 1 = -1 + C$$

$$\Rightarrow C = 2$$

$$e^y = x^2 - \cos x + 2$$

$$y = \ln(x^2 - \cos x + 2)$$

Part B. (Not related to above). Give the limit definition of the following improper integral:

$$\int_1^\infty \frac{2}{x^3}\, dx \stackrel{?}{=} \underline{\lim_{A \to \infty} \int_1^A \frac{2}{x^3}\, dx}$$

Part C. Evaluate the improper integral below showing clearly how you use limits:

$$\int_1^\infty \frac{2}{x^3}\, dx = \lim_{A \to \infty} \left[\frac{-2}{2x^2} \right]_1^A = \lim_{A \to \infty} \left[-\frac{1}{A^2} + 1 \right]$$

$$= 1 \text{ (convergent improper integral)}$$

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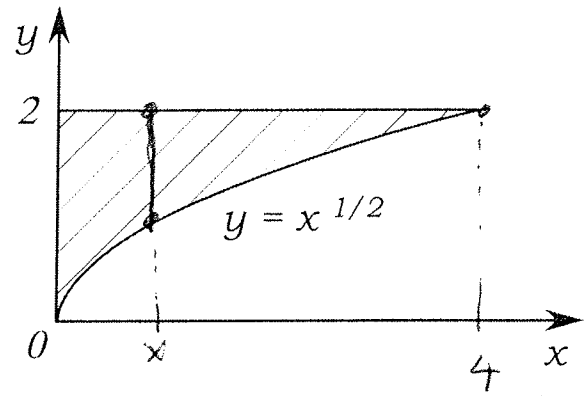
14.(12 pts.) **(Part A).** R is the shaded region as shown. Evaluate the following double integral showing all your steps.

$$\iint_R ye^x \, dA$$



$$= \int_0^4 \int_{\sqrt{x}}^2 ye^x \, dy \, dx$$

$$= \int_0^4 \frac{y^2}{2} e^x \Big|_{\sqrt{x}}^2 \, dx$$

$$= \int_0^4 2e^x - \frac{1}{2} x e^x \, dx$$

Integrate by Parts

$$= 2e^x \Big|_0^4 - \frac{1}{2} \int_0^4 \underbrace{x}_{u} \underbrace{e^x dx}_{dv}$$

$$= (2e^4 - 2) - \frac{1}{2}\left[xe^x \Big|_0^4 - \int_0^4 e^x dx \right]$$

$$= 2e^4 - 2 - \frac{1}{2}(4e^4 - 0) + \frac{1}{2}(e^4 - e^0)$$

$$= 2e^4 - 2 - 2e^4 + \frac{1}{2}e^4 - \frac{1}{2} = \frac{1}{2}e^4 - \frac{5}{2}$$

Using x-first

$0 \le x \le 4$

$\sqrt{x} \le y \le 2$

OR Using y-first

$0 \le y \le 2$

$0 \le x \le y^2$

(Part B). Evaluate the following double integral. Hint: You may use any coordinate system.

$$\int_0^2 \int_0^{\sqrt{4-x^2}} (x^2 + x + y^2) \, dy \, dx = \int_0^2 \int^{\sqrt{4-x^2}} (\underbrace{x^2+y^2}_{r^2} + \underbrace{x}_{r\cos\theta}) \, dy \, dx$$

$0 \le x \le 2$

$0 \le y \le \sqrt{4-x^2}$

$y = \sqrt{4-x^2}$

$y^2 = 4 - x^2$

$x^2 + y^2 = 4$

$$= \int_0^2 \int_0^{\pi/2} (r^2 + r\cos\theta) \cdot r \, d\theta \, dr$$

$$= \int_0^2 \int_0^{\pi/2} (r^3 + r^2\cos\theta) \, d\theta \, dr$$

$$= \int_0^2 \left[r^3\theta + r^2 \sin\theta \right]_{\theta=0}^{\pi/2} dr$$

$$= \int_0^2 \left(\frac{\pi}{2} r^3 + r^2 \sin\frac{\pi}{2} - 0 \right) dr = \int_0^2 \frac{\pi}{2} r^3 + r^2 \, dr$$

$$= \left[\frac{\pi}{8} r^4 + \frac{r^3}{3} \right]_0^2 = 2\pi + \frac{8}{3}$$