Math 10360 – Final Review

1. Find the equation of the tangent line to the curve $y = \ln\left(\dfrac{5x^2-1}{x^2+3}\right)$ at $x = 1$.

2. The density of a circular disc of 10 cm is given by the function in polar coordinates $\rho(r,\theta) = \dfrac{2\theta}{\sqrt{400-r^2}}$ gram/cm^2. Find the total mass of the disc.

3. Find the equation of the tangent line to the graph of the function $f(x) = e^x + x^e$ at $x = 1$.

4. It was observed that a radioactive substance reduces its mass from 10 grams to 2 grams after 3 hours. (a) Find a formula for its mass $y(t)$ after t hours, (b) What is the half-life of the radioactive substance?

Answer: (a) $y(t) = 10e^{\frac{t}{3}\ln(0.2)} = 10(0.2)^{t/3}$; (b) Half-life $= \dfrac{3\ln(0.5)}{\ln(0.2)}$

5. Consider the region bounded by $y = 1 - x, \quad y = e^x, \quad x = 1$.

(a) Find the area of the region.

(b) Find the volume of the solid obtained by rotating the region about (i) $x = 1$, (ii) about $y = -1$.

(c) Consider the solid whose base the given region. If the cross-sections perpendicular to the x-axis are semi-circles.

6. A colony of wasps is building an underground nest. They start by finding an **empty** hole and begin constructing cells at a rate of

$$\frac{dy}{dt} = e^{-0.5y}$$

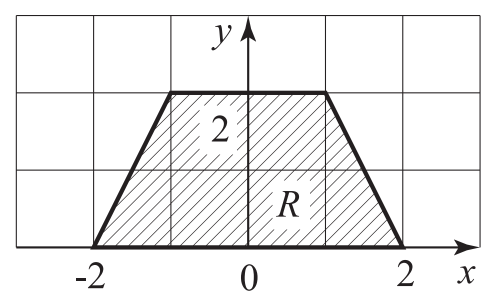
in tens of cells per day. Find the number of cells y in terms of t. Assume that $y(0) = 0$

7. An open tank has the shape of a right circular cone is filled with a fluid of weight-density 900 kg/m^3. The tank is 8 meters across the top and 6 meters high. How much work is done in emptying the tank by pumping the water over the top edge? Take the acceleration due to gravity as 10m/s^2.
Hint: Similar triangles.

8. Parametrize the shaded region R as a single region. Evaluate the following integral

$$\iint_R (x + y^2)\,dA$$

9. Consider a solid whose base is the region R. If the slice perpendicular to the $y-$axis are semicircles, find the volume of the solid.



10. Let $y(t)$ be the solution to the initial value problem $y' = y^3 - 2y, \quad y(-1) = 1$.
(a) Find $T_3(t)$, the 3rd-degree Taylor polynomial of $y(t)$ about -1. (Ans: $1 - (t+1) - \frac{1}{2}(t+1)^2 + \frac{5}{6}(t+1)^3$)
(b) Use (a) to approximate $y(-1.1)$ (Ans: $T_3(-1.1) = 1.09$)

11a. Solve the following initial value problems: $\dfrac{dy}{dx} = xy^2 + x; \qquad y(0) = 1.$

11b. Find the equation of $y = f(x)$ if its slope is given by $4\sin^2(x)$ and that it passes through the point $(\pi, 5)$.

12. Assuming that the pattern continues, which of the following series are geometric series? Find the sum of the geometric series that converges.

(a) $\dfrac{0.5}{(0.03)^2} + \dfrac{0.5}{(0.03)^3} + \dfrac{0.5}{(0.03)^4} + \cdots$ (Ans: Divergent geometric series)

(b) $5 + 6(0.03) + 7(0.03)^2 + 8(0.03)^3 + 9(0.03)^4 + \cdots$ (Ans: Not geometric)

(c) $5 - 5\left(\frac{e}{\pi}\right) + 5\left(\frac{e}{\pi}\right)^2 - 5\left(\frac{e}{\pi}\right)^3 + 5\left(\frac{e}{\pi}\right)^4 - \cdots$ (Ans: $\dfrac{5}{1+\frac{e}{\pi}} = \dfrac{5\pi}{\pi+e}$)

(d) $2 + \dfrac{4}{e} + \dfrac{8}{e^2} + \dfrac{16}{e^3} + \dfrac{32}{e^4} + \cdots$ (Ans: $\dfrac{2}{1-2/e}$)

13a. The velocity of a particle moving on a straight line is given by $v(t) = \dfrac{1}{6-5t+t^2}$ for $t > 3$. What is the total change in the position of the particle over the time interval $4 < t < 5$.

13b. Find the Nth partial sum of the series $\displaystyle\sum_{n=4}^{\infty} \frac{1}{6-5n+n^2}$. Hence find the sum of the series.

14. Evaluate the following indefinite integrals:

a. $\displaystyle\int e^{2x} - 2e^{-x} + e + x^{3/2}\,dx$

b. $\displaystyle\int \cos(4x)\cos(5x)\,dx$

c. $\displaystyle\int \frac{\arcsin(2x)}{\sqrt{1-4x^2}}\,dx$

d. $\displaystyle\int \cos^2(2x)\,dx$

e. $\displaystyle\int_0^{\sqrt{3}} \frac{x}{\sqrt{9-x^2}}\,dx$

f. $\displaystyle\int \sin^5(2x)\,dx$

g. $\displaystyle\int xe^{3x}\,dx$

h. $\displaystyle\int_1^3 x^2 \ln x\,dx$

i. $\displaystyle\int \frac{x+3}{2x^2-3x+1}\,dx$

15a. The Maclaurin series of e^x is given as follows: $\displaystyle e^x = \sum_{k=0}^{\infty} \frac{1}{k!}x^k$ for all real number x.

(i) Find the 3rd Taylor polynomial of e^x at 0 and estimate $e^{0.2}$. (Ans: 1.22133)
(ii) Write down the error of your estimate in Part (b) as an infinite series.
(iii) Use the Maclaurin polynomial for $e^{-x^2/2}$ of degree 4 to estimate $\int_0^1 e^{-x^2/2}dx$. (Ans: 0.858333)
(iv) Find the Taylor series for e^{2x-2} centered at 1. Be sure to simplify the coefficients.

15b. Use (i) Trapezoidal rule, (ii) Simpson's rule to estimate $\int_0^1 e^{-x^2/2}dx$.

16. Explain why the integral $\displaystyle\int_0^1 \frac{x\,dx}{\sqrt[3]{1-x^2}}$ is improper. Evaluate the integral.

17. The 4th-degree polynomial of the function $f(x)$ about -2 is given by

$$P_4(x) = 5 - 2(x+2) + 3(x+2)^2 - (x+2)^4.$$

(a) What is the slope of $f(x)$ at $x = -2$? Find the equation of the tangent line to the graph of $f(x)$ at $x = -2$. Give your answer in the form $y = mx + b$. (Ans: $y = -2x + 1$)
(b) Write down the values of $f(-2)$, $f''(-2)$, $f'''(-2)$, and $f^{(4)}(-2)$.
(Ans: $f(-2) = 5$, $f''(-2) = 6$, $f'''(-2) = 0$, and $f^{(4)}(-2) = -24$)

18. A chain 10 meter long weighs 4 kg per meter with a 20 kg pounds load attached at the end is hung from a platform 15 meter above the ground. How much work is required to lift the whole chain and the load to the top of the platform?

19. A tank has 100 liter of solution containing 4 kg of salt. A mixture with 0.01kg of salt per liter of water is added at a rate of 0.2 liters per minute. At the same time, 0.1 liter of well-stirred mixture are removed each minute. Find the amount of salt in the tank as a function of time.

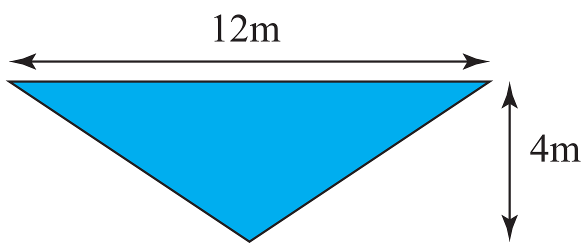
20. Consider the following initial value problem

$$\frac{dy}{dt} = t - y^2; \qquad\qquad y(1) = 1.$$

Use Euler's method with **three equal steps** to estimate $y(1.3)$.

21. Suppose a patient take 5 milligrams of a certain drug at the beginning of each day. The drug is eliminated from the patient's system in such a way that a single dose will only have 80% remaining in the patient's system after one day. (a) By the end of the third day, the patient has taken a total of three doses. How many milligrams remain in his system just before he takes the next dose? (b) Assuming the treatment is continued indefinitely, use an infinite geometric series to approximate the amount remaining in the patient's system after a very long period if measurement is take before the next dose.

22. A 20 m long tank has uniform cross-section as show below. Suppose the tank is completely filled with a liquid with weight density 800 kg/m^3. Find the work required to pump all the liquid over the top of the tank. You may take the acceleration due to gravity as $g = 10\text{m/sec}^2$.



23. Find the values of x for which the following power series is (a) convergent and (b) divergent. Write down the radius of convergence. Ignore end-point behavior.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 x^n}{5^n}$ **b.** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{(2n)!}$ **c.** $\displaystyle\sum_{n=1}^{\infty} \frac{5(x-3)^n}{2^n}$

24. Solve the equation: $\displaystyle\frac{dy}{dt} = y + t$

Name: ______________________________

Math 10360: Calculus B, Final Exam
May 6, 2029

Class Time: __________________________

- Be sure that you have all 15 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
- **The entire set of exam except the formula sheet has to be turn in.**
- **Sign the pledge.** "On my honor, I have neither given nor received unauthorized aid on this Exam":

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Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	a	b	c	d	e	14.	a	b	c	d	e
2.	a	b	c	d	e	15.	a	b	c	d	e
3.	a	b	c	d	e	16.	a	b	c	d	e
4.	a	b	c	d	e	17.	a	b	c	d	e
5.	a	b	c	d	e	18.	a	b	c	d	e
6.	a	b	c	d	e	19.	a	b	c	d	e
7.	a	b	c	d	e	20.	a	b	c	d	e
8.	a	b	c	d	e	21.	a	b	c	d	e
9.	a	b	c	d	e	22.	a	b	c	d	e
10.	a	b	c	d	e	23.	a	b	c	d	e
11.	a	b	c	d	e	24.	a	b	c	d	e
12.	a	b	c	d	e	25.	a	b	c	d	e
13.	a	b	c	d	e						

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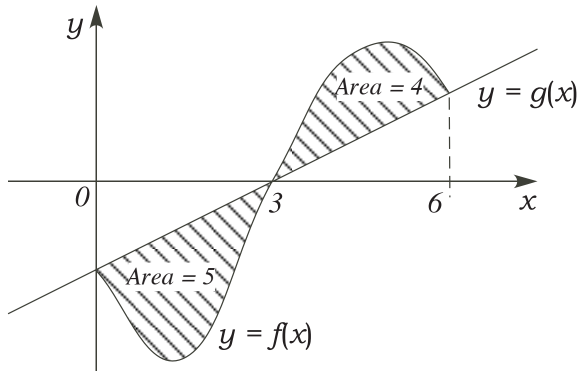
Multiple Choice

1.(6 pts.) The graphs of the functions $f(x)$ (curve line) and $g(x)$ (straight line) are shown below. What is the value of $\int_0^6 [f(x) - g(x)]\,dx$?



(a) -9

(b) 9

(c) -1

(d) $-1/6$

(e) 1

2.(6 pts.) Find the equation of the tangent line at $x = 1$ to the graph of the function

$$f(x) = 5e^{x^2-1}$$

(a) $y - 1 = 10(x - 5)$

(b) $y - 5 = 10xe^{x^2-1}(x - 1)$

(c) $y + 5 = 10xe^{x^2-1}(x + 1)$

(d) $y + 5 = 10(x + 1)$

(e) $y - 5 = 10(x - 1)$

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3.(6 pts.) Applying the integration by parts formula to the definite integral $\int_1^e t^3 \ln t \, dt$ with $u = \ln t$ and $dv = t^3 dt$ gives us the expression:

(a) $\dfrac{e^4}{4} - \displaystyle\int_1^e \frac{t^3}{4} dt$

(b) None of these.

(c) $\dfrac{e^3}{3} - \displaystyle\int_0^1 \frac{1}{3} e^{3t} dt$

(d) $\dfrac{e^5+4}{5} - \displaystyle\int_1^e t^4 \ln t \, dt$

(e) $\displaystyle\int_0^1 u e^{3u} du$

4.(6 pts.) Find the partial fraction decomposition of the function: $\dfrac{x+1}{x^2+x-2}$.

(a) $\dfrac{3}{2(x+1)} + \dfrac{3}{(x-2)}$

(b) $\dfrac{x+1}{x^2+x-2}$

(c) $\dfrac{3}{2(x-1)} + \dfrac{3}{(x+2)}$

(d) $\dfrac{2}{3(x-1)} + \dfrac{1}{3(x+2)}$

(e) $\dfrac{2}{3(x+1)} + \dfrac{1}{3(x-2)}$

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5.(6 pts.) The Taylor series for the function $\arctan(1-x)$ centered at 1 is:

$$\sum_{k=0}^{\infty}(-1)^{k+1}\frac{(x-1)^{2k+1}}{2k+1}.$$

Use the 3rd Taylor polynomial for $\arctan(1-x)$ centered at 1 to estimate the value of $\arctan(0.8)$

(a) $(0.8) - \dfrac{(0.8)^3}{3}$

(b) $(0.2) - \dfrac{(0.2)^3}{3} + \dfrac{(0.2)^5}{5}$

(c) $(0.8) - \dfrac{(0.8)^3}{6} + \dfrac{(0.8)^5}{120}$

(d) $(0.8) - \dfrac{(0.8)^3}{3} + \dfrac{(0.8)^5}{5}$

(e) $(0.2) - \dfrac{(0.2)^3}{3}$

6.(6 pts.) Using summation notation, write the **ERROR** for the estimate in the above problem as an infinite sum.

(a) $\displaystyle\sum_{k=3}^{\infty}(-1)^{k}\frac{(0.8)^{2k+1}}{2k+1}$

(b) $\displaystyle\sum_{k=3}^{\infty}(-1)^{k}\frac{(0.2)^{2k+1}}{2k+1}$

(c) $\displaystyle\sum_{k=4}^{\infty}(-1)^{k}\frac{(0.2)^{2k+1}}{2k+1}$

(d) $\displaystyle\sum_{k=2}^{\infty}(-1)^{k}\frac{(0.8)^{2k+1}}{2k+1}$

(e) $\displaystyle\sum_{k=2}^{\infty}(-1)^{k}\frac{(0.2)^{2k+1}}{2k+1}$

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7.(6 pts.) Convert the following integral to an integral with variable θ using the substitution $x = 2\sin\theta$.

$$\int_0^2 \frac{1}{x^2\sqrt{4-x^2}}\,dx$$

(a) $\displaystyle\int_0^2 \frac{1}{8\sin^2\theta\cos\theta}\,d\theta$

(b) $\displaystyle\int_0^{\pi} \frac{\cos\theta}{4\sin^2\theta}\,d\theta$

(c) $\displaystyle\int_0^{\pi/2} \frac{\csc^2\theta}{4}\,d\theta$

(d) $\displaystyle\int_0^{\pi/2} \frac{1}{8\sin^2\theta\cos\theta}\,d\theta$

(e) $\displaystyle\int_0^2 \frac{\csc^2\theta}{4}\,d\theta$

8.(6 pts.) Solve for x in terms of t if

$$\ln(x-1) - \ln(x+1) = 2t.$$

(a) $x = \dfrac{-e^t - 2}{e^t - 2}$

(b) $x = \dfrac{-e^{2t} - 1}{e^{2t} - 1}$

(c) $x = \dfrac{e^{2t} - 1}{e^{2t} + 1}$

(d) $x = \dfrac{e^t + 2}{e^t - 2}$

(e) $x = \dfrac{e^{2t} + 1}{e^{2t} - 1}$

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9.(6 pts.) Find all values of x for which the series $\sum_{n=1}^{\infty} 2\left(\frac{x+2}{4}\right)^n$ is convergent.

(a) $-4 < x < 0$

(b) $-2 < x < 6$

(c) $x < 2$

(d) $x < 0$

(e) $-6 < x < 2$

10.(6 pts.) Find the sum of the series: $\sum_{n=3}^{\infty} (\ln(n) - \ln(n+1))$.

(a) 1

(b) 0

(c) $\ln 2$

(d) $-\infty$

(e) $\ln 3$

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11.(6 pts.) Perform the integral:

$$\int \frac{\sqrt{x}-3x}{x^2}\,dx$$

(a) $\dfrac{\dfrac{2x^{3/2}}{3}-\dfrac{3x^2}{2}}{\dfrac{x^3}{3}}+C$

(b) $\dfrac{\dfrac{2x^{3/2}}{3}-\dfrac{3x^2}{2}+C}{\dfrac{x^3}{3}+C}$

(c) $\dfrac{\dfrac{x^{-1/2}}{2}-3}{2x}+C$

(d) $-\dfrac{3}{2}x^{-5/2}-3x^{-2}+C$

(e) $-2x^{-1/2}-3\ln|x|+C$

12.(6 pts.) Consider the volume of solid formed when the region enclosed by the graphs of $y=2x$ and $y=x^2$ is revolved about the x-axis. If you use **WASHERS method** to find this volume, which of the following integrals will you obtain?

(a) $\displaystyle\int_0^4 2\pi y(-\sqrt{y}+y)\,dy.$

(b) $\displaystyle\int_0^2 \pi(-x-x^2)^2\,dx.$

(c) $\displaystyle\int_0^2 \pi(4x^2-x^4)\,dx.$

(d) $\displaystyle\int_0^4 \pi(-x-x^2)^2\,dx.$

(e) $\displaystyle\int_0^4 2\pi(x^4-4x^2)\,dx.$

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13.(6 pts.) Suppose that the 6th Taylor polynomial for $f(x)$ at 2 is

$$P_6(x) = 3 - 2(x-2)^3 - (x-2)^4 + (x-2)^6.$$

What is the value of $f^{(4)}(2)$?

(a) -12

(b) -24

(c) -720

(d) 24

(e) 720

14.(6 pts.) Consider the initial value problem:

$$y' = 3x + y, \qquad y(0) = 1.$$

Using Euler's method with **TWO** steps of equal size estimate $y(0.2)$

(a) 1.424

(b) 1.24

(c) 2

(d) 1.1

(e) 4.3

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15.(6 pts.) Consider the initial value problem:

$$y' = x\sin^2(y); \qquad y(-1) = \frac{\pi}{4}$$

Find the 2nd Taylor polynomial for $y(x)$ centered at -1.

(a) $T_2(x) = \frac{\pi}{4} - \frac{1}{4}(x-1) + \frac{1}{6}(x-1)^2$

(b) $T_2(x) = \frac{\pi}{4} - \frac{1}{2}x + \frac{1}{2}x^2$

(c) $T_2(x) = \frac{\pi}{4} - \frac{1}{4}(x+1) + \frac{1}{6}(x+1)^2$

(d) $T_2(x) = \frac{\pi}{4} - \frac{1}{2}(x+1) + \frac{1}{2}(x+1)^2$

(e) $T_2(x) = \frac{\pi}{4} - \frac{1}{2}(x-1) + \frac{1}{2}(x-1)^2$

16.(6 pts.) Compute the limit: $\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x$.

(a) 2

(b) -2

(c) e^2

(d) e

(e) Does not exist.

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17.(6 pts.) Find the sum of the **first 10 terms** of the geometric series $\sum_{n=2}^{\infty} 5(-2)^n$.

(a) $\dfrac{5(1-2^{10})}{3}$

(b) $\dfrac{20}{3}$

(c) $\dfrac{20(1-2^{10})}{3}$

(d) $\dfrac{5(1+2^{11})}{3}$

(e) $\dfrac{20(1+2^{11})}{3}$

18.(6 pts.) A rope 5 meter long weighs 2 kg per meter is hung from a platform 10 meters above the ground. How much work is required to lift the whole rope to the top of the platform? You may assume that the acceleration due to gravity is 10m/s^2.

(a) 125 J

(b) 750 J

(c) 500 J

(d) 1000 J

(e) 250 J

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19.(6 pts.) Perform the integration $\int \frac{\sec^2 x}{\tan x}\,dx$.

(a) $\ln|\tan x| + C$

(b) $\tan x + C$

(c) $-\frac{1}{\tan^2 x} + C$

(d) $-\frac{1}{\cos x \sin x} + C$

(e) $-\frac{\tan x}{\ln|\cos x|} + C$

20.(6 pts.) A patient is injected once a day with 10 units of a certain drug. Suppose this drug is eliminated by the body exponentially with any single injection leaving an amount of $10e^{-0.5t}$ remaining after t days. Approximate the number of units of the drug remaining in the patient's system after a very long time **if the measurement is done before the next dose is given.**

(a) $\frac{10e^{-0.5}}{1-10e^{-0.5}}$

(b) $\frac{10e^{-0.5}}{1+e^{-0.5}}$

(c) $\frac{10}{1+e^{-0.5}}$

(d) $\frac{10}{1-e^{-0.5}}$

(e) $\frac{10e^{-0.5}}{1-e^{-0.5}}$

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21.(6 pts.) Consider the initial value problem:

$$y' = ye^{y-2}; \qquad y(1) = 2$$

Find the 2nd Taylor polynomial for $y(t)$ centered at 1.

(a) $P_2(t) = 2 + 2(t-1) + 6(t-1)^2$

(b) $P_2(t) = 2 + 2(t+1) + 6(t+1)^2$

(c) $P_2(t) = 2 + 2(t+1) + 3(t+1)^2$

(d) $P_2(t) = 2 + 2t + 3t^2$

(e) $P_2(t) = 2 + 2(t-1) + 3(t-1)^2$

22.(6 pts.) The rate of change in the height of a person sitting on a ferris wheel measured from the ground is given by

$$h'(t) = 4\sin^2 t$$

What is the **total change** in the height of the person over the time interval $0 \le t \le \frac{\pi}{4}$?

(a) $\frac{\pi}{4} + \frac{1}{2}$

(b) $\frac{\pi}{4} - \frac{1}{2}$

(c) $\frac{\sqrt{2}}{3} - \frac{4}{3}$

(d) $\frac{\pi}{2} - 1$

(e) $\frac{\pi}{2} + 1$

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23.(6 pts.) Solve the initial value problem:

$$\frac{dy}{dx} = 2xy + x; \qquad y(0) = 1$$

(a) $y = \dfrac{3e^{x^2} - 1}{2}$

(b) $y = \dfrac{3e^{x^2/2} - 1}{2} + C$

(c) $y = \dfrac{3e^{x^2} + 1}{4}$

(d) $y = \dfrac{3e^{x^2/2} - 1}{2}$

(e) $y = \dfrac{3e^{x^2/2} + 1}{4}$

24.(6 pts.) Perform the integral $\displaystyle\int \sin(5x)\cos(2x)\,dx$.

(a) $\dfrac{\sin(7x)}{14} + \dfrac{\sin(3x)}{6} + C$

(b) $-\dfrac{\cos(7x)}{14} - \dfrac{\cos(3x)}{6} + C$

(c) $\dfrac{\cos(7x)}{14} + \dfrac{\cos(3x)}{6} + C$

(d) $-\dfrac{\sin(7x)}{14} - \dfrac{\sin(3x)}{6} + C$

(e) $\dfrac{\sin(7x)}{14} - \dfrac{\sin(3x)}{6} + C$

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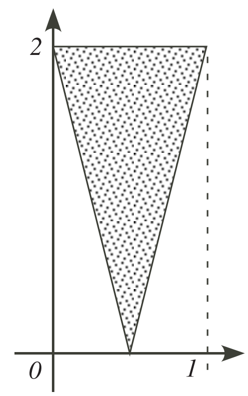
25.(6 pts.) The end of a 20 m tank containing a fluid (weight density 400 kg/m^3) is vertical and has a triangular shape shown below. If all dimensions are in meters, find the amount of need to pump all the fluid over the top of the tank. Take the acceleration due to gravity as 10m/s^2.

(a) 40000 J (b) 160000/3 J (c) 50000 J

(d) 200000/3 J (e) 80000/3 J



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You may find these identiies helpful in the test:

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right]$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A+B)+\cos(A-B)\right]$$

$$\sin A\sin B = -\frac{1}{2}\left[\cos(A+B)-\cos(A-B)\right]$$

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Math 10360: Calculus B, Final Exam Class Time: ANSWERS

May 6, 2029

- Be sure that you have all 15 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
- **The entire set of exam except the formula sheet has to be turn in.**
- **Sign the pledge.** "On my honor, I have neither given nor received unauthorized aid on this Exam":

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Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	a	b	•	d	e
2.	a	b	c	d	•
3.	•	b	c	d	e
4.	a	b	c	•	e
5.	•	b	c	d	e
6.	a	b	c	•	e
7.	a	b	•	d	e
8.	a	•	c	d	e
9.	a	b	c	d	•
10.	a	b	c	•	e
11.	a	b	c	d	•
12.	a	b	•	d	e
13.	a	•	c	d	e

14.	a	•	c	d	e
15.	a	b	c	•	e
16.	a	b	•	d	e
17.	a	b	•	d	e
18.	a	b	c	d	•
19.	•	b	c	d	e
20.	a	b	c	d	•
21.	a	b	c	d	•
22.	a	b	c	•	e
23.	•	b	c	d	e
24.	a	•	c	d	e
25.	a	•	c	d	e

Name: ______________________________

Math 10360: Calc B, Final Exam
December 17, 2029

Class Time: __________________________

- Be sure that you have all 15 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
- You may remove this answer sheet if you prefer. **Return the entire exam** including questions and the answer sheet properly attached.
- **Sign the pledge.** "On my honor, I have neither given nor received unauthorized aid on this Exam":

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Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	a	b	c	d	e	14.	a	b	c	d	e
2.	a	b	c	d	e	15.	a	b	c	d	e
3.	a	b	c	d	e	16.	a	b	c	d	e
4.	a	b	c	d	e	17.	a	b	c	d	e
5.	a	b	c	d	e	18.	a	b	c	d	e
6.	a	b	c	d	e	19.	a	b	c	d	e
7.	a	b	c	d	e	20.	a	b	c	d	e
8.	a	b	c	d	e	21.	a	b	c	d	e
9.	a	b	c	d	e	22.	a	b	c	d	e
10.	a	b	c	d	e	23.	a	b	c	d	e
11.	a	b	c	d	e	24.	a	b	c	d	e
12.	a	b	c	d	e						
13.	a	b	c	d	e						

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Multiple Choice

1.(6 pts.) The population density of a city measured from the city center is given by the radial function

$$\rho(r) = \frac{1}{1+r^2},$$

where r is the distance from the city center measured in kilometers, and ρ is in thousands per km^2. Find the the number of people (in thousand) within a 5km radius from the city center.

(a) $\dfrac{25\pi}{26}$ thousand.

(b) $\tan(26)$ thousand.

(c) $\dfrac{1}{2}\ln(26)$ thousand.

(d) $\pi\ln(26)$ thousand.

(e) $2\pi\tan(26)$ thousand.

2.(6 pts.) Evaluate the integral $\displaystyle\int_1^2 \frac{10}{x(5-x)}dx$.

(a) $10\ln\left(\dfrac{8}{3}\right)$

(b) $\dfrac{1}{5}\ln\left(\dfrac{8}{3}\right)$

(c) $2\ln\left(\dfrac{8}{3}\right)$

(d) $2\ln\left(\dfrac{3}{2}\right)$

(e) $10\ln\left(\dfrac{3}{2}\right)$

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3.(6 pts.) Let $f(x, y) = \ln(3x - 2y)$. Find the value of the limit

$$\lim_{h \to 0} \frac{f(1, 1+h) - f(1, 1)}{h}.$$

(a) -3 (b) 2 (c) Does not exist

(d) 3 (e) -2

4.(6 pts.) Find the Nth partial sum of the infinite series: $\displaystyle\sum_{n=5}^{\infty} \ln\left(\frac{n+1}{n+2}\right)$.

(a) $\ln(6) - \ln(N+2)$

(b) $\ln(6)$

(c) $-\infty$

(d) $\ln(6) - \ln(N+6)$

(e) $-\ln(N+6)$

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5.(6 pts.) The Taylor series for the function e^x centered at 0 is: $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$.

Use the given series to find the Taylor series for the function $e^{(2-x)}$ centered at 2.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}(x-2)^n$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n!}(x-2)^n$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}(2-x)^n$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}(x+2)^n$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}x^n$

6.(6 pts.) Suppose that the 6th Taylor polynomial for $f(x)$ at -1 is

$$P_6(x) = 5 - (x+1) - 3(x+1)^3 + 2(x+1)^5 + (x+1)^6.$$

What is the value of $f^{(5)}(-1)$?

(a) 120

(b) 720

(c) 2

(d) 240

(e) 1

Name: ______________________________

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7.(6 pts.) Consider the initial value problem:

$$y' = (y - t)^2, \qquad y(1) = 0.$$

Using Euler's method, with **TWO**, steps of equal size estimate $y(1.2)$

(a) 1.1

(b) 0.2

(c) 0.1

(d) 0.3

(e) 1.0

8.(6 pts.) Evaluate the integral $\displaystyle\int xe^{3x}\,dx$.

(a) $\dfrac{x}{3}e^{3x} - \dfrac{1}{9}e^{3x} + C$

(b) $xe^{3x} - e^{3x} + C$

(c) $\dfrac{x^2}{6}e^{3x} + C$

(d) $3xe^{3x} - 9e^{3x} + C$

(e) None of these.

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9.(6 pts.) Consider the volume of solid formed when the region enclosed by the graphs of the x-axis, y-axis, $y = e^x$ and $x = 1$ is revolved about the line $x = 1$. If you use **SHELL method** to find this volume, which of the following integrals will you obtain?

(a) $\displaystyle\int_0^e \pi(1 - e^y)^2 \, dy.$

(b) $\displaystyle\int_0^1 2\pi x(1 - e^x) \, dx.$

(c) $\displaystyle\int_0^1 2\pi(1 - x)e^x \, dx.$

(d) $\displaystyle\int_0^1 \pi e^{2x} \, dx.$

(e) $\displaystyle\int_1^e 2\pi y(1 - \ln y) \, dy.$

10.(6 pts.) The rate of change of temperate in a weather simulation chamber is given by

$$r(t) = 20\sin(2t)\sin(t)$$

where r is ^{o}C/hour. What is the total change in temperature over the time duration $0 \le t \le \dfrac{\pi}{2}$?

(a) $\dfrac{10}{3}$ ^{o}C

(b) $\dfrac{40}{3}$ ^{o}C

(c) 0 ^{o}C

(d) $-\dfrac{20}{3}$ ^{o}C

(e) $\dfrac{20}{3}$ ^{o}C

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11.(6 pts.) Convert the following integral to an integral with variable θ using the substitution $x = 3\tan\theta$.

$$\int_0^3 \frac{x^3}{\sqrt{9+x^2}}\,dx$$

(a) $\displaystyle\int_0^3 \frac{9\tan^3\theta}{\sec\theta}\,d\theta$

(b) $\displaystyle\int_0^{\pi/4} \frac{9\tan^3\theta}{\sec\theta}\,d\theta$

(c) $\displaystyle\int_0^3 \frac{9\tan^3\theta}{\csc\theta}\,d\theta$

(d) $\displaystyle\int_0^3 27\tan^3\theta\sec\theta\,d\theta$

(e) $\displaystyle\int_0^{\pi/4} 27\tan^3\theta\sec\theta\,d\theta$

12.(6 pts.) A tank contains 100 gal of syrup with sugar concentration 0.5 lb/gal. The tank is kept well stirred as syrup with sugar concentration 0.1 lb/gal is pumped in at a rate of 2 gal/min. If syrup is pumped out of the tank at rate 4 gal/min, and $x(t)$ denotes the amount of sugar in the tank, in pounds, which one of the following initial value problems models the the amount of sugar in the tank.

(a) $\displaystyle\frac{dx}{dt} = 0.4 - \frac{2x}{100-4t}, \quad x(0) = 50$

(b) $\displaystyle\frac{dx}{dt} = 0.1 - \frac{x}{100+2t}, \quad x(0) = 0.5$

(c) $\displaystyle\frac{dx}{dt} = 0.2 - \frac{4x}{100-2t}, \quad x(0) = 50$

(d) $\displaystyle\frac{dx}{dt} = 0.2 - \frac{4x}{100+2t}, \quad x(0) = 50$

(e) $\displaystyle\frac{dx}{dt} = 0.1 - \frac{x}{100-2t}, \quad x(0) = 0.5$

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13.(6 pts.) Evaluate the following geometric series

$$1 + \frac{\pi}{3} + \frac{\pi^2}{9} + \frac{\pi^3}{27} + \cdots$$

(a) $\dfrac{3}{3-\pi}$

(b) The series is divergent.

(c) $\dfrac{1}{3-\pi}$

(d) $\dfrac{22}{7}$

(e) $\dfrac{3}{3+\pi}$

14.(6 pts.) The series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(x-1)^n}{n+1}$ converges on which of the following interval?.

(a) $-1 < x < 1$

(b) $-\dfrac{1}{2} < x < \dfrac{1}{2}$

(c) $\dfrac{1}{2} < x < \dfrac{3}{2}$

(d) $-\infty < x < \infty$

(e) $x = 1$

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15.(6 pts.) Solve for the general solution of the equation below for $0 < x < \pi/2$:

$$x\frac{dy}{dx} - y = x^2 \sec^2 x$$

(a) $y = -Cx\tan x$.

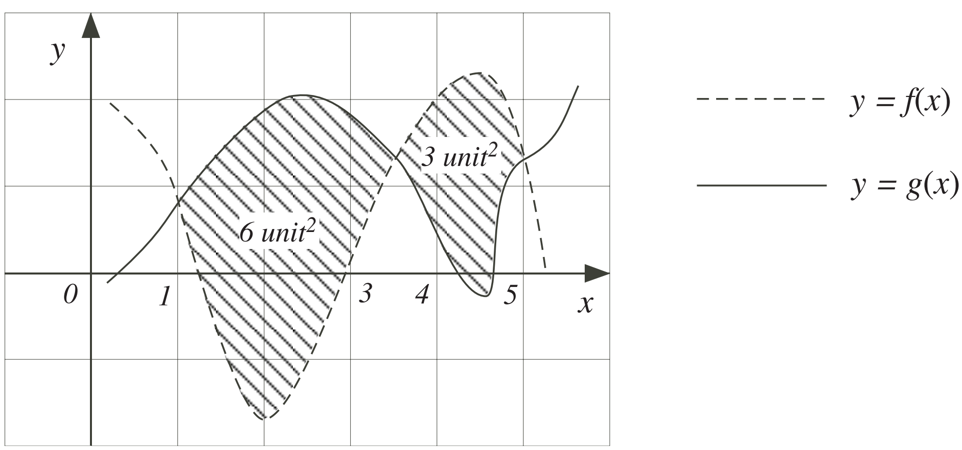
(b) $y = x\sec x\tan x + Cx$.

(c) $y = Cx\tan x$.

(d) $y = -x\tan x + Cx$.

(e) $y = x\tan x + Cx$.

16.(6 pts.) Areas bounded between the curves $y = f(x)$ and $y = g(x)$ over $1 \le x \le 5$ are as given below. Find the value of the definite integral $\int_1^5 [f(x) - g(x)]dx$.



(a) 9

(b) 2

(c) -2

(d) 3

(e) -3

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17.(6 pts.) A patient is injected once a day with 20 units of a certain drug. Suppose only 70% of each dose remains at the end of each day. Using a geometric series, approximate the number of units of the drug remaining in the patient's system after a very long time **if the measurement is done before the next dose is given.**

(a) $\dfrac{20}{1-e^{-0.7}}$

(b) 70 units.

(c) $\dfrac{200}{17}$ units.

(d) $\dfrac{140}{3}$ units.

(e) $\dfrac{140}{17}$ units.

18.(6 pts.) Let (x, y) denote the usual rectangular coordinates. Write the following function $f(x, y)$ in polar coordinates (r, θ).

$$f(x, y) = 2x^2 + 2y^2 + \frac{x}{y}.$$

(a) $f(r, \theta) = 2r^2 + \cot\theta$.

(b) $f(r, \theta) = 2r^2 + \tan\theta$.

(c) $f(r, \theta) = r^2\cos(2\theta) + \cot\theta$.

(d) $f(r, \theta) = 2\sqrt{r} + \cot\theta$.

(e) $f(r, \theta) = 2r + \tan\theta$.

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19.(6 pts.) A 10 meter long uniform chain weighing 30 kg (in total) is hung from the top of a 50 meter building. How much work is required to lift the whole chain to the top of the platform? Take acceleration due to gravity as 10m/s^2.

(a) 1500 J

(b) 37500 J

(c) 3000 J

(d) 5000 J

(e) 15000 J

20.(6 pts.) Perform the integral $\int \cos^3(4x)\,dx$.

(a) $\dfrac{\sin(4x)}{16} - \dfrac{\sin^3(4x)}{48} + C$

(b) $-\dfrac{\sin(4x)}{4} + \dfrac{\sin^3(4x)}{12} + C$

(c) $-\sin(4x) + \dfrac{\sin^3(4x)}{3} + C$

(d) $\sin(4x) - \dfrac{\sin^3(4x)}{3} + C$

(e) $\dfrac{\sin(4x)}{4} - \dfrac{\sin^3(4x)}{12} + C$

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21.(6 pts.) Consider the initial value problem:

$$y' = \sin(y) - x^2; \qquad y(-1) = 0$$

Find the 2nd Taylor polynomial for $y(x)$ centered at -1.

(a) $T_2(x) = -(x-1) + \frac{1}{2}(x-1)^2$

(b) $T_2(x) = -(x+1) + \frac{1}{2}(x+1)^2$

(c) $T_2(x) = -\frac{1}{2}(x-1) + \frac{1}{6}(x-1)^2$

(d) $T_2(x) = -\frac{1}{2}(x+1) + \frac{1}{6}(x+1)^2$

(e) $T_2(x) = -x + \frac{1}{2}x^2$

22.(6 pts.) Using Trapezoidal rule with 4 equal subintervals, estimate the area under the curve $y = f(t)$ over the interval $0 \leq t \leq 4$.

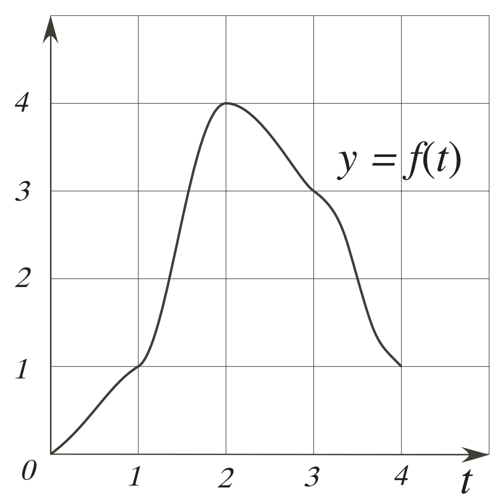
(a) 9

(b) $\frac{17}{2}$

(c) 8

(d) 17

(e) 16



Name: ______________________________

Class Time: __________________________

23.(6 pts.) Solve the initial value problem:

$$\frac{dy}{dx} = \frac{y}{\sqrt{1-x^2}}; \qquad y(0) = 3e$$

(a) $y = e^{\arcsin(x)} + 3e$

(b) $y = e^{\arctan(x)} + 3e$

(c) $y = 3e^{\arctan(x)+1}$

(d) $y = 3e^{-\arccos(x)}$

(e) $y = 3e^{\arcsin(x)+1}$

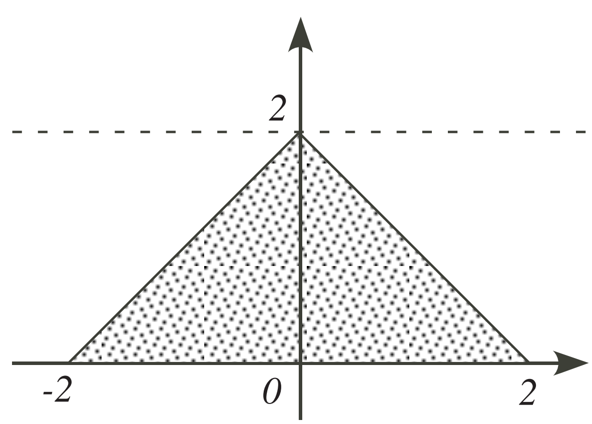
24.(6 pts.) The density of a metal plate as show below is given by

$$\rho(x, y) = y \text{ kg/m}^2.$$

Find the total mass of the plate in kg.

(a) 8 kg. (b) $\frac{16}{3}$ kg. (c) 2 kg.

(d) $\frac{8}{3}$ kg. (e) $\frac{4}{3}$ kg.



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25. (a) Find the Nth partial sum of the series: $\displaystyle\sum_{n=2}^{\infty}\left(\frac{n+1}{n+2}-\frac{n+2}{n+3}\right)$.

(b) What is the sum of the series?

Answers: (a) $\frac{3}{4}-\frac{N+3}{N+4}$; (b) $-\frac{1}{4}$

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You may find these identiies helpful in the test:

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A\sin B = -\frac{1}{2}\left[\cos(A+B) - \cos(A-B)\right]$$

Name: ____________________________

Math 10360: Calc B, Final Exam
December 17, 2029

Class Time: ANSWERS

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6.	a	b	c	•	e	19.	•	b	c	d	e
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8.	•	b	c	d	e	21.	a	•	c	d	e
9.	a	b	•	d	e	22.	a	•	c	d	e
10.	a	•	c	d	e	23.	a	b	c	d	•
11.	a	b	c	d	•	24.	a	b	c	•	e
12.	a	b	•	d	e						
13.	a	•	c	d	e						