Math 10360 – Example Set 02A
Derivative of Exponential & Logarithmic Functions: Section 3.9
Derivative of Inverse Trig Function: Section 5.8

1. By restricting the domain of $\sin x$, $\cos x$, and $\tan x$ define their inverse functions ($\arcsin x$, $\arccos x$, and $\arctan x$). Sketch the graph of each of the inverse functions stating their range and domain.

2. Using chain rule, obtain the derivative of $\arcsin(x)$, $\arccos(x)$, and $\arctan(x)$.

Key Formulas:

$\frac{d}{dx}(\arcsin x) \stackrel{?}{=}$ $\quad$ $\frac{d}{dx}(\arccos x) \stackrel{?}{=}$ $\quad$ $\frac{d}{dx}(\arctan x) \stackrel{?}{=}$

$\int \frac{1}{\sqrt{1-x^2}}\,dx \stackrel{?}{=}$ $\quad$ $\int \frac{1}{1+x^2}\,dx \stackrel{?}{=}$

3. Using the log function, find the derivative of $y = (1+2x)^{\arctan x}$.

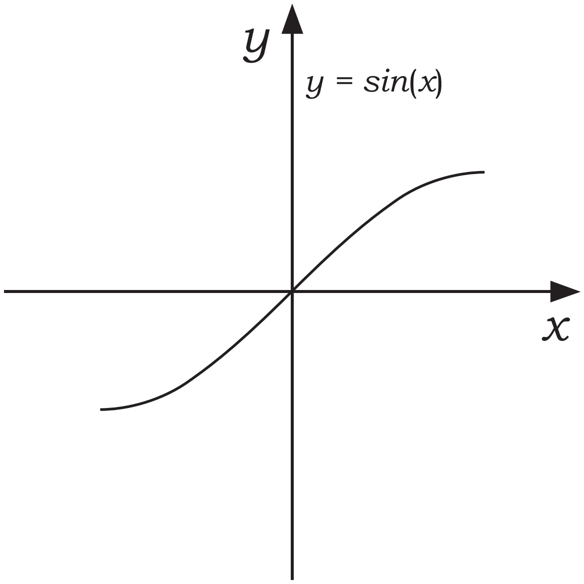
4a. Find the derivative of $\arcsin(2x+y^2)$ with respect to x treating y as a constant.

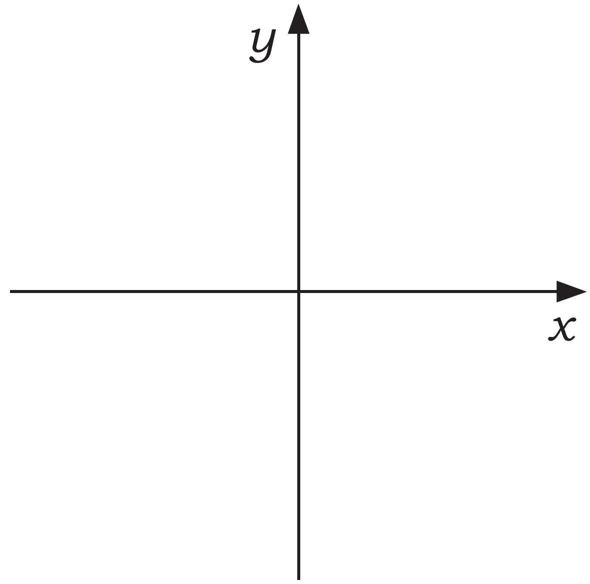
4b. Find the derivative of $\arcsin(2x+y^2)$ with respect to y treating x as a constant.

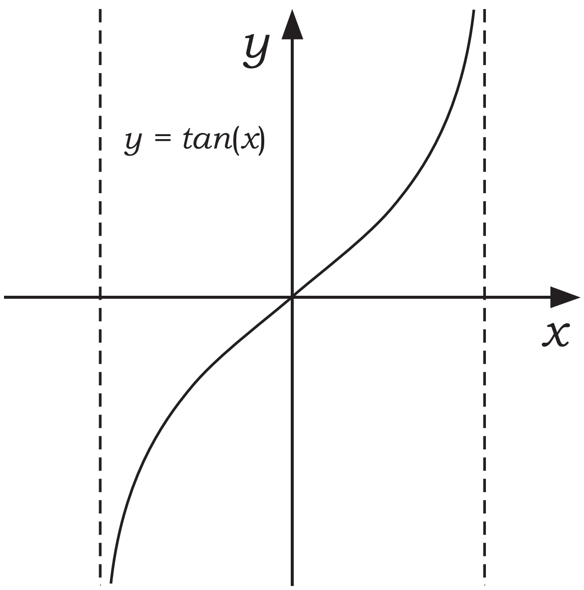
5. A population $y(t)$ (in units of millions) of bacteria grows according to the rate $\frac{dy}{dt} = \frac{1}{1+4t^2}$. Find the total change in the size of the population over the time duration $0 \le t \le 1/2$.

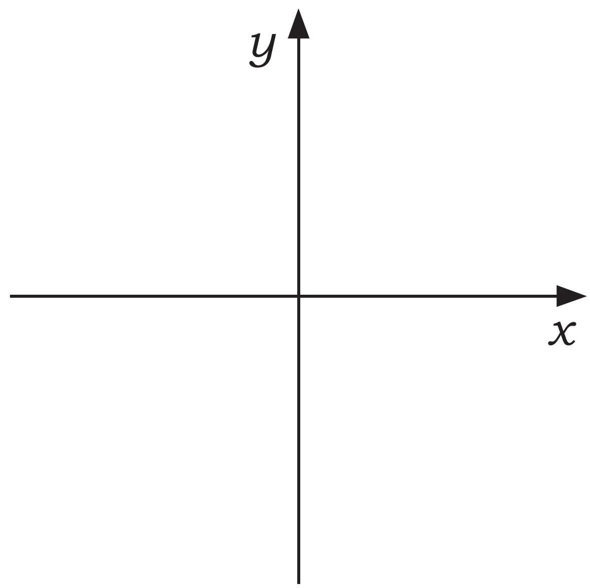
6. (Review) Without using a calculator, find the value of each of the following expressions:

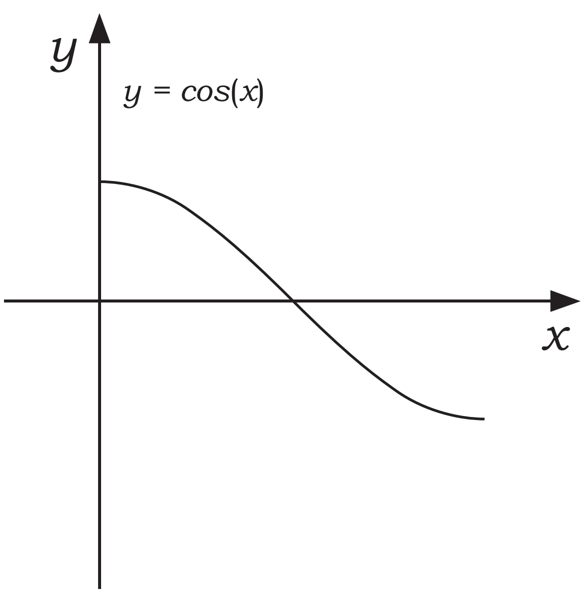
a. $\arcsin(\sqrt{3}/2)$ $\quad$ **b.** $\arcsin(-\sqrt{3}/2)$ $\quad$ **c.** $\arccos(0.5)$ $\quad$ **d.** $\arctan(-1)$

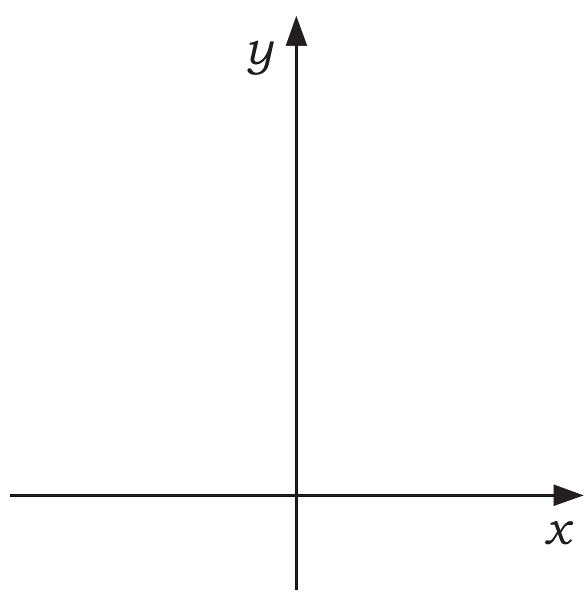

y
y = sin(x)
x


y
x


y
y = tan(x)
x


y
x


y
y = cos(x)
x


y
x

Math 10360 – Example Set 02B
Method of Substitution: Section 5.7
Derivative of Inverse Trig Function: Section 5.8

1. Complete the following important integral formulas:

(i) $\int x^n \, dx \stackrel{?}{=}$ if $n \neq -1$. **(ii)** $\int \frac{1}{x} \, dx = \int x^{-1} \, dx \stackrel{?}{=}$

(iii) $\int \frac{1}{ax+b} \, dx \stackrel{?}{=}$ **(iv)** $\int e^{ax+b} \, dx \stackrel{?}{=}$

(v) $\int \frac{1}{\sqrt{1-x^2}} \, dx \stackrel{?}{=}$ **(vi)** $\int \frac{1}{1+x^2} \, dx \stackrel{?}{=}$

2. If the **slope** at each point of the graph of $f(x)$ is given by

$$\frac{2x+1}{4+x^2}.$$

Find a formula for $f(x)$ if its graph passes through $(2, 0)$.

3. Perform the following integrals:

a. $\int_0^1 \frac{x+2x^3}{1+x^2+x^4} \, dx$ **b.** $\int \frac{1}{\sqrt{4-9x^2}} \, dx$ **c.** $\int \frac{4+x}{\sqrt{1-9x^2}} \, dx$

d. $\int_0^{\ln 2} \frac{e^t}{1+e^{2t}} \, dt$ **e.** $\int \frac{e^{2t}}{1+e^{2t}} \, dt$

Math 10360 – Example Set 02C
Sections 3.9: Application of Exponential and Logarithm Functions

1. The population $P(t)$, at time t, hours of a bacteria is given by $P(t) = 5e^{2t}$ in thousands.

(a) What is the initial population of the bacteria?

(b) Give a formula for the growth rate of the population of the bacteria.

(c) What did you observe about the growth rate?

(d) Explain what is meant by the doubling time for the population. Find this time.

Exponential Growth and Decay (5.8). A quantity y is said to grow or decay exponentially with growth constant k if y satisfies the following differential equation:

$$\frac{dy}{dt} = \underline{\hspace{6cm}}$$

Moreover, if C is the initial value of y, $y(t) =$ ____________________.

Doubling time and Half life.

The **doubling time** of a quantity growing exponentially as time progress is the amount of time needed for

____

The **half life** of a quantity decaying exponentially as time progress is the amount of time needed for

____

2. Recent experiments on viability of the coronavirus indicates that it reduces exponentially on various surfaces. The half life of the coronavirus on glass is estimated to be about 14 hours. (a) Starting with 100% initially, find a formula in the form $A \cdot e^{rt}$ for the percentage of the virus on glass after t hours. (b) If we consider the virus no longer infectious (or viable) after it is reduced to 1% or less, estimate how long will the virus remain infectious on glass. $A = 100, r = -\frac{\ln(2)}{14}$

Reference:
Aerosol and Surface Stability of SARS-CoV-2 as Compared with SARS-CoV-1, N Engl J Med April 2020

Stability of SARS-CoV-2 in different environmental conditions, Lancet April 2020.

3. A cypress beam found in the tomb of Sneferu in Egypt contained 55% of the amount of Carbon-14 found in living cypress wood. Estimate the age of the tomb given that Carbon-14 has a half-life of 5730 years.