Math 10360 – Example Set 05A
Section 7.1 - Integration by Parts

1. A 10m chain with non-uniform linear mass density $\rho(y) = e^y$ kg/m for $0 \leq y \leq 10$ is coiled on the ground is lifted straight up from its heavier end (labelled A) so that End A is 10m above the ground and the rest of the chain dangles free below. Find the work done in lifting the chain. You may take the acceleration due to gravity as $g = 10\text{m/s}^2$.

Integration by Parts

IDEA: Recall that Integration by Substitution "reverses" chain rule. Today we learn another technique, called *integration by parts*, which "reverses" the product rule.

Let $u(x)$ and $v(x)$ be two differentiable functions. Applying product rule, we have:

$$\frac{d}{dx}(u(x)v(x)) = u(x)v'(x) + u'(x)v(x)$$

By definition of anti-derivative:

$$u(x)v(x) = \underline{\hspace{6cm}} = \int u(x)v'(x)\,dx + \int u'(x)v(x)\,dx.$$

Rearranging terms, we have:

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int v(x)u'(x)\,dx$$

Note $\dfrac{du}{dx} = u'(x) \quad \Rightarrow \quad du = \underline{\hspace{2cm}}$. Also $\dfrac{dv}{dx} = v'(x) \quad \Rightarrow \quad dv = \underline{\hspace{2cm}}$.

Suppressing variable x, we get:

$$\boxed{\int u\ dv = \hspace{6cm} .} \rightarrow \textbf{Integration by Parts Formula}$$

As a definite integral, we have:

$$\boxed{\int_a^b u\ dv = \hspace{6cm} .}$$

2. Evaluate the following integrals:

(a) $\int_0^{10} ye^y \, dy$

(b) $\int x^3 \ln x \, dx$

(c) $\int xe^{x^2} \, dx$

(d) $\int \arctan x \, dx$

2e. $\int \sin^4 x \cos x \, dx \stackrel{?}{=}$

Math 10360 – Example Set 05B
Section 7.2 Trigonometric Integrals

1. Use the following identities to complete the blanks below:

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$\sin A\cos B =$ ______________________________

$\cos A\cos B =$ ______________________________

$\sin A\sin B =$ ______________________________

$\sin(2A) =$ ______________________________

The Pythagorean Identities and one of the given identities above, write the following in terms of $\cos(2A)$:

$\cos^2 A =$ ______________________

$\sin^2 A =$ ______________________

2. Using appropriate identities, evaluate the following definite integrals:

a. $\displaystyle\int \sin(2x)\cos(3x)\,dx$ **b.** $\displaystyle\int \cos^2 2z\,dz$ **c.** $\displaystyle\int_0^{\pi/3} \sin^5 x\,dx$

d. $\displaystyle\int_0^{\pi} \sin^4(x)\,dx$ **e.** $\displaystyle\int_0^{\pi/2} \cos(5x)\cos(x)\,dx$

You may find these formulae helpful in the test:

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

$$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$$

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A\sin B = -\frac{1}{2}\left[\cos(A+B) - \cos(A-B)\right]$$

Math 10360 – Example Set 05C
Section 7.5 Partial Fraction

1. Evaluate the integral $\int_5^8 \frac{3x-7}{x^2-5x+6}\,dx$.

2. Determine if the following rational functions proper. If not, apply long division to write the rational expression as a sum of a polynomial and a proper rational function.

a. $\frac{x^2+x+1}{(x+1)(x+4)^2}$

b. $\frac{2x^4+x^3+4x^2+1}{x^3+x}$