

Math 10360 – Example Set 06A
Section 7.5 Partial Fraction

Partial Fraction Decomposition – Definition

Definition. Let $\frac{p(x)}{q(x)}$ be a _____ rational function. The partial fraction decomposition of $\frac{p(x)}{q(x)}$ is a procedure that write $\frac{p(x)}{q(x)}$ as a sum of “simpler” rational expressions in the form $\frac{r(x)}{(s(x))^n}$ where

- (a) $s(x)$ is an _____ of $q(x)$,
- (b) n is an integer from 1 to _____ the factor $s(x)$ occurs in $q(x)$, and
- (c) $r(x)$ is a polynomial of degree _____ the degree of $s(x)$.

Remark: A rational function is a function of the form $\frac{p(x)}{q(x)}$ where both $p(x)$ and $q(x)$ are polynomials.

Here are some examples:

$$\frac{1}{x^2 - 4} \qquad \frac{x - 1}{x^2 - 4} \qquad \frac{x^2 - 1}{x^2 - 4} \qquad \frac{x^3 - 1}{x^2 - 4} \qquad \frac{x^3 - 1}{(x^2 - 4)^2}$$

Can you tell which of the above rational functions are proper?

- | | | |
|-----------------------------------|--------|------------|
| (1) $\frac{1}{x^2 - 4}$ | Proper | Not Proper |
| (2) $\frac{x - 1}{x^2 - 4}$ | Proper | Not Proper |
| (3) $\frac{x^2 - 1}{x^2 - 4}$ | Proper | Not Proper |
| (4) $\frac{x^3 - 1}{x^2 - 4}$ | Proper | Not Proper |
| (5) $\frac{x^3 - 1}{(x^2 - 4)^2}$ | Proper | Not Proper |

Partial Fraction Decomposition – Definition

Give the partial fraction decomposition of each of the rational functions below. (Examples 01 to 03). You do not need to find the coefficients; just give the form of the required partial fraction.

Example 1 $\frac{x^2 + x + 1}{(x + 1)(x + 4)^2} =$

Example 2 $\frac{x^2 + x + 1}{x^3(x - 1)(x^4 - 1)} =$

Example 3. Find the form of the partial fraction decomposition of the following rational function. You do not need to solve for the resulting coefficients.

$$\frac{100x^7 + x^3 + 5000}{(x^2 + 4)^2(x^4 + 5x^3 + 6x^2)(16 - x^4)} =$$

Example 4. Find the partial fraction decomposition of the following rational functions and integrate the rational functions:

a. $\frac{x^2 + x + 1}{(x + 1)(x + 4)^2}$

b. $\frac{2x^4 + x^3 + 4x^2 + 1}{x^3 + x}$

Math 10360 – Example Set 06B
Sections 7.7 Improper Integrals

1. Consider the function $f(x) = \frac{1}{x^{2/3}}$ for $x > 0$. Give a sketch of the graph of $f(x)$.
- a. Give a sketch of the region under the graph of $f(x)$ over the interval $[1, \infty)$. How would you find the area of this region (convergent or divergent)?
- b. Give a sketch of the region under the graph of $f(x)$ over the interval $(0, 1]$. How would you find the area of this region (convergent or divergent)?
- c. There are two types of improper integrals in (a) and (b) above. How would you define an improper integral in general?
2. State whether each of the following integrals are improper or not. Find the values of all improper integrals below, and state whether they are convergent or divergent.

a. $\int_{-1}^1 \frac{1}{x^{2/3}} dx$

b. $\int_{-\infty}^{\infty} \frac{1}{16 + x^2} dx$