

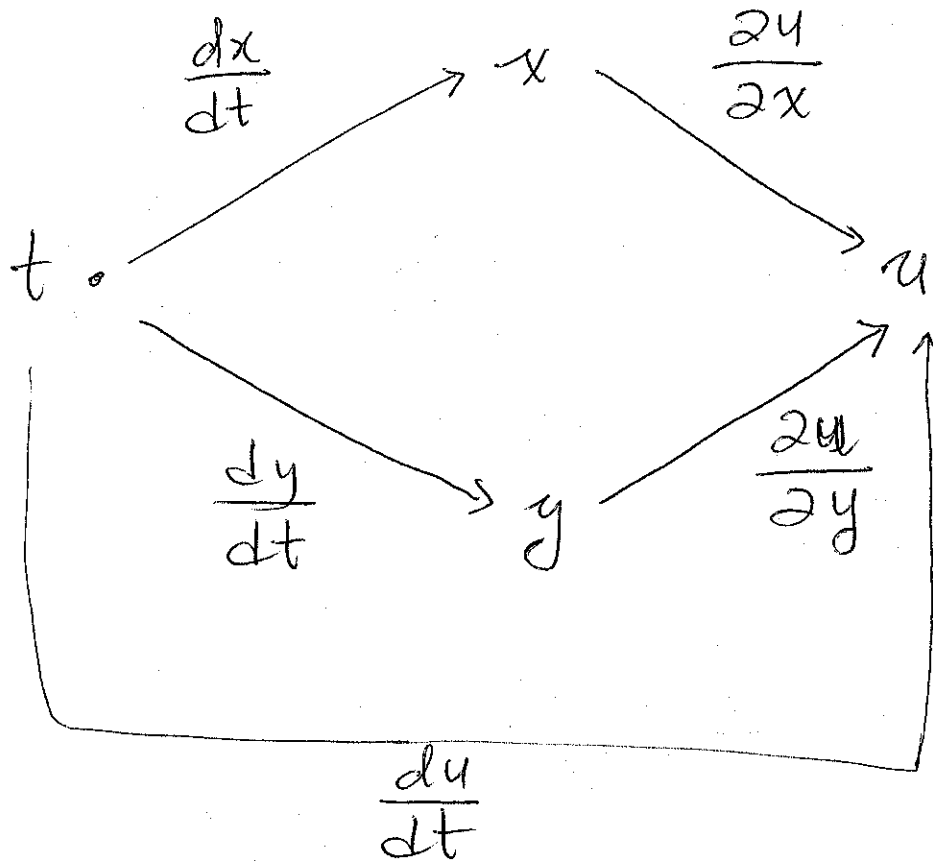
Chain Rule - Example 01
Sections 14.6 Online Text

1. Find a formula for the following derivative by first drawing a tree diagram to connect all related quantities:

$\frac{du}{dt}$ where $u = \ln(x^2 + y^2)$; $x = \cos 2t$ and $y = \sin t$.

$$\frac{du}{dt} = ?$$

$$u = \ln(x^2 + y^2) ; x = \cos 2t ; y = \sin t$$



$$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt}$$

①

$$\frac{\partial u}{\partial x} = \frac{2x}{x^2 + y^2} ; \quad \frac{\partial u}{\partial y} = \frac{2y}{x^2 + y^2}$$

$$\frac{dx}{dt} = -2 \sin 2t ; \quad \frac{dy}{dt} = \cos t$$

$$\frac{du}{dt} = \frac{2x}{x^2+y^2} \cdot (-2\sin 2t) + \frac{2y}{x^2+y^2} (\cos t)$$

$$= \frac{-4x \sin 2t}{x^2+y^2} + \frac{2y \cos t}{x^2+y^2}$$

2

$$\text{Find } z_x = \frac{\partial z}{\partial x} \text{ if } x^3 y z = y^2 + 3x - 2xz^3 - 7$$

$$x^3 y z(x, y) = y^2 + 3x - 2x(z(x, y))^3 - 7$$

$$\frac{\partial}{\partial x} (x^3 y z(x, y)) = \frac{\partial}{\partial x} (y^2 + 3x - 2x(z(x, y))^3 - 7)$$

$$x^3 y \frac{\partial z}{\partial x} + 3x^2 y z$$

$$= 0 + 3 - (2x \cdot 3(z)^2 \frac{\partial z}{\partial x} + 2z^3) - 0$$

$$= 3 - 6xz^2 \frac{\partial z}{\partial x} - 2z^3$$

$$x^3 y \frac{\partial z}{\partial x} + 6xz^2 \frac{\partial z}{\partial x} = 3 - 2z^3 - 3x^2 y z$$

$$(x^3 y + 6xz^2) \frac{\partial z}{\partial x} = 3 - 2z^3 - 3x^2 y z$$

$$\frac{\partial z}{\partial x} = \frac{3 - 2z^3 - 3x^2 y z}{x^3 y + 6xz^2}$$

$$z_x(1, 1, -1) = \frac{\partial z}{\partial x} \Big|_{\substack{x=1 \\ y=1 \\ z=-1}} = \frac{3 - 2(1)^3 - 3(1)^2(1)(-1)}{(1)^3(1) + 6(1)(-1)^2}$$

$$= 1$$

Q2/13A