Chain Rule - Example 01
Sections 14.6 Online Text

1. Find a formula for the following derivative by first drawing a tree diagram to connect all related quantities:

$\dfrac{du}{dt}$ where $u = \ln(x^2 + y^2)$; $x = \cos 2t$ and $y = \sin t$.

$$\frac{du}{dt} = ?$$

$$u = \ln(x^2+y^2) \; ; \; x = \cos 2t \; ; \; y = \sin t$$

$$\begin{tikzcd}
 & x \arrow[dr, "\frac{\partial u}{\partial x}"] & \\
t \arrow[ur, "\frac{dx}{dt}"] \arrow[dr, "\frac{dy}{dt}"'] \arrow[rr, bend right=90, "\frac{du}{dt}"'] & & u \\
 & y \arrow[ur, "\frac{\partial u}{\partial y}"'] &
\end{tikzcd}$$

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dt} \qquad 1$$

$$\frac{\partial u}{\partial x} = \frac{2x}{x^2+y^2} \; ; \quad \frac{\partial u}{\partial y} = \frac{2y}{x^2+y^2}$$

$$\frac{dx}{dt} = -2\sin 2t \; ; \quad \frac{dy}{dt} = \cos t$$

$$\frac{dy}{dt} = \frac{2x}{x^2+y^2} \cdot (-2\sin 2t) + \frac{2y}{x^2+y^2}(\cos t)$$

$$= \frac{-4x\sin 2t}{x^2+y^2} + \frac{2y\cos t}{x^2+y^2}.$$

2

Find $z_x = \frac{\partial z}{\partial x}$ if $x^3yz = y^2 + 3x - 2xz^3 - 7$

$$x^3 y z(x,y) = y^2 + 3x - 2x(z(x,y))^3 - 7$$

$$\frac{\partial}{\partial x}\left(x^3 y z(x,y)\right) = \frac{\partial}{\partial x}\left(y^2 + 3x - 2x(z(x,y))^3 - 7\right)$$

$$x^3 y \frac{\partial z}{\partial x} + 3x^2 y z$$

$$= 0 + 3 - \left(2x \cdot 3(z)^2 \frac{\partial z}{\partial x} + 2z^3\right) - 0$$

$$= 3 - 6xz^2 \frac{\partial z}{\partial x} - 2z^3$$

$$x^3 y \frac{\partial z}{\partial x} + 6xz^2 \frac{\partial z}{\partial x} = 3 - 2z^3 - 3x^2 y z$$

$$(x^3 y + 6xz^2)\frac{\partial z}{\partial x} = 3 - 2x^3 - 3x^2 y z$$

$$\frac{\partial z}{\partial x} = \frac{3 - 2x^3 - 3x^2 y z}{x^3 y + 6xz^2}$$

$$z_x(1,1,-1) = \left.\frac{\partial z}{\partial x}\right|_{\substack{x=1\\ y=1\\ z=-1}} = \frac{3 - 2(1)^3 - 3(1)^2(1)(-1)}{(1)^3(1) + 6(1)(-1)^2}$$

$$= 1$$

Q2 | 13A