Review: Derivative Estimation for a Single Variable smooth function.

There are generally three ways to estimate the derivative of a function $f(x)$ at $x = a$.

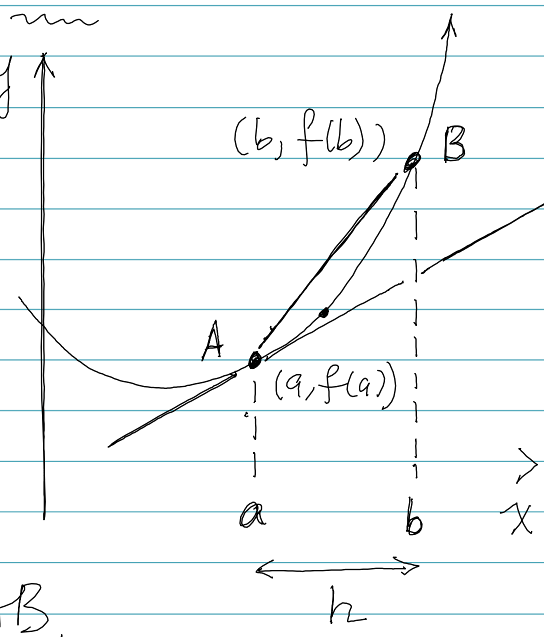
All three ways uses:

(a) points (or values of) x near to a.

(b) secant lines (chords) estimate the tangent line to $f(x)$ at $x = a$.

(1) Forward Difference Estimate.

If B is close to A, then the secant line AB is almost parallel to the tangent line at $x = a$.



So $f'(a) \simeq \underbrace{\dfrac{f(b) - f(a)}{b - a}}_{\text{slope of the secant AB.}}$

slope at $x = a$

This estimate is called the forward difference estimate since b is $> a$.

If we write $b = a+h$

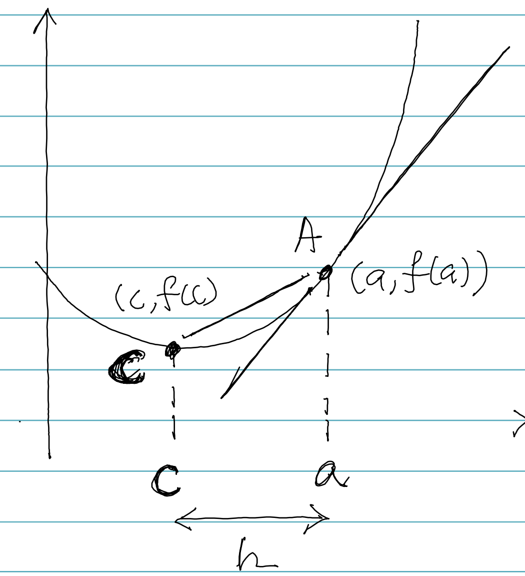
Then $$f'(a) \simeq \frac{f(b)-f(a)}{b-a}$$

$$= \frac{f(a+h)-f(a)}{a+h-a}$$

So $$\underbrace{f'(a) \simeq \frac{f(a+h)-f(a)}{h}}_{\text{forward difference formula.}}$$

(2) Backward Difference Estimate

If C is close to A then the secant line AC is almost parallel to the tangent line of $f(x)$ at $x = a$.



$$\text{So } \underset{\substack{\uparrow \\ \text{slope at} \\ x=a}}{f'(a)} \approx \underbrace{\frac{f(a) - f(c)}{a - c}}_{\text{slope of the secant line } AC.}$$

This estimate is called the backward difference estimate since c is $< a$.

If we write $c = a - h$,

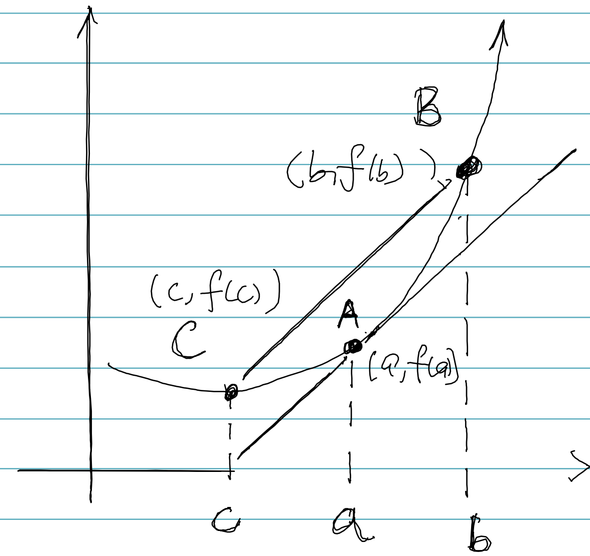
then $f'(a) \approx \dfrac{f(a) - f(c)}{a - c}$

$$= \frac{f(a) - f(a-h)}{a - (a-h)}$$

$$\text{So } f'(a) \approx \underbrace{\frac{f(a) - f(a-h)}{h}}_{\text{backward difference formula}}$$

(3) Central Difference Estimates.

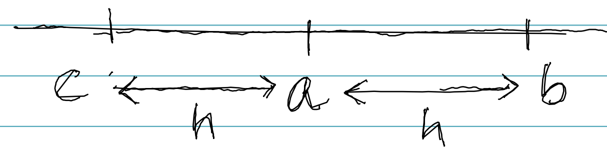
If both B and C are close to A then the slope of the tangent line at $x=a$ is almost the slope of chord BC.



$$\text{So } f'(a) \simeq \underbrace{\frac{f(b)-f(c)}{b-c}}_{\text{slope of chord } BC}$$

This estimate is called the central difference estimate since a is between b and c.

If a is exactly ~~in~~ the mid-point between b and c



Then $b=a+h$ and $c=a-h$

Then we have :

$$f'(a) \simeq \frac{f(b) - f(c)}{b - c} \leftarrow$$

$$= \frac{f(a+h) - f(a-h)}{a+h - (a-h)}$$

$$= \frac{f(a+h) - f(a-h)}{a+h - a + h}$$

$$\underbrace{f'(a) \simeq \frac{f(a+h) - f(a-h)}{2h}}$$

central difference formula

This only works if both $\underline{\underline{b}}$ & $\underline{\underline{c}}$ are <u>equidistance</u> from $\underline{\underline{a}}$

If not we just use the slope of chord BC at the beginning.