Q1. $x^2+y^2 = 4$

$\underbrace{x^2+y^2-4}_{g(x,y)} = 0 \leftarrow$ constraint curve.

$h(x,y) = 2x+4y+20.$

$\frac{\partial h}{\partial x} = 2$; $\frac{\partial h}{\partial y} = 4$

$\frac{\partial g}{\partial x} = 2x$; $\frac{\partial g}{\partial y} = 2y$

Solve the equations:

$$\begin{cases} \frac{\partial h}{\partial x} = \lambda \frac{\partial g}{\partial x} \Rightarrow 2 = \lambda(2x) \Rightarrow 1 = \lambda x & \text{--- (1)} \\ \frac{\partial h}{\partial y} = \lambda \frac{\partial g}{\partial y} \Rightarrow 4 = \lambda(2y) \Rightarrow 2 = \lambda y & \text{--- (2)} \\ x^2+y^2-4 = 0 & \text{--- (3)} \end{cases}$$

From (1): $x = 1/\lambda$ and (2): $y = 2/\lambda$

Substitute into (3): $\frac{1}{\lambda^2} + \frac{4}{\lambda^2} - 4 = 0$

$\Rightarrow \frac{5}{\lambda^2} = 4 \Rightarrow \lambda^2 = \frac{5}{4} \Rightarrow \lambda = \pm\frac{\sqrt{5}}{2}.$ (1)

Case : $\lambda = \sqrt{5}/2$

$$x = \frac{1}{\lambda} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$y = \frac{2}{\lambda} = \frac{4}{\sqrt{5}} = \frac{4\sqrt{5}}{5}$$

$\left(\frac{2\sqrt{5}}{5}, \frac{4\sqrt{5}}{5}\right)$ is a critical point.

Case : $\lambda = -\sqrt{5}/2$

$$x = \frac{1}{\lambda} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$y = \frac{2}{\lambda} = -\frac{4}{\sqrt{5}} = -\frac{4\sqrt{5}}{5}$$

$\left(-\frac{2\sqrt{5}}{5}, -\frac{4\sqrt{5}}{5}\right)$ is another critical point.

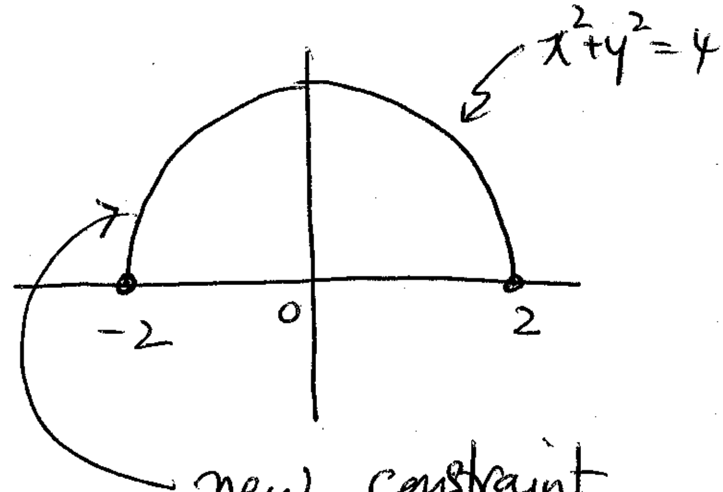
$$h\left(\frac{2\sqrt{5}}{5}, \frac{4\sqrt{5}}{5}\right) = \frac{4\sqrt{5}}{5} + \frac{16\sqrt{5}}{5} + 20$$
$$= 4\sqrt{5} + 20 \text{ m.}$$

$$h\left(\frac{-2\sqrt{5}}{5}, -\frac{4\sqrt{5}}{5}\right) = -\frac{4\sqrt{5}}{5} - \frac{16\sqrt{5}}{5} + 20$$
$$= -4\sqrt{5} + 20 \text{ m.}$$

max ht $= (4\sqrt{5} + 20)$ m

min. ht $= (-4\sqrt{5} + 20)$ m.

Q2.



We check end-points and critical points on the new constraint

<u>End points</u>

$h(-2,0) = -2(2)+0+20 = 16$

$h(2,0) = 2(2)+0+20 = 24.$

<u>Critical points</u>

These are solve just like in Q1, using equations ①, ②, and ③.
But only one point $\left(\frac{2\sqrt{5}}{5}, \frac{4\sqrt{5}}{4}\right)$ is on the ~~curve~~ constraint curve.

$h\left(\frac{2\sqrt{5}}{5}, \frac{4\sqrt{5}}{4}\right) = 4\sqrt{5}+20$

So max $= (4\sqrt{5}+20)$ m ; min $= 16$ m

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