# Hw 12 Solutions [Check your answers]

Note Title 5/13/2009

## 22.3 (2) [5 points]

**Claim:** $\sum_{0}^{\infty} e^{-nx}\sin(nx)$ converges uniformly

on $(a, \infty)$ for $a > 0$

We have, for $x \in (a,\infty)$

$$\left|e^{-nx}\sin(nx)\right| \leq \left|e^{-nx}\right| \leq e^{-na} = M_n$$

$$\sum M_n = \sum \left(e^{-a}\right)^n \text{ converges } \left(\text{since } e^{-a} < 1\right) \text{ (geometric series)}$$

$\xRightarrow{\text{Weierstrass M-test}}$ $\sum e^{-nx}\sin(nx)$ converges uniformly on $(a,\infty)$

Since $e^{-nx}\sin(nx)$ is continuous,

$\xRightarrow{\text{continuity of unif. limits}}$ $\sum e^{-nx}\sin(nx)$ is continuous on $(a,\infty)$.

Pick $x_0 \in (0, \infty)$.

Choose $0 < a < x_0$

Since $\sum_1^\infty e^{-nx} \sin(nx)$ is continuous on $(a, \infty)$,

it is continuous at $x_0$.

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Since this works for every $x_0 \in (0, \infty)$,

$\sum_1^\infty e^{-nx} \sin(nx)$ is continuous on $(0, \infty)$. $\square$

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26.5(5) [3 points for each part]

(a) $\frac{d}{dx} x^n = n x^{n-1}$

The power series $\left(1 + x + x^2 + \cdots\right) = \frac{1}{1-x}$ has radius of

convergence 1.

$\Longrightarrow \left(1 + 2x + 3x^2 + \cdots\right)$

Thm 22.6 converges to $\frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{1}{(1-x)^2}$

for $|x| < 1$.

So $\quad 1 + 2x + 3x^2 + \cdots = \dfrac{1}{(1-x)^2}$

for $|x| < 1$.

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(b) $\quad 1 + 4x + 9x^2 + 16x^3 + \cdots\cdots$

$$= 1 + (2^2)x + (3^2)x^2 + (4^2)x^3 + \cdots\cdots$$

Now: $\quad \dfrac{d}{dx}\, n x^n = n^2 x^{n-1}$

In part (a), we showed

$$\sum_{n \geq 1} n x^{n-1} = \frac{1}{(1-x)^2}, \quad |x| < 1$$

$$\Longrightarrow \quad x\left(\sum_{n \geq 1} n x^{n-1}\right) = \frac{x}{(1-x)^2}$$

$$\|$$

$$\sum_{n \geq 1} n x^n$$

$$\Rightarrow \sum_{n\geq 1} n^2 x^{n-1} = \frac{d}{dx}\left(\frac{x}{(1-x)^2}\right)$$

Thm 22.6

$$= \frac{(1-x)^2 + x\,2(1-x)}{(1-x)^4}$$

$$= \frac{(1-x) + 2x}{(1-x)^3} = \frac{x+1}{(1-x)^3}$$

for $|x| < 1$