

12 - Classifying spaces

Note Title

3/9/2010

apply to $\tilde{H}^n(-; \pi)$

get $K = K(\pi, n)$

Remark: This is overkill: can construct $K(\pi, n)$ manually - assume π is abelian, $n \geq 2$

Moore Spaces:

$$\begin{aligned} \left[\underset{J}{V S^n}, \underset{I}{V S^n} \right] &= \prod_J \left[S^n, \underset{I}{V S^n} \right] \\ &= \prod_J \bigoplus_I \underbrace{\pi_n(S^n)}_{\mathbb{Z}} \\ &= \prod_J \text{Hom}(\mathbb{Z}, \bigoplus_I \mathbb{Z}) \\ &= \text{Hom}\left(\bigoplus_J \mathbb{Z}, \bigoplus_I \mathbb{Z}\right) \end{aligned}$$

$$0 \rightarrow \bigoplus_J \mathbb{Z} \xrightarrow{d} \bigoplus_I \mathbb{Z} \longrightarrow \pi_0 \rightarrow 0$$

$$\underset{J}{V S^n} \xrightarrow{\tilde{z}} \underset{I}{V S^n} \longrightarrow C(d) =: M(\pi, n)$$

$$\tilde{H}_K(M(\pi, u)) = \begin{cases} \pi, & u = m \\ 0, & \text{o/w} \end{cases}$$

$$K(\pi, u) \quad \pi_n \quad M(\pi, u) = \pi$$

inductively will higher levels sgs

$$\begin{array}{ccc} V S^{n+1} & \longrightarrow & K^0 = M(\pi, u) \\ \pi_{n+1}(K^0) & & \downarrow \\ V S^{n+2} & \longrightarrow & K^1 \\ \pi_{n+2}(K^1) & & \downarrow \\ & & K^2 \\ & & \vdots \\ & & \vdots \end{array} \quad \xrightarrow{\text{lim}} K^i = K(\pi, u)$$

Suppose X is a $K(\pi, u)$

$$\begin{array}{ccc} V S^1 \xrightarrow{\cong} V S^2 \longrightarrow M(\pi, u) & \text{Inductively} & V S^{n+1} \rightarrow K^{n+1} \rightarrow K^n \\ \downarrow \tau & \swarrow \text{is } \cong \text{ on } \pi_n & \downarrow \tau_{n+1}(K^{n+1}) \\ X & & X \end{array}$$

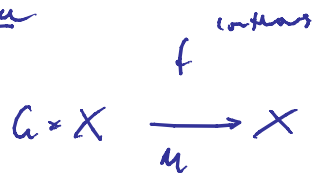
set $K(\pi, u) \rightarrow X$ w.e. (h.e. if X)
c.w.

Prop' $\Omega K(\pi, u_1) \cong K(\pi, u)$

Principle G-bundles

$G = \text{top'l}$ gp

A G-space



$\mu(L, x) = x$

$\mu(gL, x) = \mu(L, \mu(L, x))$

A principle G-bundle / B

$E \leftarrow G\text{-space}$

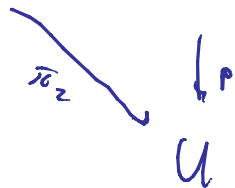


$B = E/G$

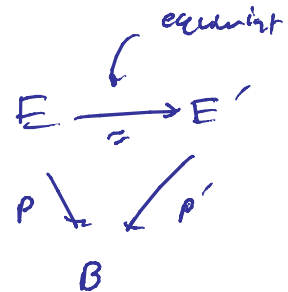
s.t. $\forall x \in B$

\exists u s.t. u of x

$G \times U \xrightarrow{\cong} p^{-1}(u)$ *equivariant homeo*



iso



E, E' princ. G-bundles / B

Prop:

any equivariant map $E \rightarrow E'$

is an iso.

