HOMEWORK 7

DUE DATE: THURSDAY, APRIL 3 (AFTER SPRING VACATION)

1. (Hatcher) Given abelian groups G and H, and CW complexes $K(G,n)$ and $K(H,n)$, show that the map

$$[K(G,n), K(H,n)]_* \to \mathrm{Hom}(G,H)$$

given by sending a homotopy class $[f]$ to the induced homomorphism $f_* : \pi_n(K(G,n)) \to \pi_n(K(H,n))$ is a bijection.

2. (This may be useful for the next problems) Let $f : X \to Y$ be a pointed map. Show that the cofiber of

$$f \wedge 1 : X \wedge Z \to Y \wedge Z$$

is given by $C(f) \wedge Z$.

3. Let n be greater than 1. Show that there is a there is a natural isomorphism

$$\widetilde{H}_{k+n}(X \wedge M(\pi,k)) \cong \widetilde{H}_n(X,\pi).$$

(X a CW complex, say).

4. Show that the universal coefficient theorem follows from the last problem and the long exact sequence of the cofiber sequence

$$\bigvee_I S^n \to \bigvee_J S^n \to M(\pi,n).$$

5. Show that $p : E \to B$ is a principle G-bundle, and $f : X \to B$ is a map, then the pullback $f^*E = E \times_B X \to X$ is a principle G-bundle. Show that if $g : Y \to X$ is another map, then there is an isomorphism of G-bundles

$$g^* f^* E \cong (f \circ g)^* E.$$