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Bill McDonald

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ESTIMATING MARKET MODEL BETAS: A COMPARISON OF RANDOM COEFFICIENT METHODS AND THEIR ABILITY TO CORRECTLY IDENTIFY RANDOM VARIATION*

BILL McDONALD
College of Business, University of Notre Dame, Notre Dame, Indiana 46556

When estimating market model betas using random coefficient methods, the rather fine distinction between significance or insignificance, as argued in recent studies, overlooks two important factors. First, the maximum likelihood method has not been tested in comparison to the generalized least squares approximation. Second, and more importantly, the ability of these methodologies to correctly identify a known random coefficient process has not been examined in the context of the market model. Using both simulations and subsequent empirical tests, this study shows that, for reasonable levels of variation in beta, neither method can consistently identify a random coefficient process. These results suggest that the nominal level of significant random coefficients previously observed could be indicative of a much more predominant phenomenon.
(MARKET MODEL; BETAS; RANDOM COEFFICIENTS)

1. Introduction

The use of beta as a measure of systematic risk permeates virtually all levels of financial analysis. Evidence of nonstationarities in beta has led to increasing interest in stochastic parameter methodologies. Of the various stochastic forms, the random coefficient specification has received considerable attention when attempting to characterize the variation in beta. Fabozzi and Francis (1978) conclude that beta is random for a significant minority of securities. Alexander and Benson (1982) suggest improvements in the Fabozzi and Francis methodology and conclude that the findings of significance were overstated. Sunder (1980) introduces a model to allow for cases where the intertemporal variation in beta is not independent. His empirical tests, however, are based on the standard random coefficient model. Sunder's results indicate a higher level of beta nonstationarity during the 1926–50 period, with much lower levels in subsequent periods. McDonald (1983), in tests of a model equivalent to the random coefficient specification, reports a nominal level of significance for the 1976–80 time period. In these studies the distinction between a significant minority and insignificance is a rather fine point. Therefore, the empirical characteristics of the methodology become critical in resolving the random coefficients issue.

The purpose of this research is to examine the ability of random coefficient methods to detect significance under controlled conditions that parallel market model applications. Using a series of simulations, the Theil–Mennes (1959) generalized least squares (GLS) method is first compared to the maximum likelihood (ML) approach to measure the relative effectiveness of these two estimation techniques. The simulations allow a second and more important point to be addressed. That is, the ability of these methods to detect random variation in beta. The results suggest that both methodologies have difficulties in detecting the presence of a stochastic beta. The maximum likelihood method is shown to dominate the GLS approach in identifying random variation in beta.

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A brief description of the statistical models is presented in §2. (The random coefficient models have been derived in detail in previous literature. See Alexander and Benson 1982 and Froelich 1973.) §2 also describes the simulation parameters. The simulation results are presented in §3. In §4, the ML method is applied to the sample tested in Fabozzi and Francis for comparison with previous studies. The simulated and empirical results are summarized in §5.

2. Methodologies

As in previous studies, the analysis is developed from the linear relationship posited in the single-index market model, expressed in deviation form as:

$$R'_{it} = \beta_{it} R'_{mt} + \mu_{it}, \qquad \text{where} \tag{1}$$

R'_{it} = the return on security i in time t expressed in deviation form,
R'_{mt} = the value-weighted market return in time t expressed in deviation form,
β_{it} = the beta for security i.

In addition, for a random coefficient model with stationary mean we assume

$$\beta_{it} \sim N(\beta_i, \sigma_{i1}^2), \tag{2}$$

$$\mu_{it} \sim N(0, \sigma_{i0}^2), \quad \text{and} \tag{3}$$

$$\operatorname{cov}(\mu_{it}, \beta_{it}) = 0. \tag{4}$$

Dropping the i subscript for notational convenience and restating equation (1) provides:

$$R'_t = \beta R'_{mt} + \nu_t, \qquad \text{where} \tag{5}$$

$$\nu_t = (\beta_t - \beta) R'_{mt} + \mu_t, \tag{6}$$

$$\sigma_{\nu_t}^2 = \sigma_0^2 + \sigma_1^2 R_{mt}'^2. \tag{7}$$

The presence of a random coefficient process is empirically tested by the statistical significance of σ_1^2.

2.1. *The GLS Method*

The GLS method tested in this study is identical to the modified Theil–Mennes (1959) approach used in Alexander and Benson (1982).[1] Difficulties arise in applying this two-stage process if the unconstrained variance estimates of the first stage result in negative values. When this occurs, a quadratic-programming algorithm is used to satisfy the nonnegativity constraints on the estimates of σ_0^2 and σ_1^2. Alexander and Benson show that the application of quadratic-programming is generally unnecessary if the second stage estimates are negative. In this case, the second-stage quadratic estimate of σ_1^2 is zero, and thus σ_1^2 will not be identified as statistically significant.

2.2. *The Maximum Likelihood Model*

For a model with the variance structure of equation (7), the appropriate maximum likelihood function is given by (see Froehlich 1973):

$$L(\beta, \sigma_0^2, \sigma_1^2) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi}\sqrt{\sigma_0^2 + \sigma_1^2 R_{mt}'^2}} \exp\left\{ -\frac{1}{2} \frac{(R'_t - \beta R'_{mt})^2}{\sigma_0^2 + \sigma_1^2 R_{mt}'^2} \right\}. \tag{8}$$

[1] Alexander and Benson (1982) provide a detailed development of the procedure which will not be repeated here.

The statistical significance of σ_1^2 can be determined using the likelihood ratio test. Under general conditions, the difference between the log-likelihood value of equation (8) and a restricted function where $\sigma_1^2 = 0$ is distributed as $\chi^2_{(1)}$.

Since there is no analytical expression for the maximum of this likelihood function, nonlinear optimization routines must be used to derive the parameter estimates. The Davidon variable metric algorithm is used and, in cases where it failed to converge, quadratic hill-climbing was also tried. (A detailed description of these methods and their performance appears in Goldfeld and Quandt 1972.)

2.3. *Simulation Parameters*

The market model of equation (5) is generated for a 72-period time interval using a standardized random variate to simulate the error term of equation (6).[2] This process is then repeated to create 1,000 cross-sectional observations for each level of coefficient variation tested. The parameters required in the simulation are the value of β, the level of R^2, a vector of independent variables, and the relative levels of σ_0^2 and σ_1^2. The results are insensitive to the β and R^2 parameters, so without loss of generality β is set equal to 1 and the coefficient of determination is set equal to 0.3. The values for R_{mt} are standardized value-weighted market returns from the monthly CRSP tapes over the 1966–1971 time interval. The resulting sample consists of the simulated returns of 1,000 securities over 72 periods, generated by a market model with known parameters. To test the discriminating power of the GLS and ML methodologies, this sample is recreated for a series of prespecified levels of random coefficient variation. The relationship between the parameter error variance and the pure-residual error variance is defined as:

$$\lambda = \sigma_1^2/\sigma_0^2. \tag{9}$$

In equation (7), the expected value of the standardized variable R'^2_{mt} is equal to one. Therefore $[\lambda/(1+\lambda)] \times 100$ represents the percent of total error variance attributable to the variation in beta. The simulations are run for values of λ ranging from 0 to 10.[3] At each level of λ, the GLS and ML methods are applied to the 1,000 observations to test the null hypothesis, $H_0: \sigma_1^2 = 0$. A priori, it would be expected that the variation in beta is at least less than the purely unsystematic component. Therefore, the upper limit of test values ($\lambda = 10$), where the variation in beta accounts for more than 90 percent of the total variability, was considered sufficiently large to encompass most variational patterns.[4]

3. Simulation Results

Froehlich (1973) provides a detailed comparison of the various random coefficient methodologies. He reports that the two-stage estimates of the Theil–Mennes procedure deteriorate substantially when the variance estimates are negative. The ML method is shown to be effective but has some difficulties when σ_1^2 is close to zero. Goldfeld and Quandt (1972), applying a ML method of similar form in a different context, report that the ML method dominates other two-stage processes. This research extends these previous tests in the context of estimating the single-index market model.

The ability of the optimization methods to converge in estimating equation (8) is, as expected, a direct function of λ. The likelihood function is undefined for negative

[2] The temporal parameters throughout this paper are structured to parallel the sample of Fabozzi and Francis (1978), i.e., the 72-month period from 1966–1971.

[3] The actual values tested for λ are: 0, 0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, and 10.

[4] Using the simulation parameters and a λ of 10 implies a standard deviation for the simulated error in beta of approximately 1.5.

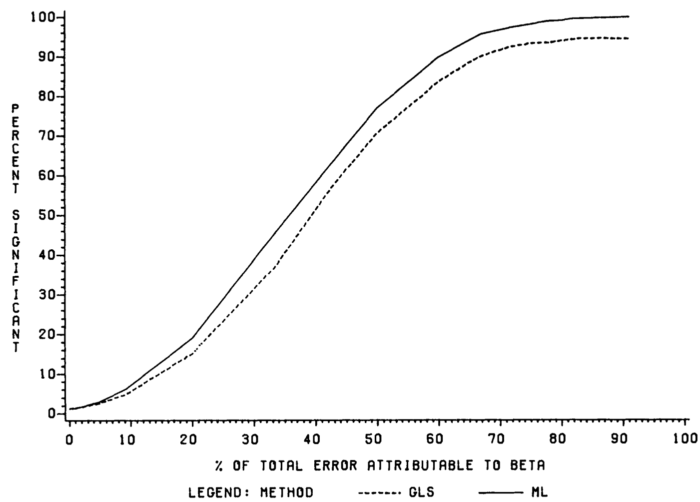

FIGURE 1

variances. Thus, for methods where σ_1^2 is close to zero, the routines typically identify a nearby solution, but are unable to assure an optimum. Approximately 47 percent of the models failed to converge when the variation in beta accounted for only 5 percent of the total variation. When the random coefficient accounts for 20 percent of the total variance, approximately 18 percent of the cases failed to converge. All estimates successfully converged for tests where the parameter variance was more than 50 percent of the total. In all cases of nonconvergence the function was at the boundary where $\sigma_1^2 = 0$. Thus, in subsequent results, when the estimation process did not converge it is assumed that $\sigma_1^2 = 0$.

The likelihood function appeared to be well defined for cases that did converge. Less than 13 iterations and 200 function evaluations were needed to identify the optimum. The results are not sensitive to the starting values selected.

The number of cases where the GLS method initially identified a negative variance was similarly a function of the level of λ. Approximately 42 percent of the cases had negative estimates when λ was equal to 0.05. This value rapidly stabilized to levels of 2–4 percent when λ was greater than 0.75, although the number of negative estimates did not converge to zero as in the ML case. Consistent with Alexander and Benson, when the second stage estimate of the parameter variance is negative, σ_1^2 is assumed to be equal to zero.

The simulation results are presented in Figures 1 and 2. Figure 1 presents the percent of cases where σ_1^2 is found to be significant at the 0.05 level for each of the methodologies given the various degrees of variation in beta (where, except for the case of $\lambda = 0$, the percent of observations with actual variation in beta is 100). For ease of interpretation the x-axis in each of the figures is reported in terms of $[\lambda/(\lambda + 1)] \times 100$, which is the percent of total error variation attributable to the random beta. In the simulated observations where $\lambda = 0$ (i.e., beta is actually stationary), the GLS and ML methods both incorrectly identify 1.2 percent of the cases as significant at the 0.05 level. This low level of spurious significance indicates that neither method exhibits a tendency toward Type I errors. For the remaining range of λ's tested, the ML method is consistently more successful than GLS in identifying cases with significant parameter variation.

Figure 2 presents the mean absolute deviation of the estimated σ_1^2 from the actual values. The ML estimates are somewhat less accurate than the GLS estimates for cases where σ_1^2 is very close to zero, paralleling the results of Froehlich. However, for other

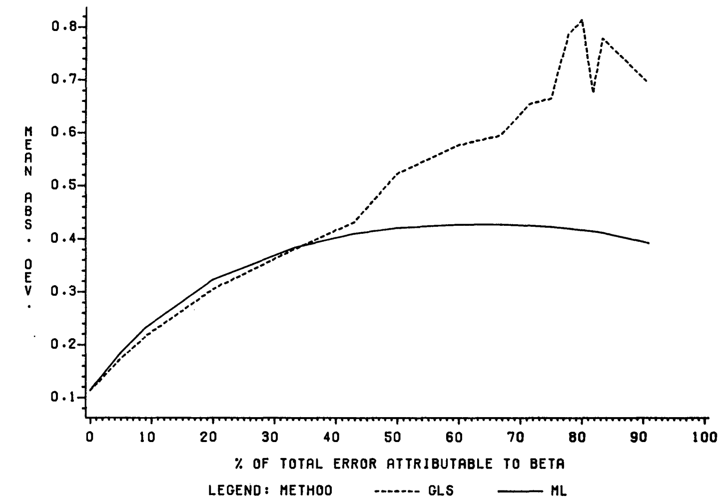

FIGURE 2

values of λ, the mean absolute deviation for the ML method is less than that of the GLS approach, and appears to be relatively stable. Since the value of σ_1^2 is increasing along the x-axis, the stability of the ML error implies that the relative error is decreasing. The inferiority of the GLS method in these comparisons possibly could be reduced by iterating the second stage; however, this approach has not been applied in previous studies.

There are two major conclusions that can be drawn from the simulation results. First, the ML approach dominates in its ability to detect random variation in beta and accurately partition the error variance. Second, for cases where the parameter error variance is not at least as large as the pure residual variance, the Type II error is substantial. That is, neither methodology is capable of accurately detecting the presence of a stochastic parameter for levels of variation that could be reasonably expected in the market model context.

4. Empirical Comparisons

Using GLS methods, a rather fine distinction has been presented in the literature arguing that beta is a random coefficient in a significant minority of cases (15%—Fabozzi and Francis) versus the conclusion of insignificance (6.4%—Alexander and Benson). Given the dominance of the ML method found from the simulations, it is of interest to apply this method to the sample in question. Using the CRSP monthly data, the 6-year time interval from 1966 to 1971 is analyzed as in the previous studies. This process provided a sample of 703 securities (compared to 700 in Fabozzi and Francis).

The ML method is applied, as described in §2, to test the significance of the random coefficients model. Using the ML method, 10 percent (70 out of 703) of the securities indicate a significant σ_1^2 at the 0.05 level. Thus, in comparison to the two previous studies, the conclusion does not provide a positive resolution. Instead, it appears that the observed level of significance is more than expected by chance, however, somewhat less than the level reported in Fabozzi and Francis. If this were the only issue, the empirical results would be arguing a rather fine point. That is, at what level does the presence of random coefficients become relevant to empirical and theoretical applications?

The simulation results in the context of this empiricism make a much more important point. Given the rather poor discriminating abilities of the random coefficient methodologies in market model applications, the existence of a "significant minority" could reveal more of a pandemic problem than first realized. For example, assume that in the actual market process half of the securities have random coefficients and that the variation in beta for these securities is 20 percent of the total variance. In this case, from the simulation results of Figure 1, we would expect to observe only 10 percent of the random coefficient models as significant using current methodologies.

5. Conclusions

The hypothesis of beta as a random coefficient is re-examined to suggest an alternative methodology and to test the discriminating power of the competing methodologies. The findings indicate that the maximum likelihood method of estimating beta as a random coefficient generally dominates the GLS approximation. More importantly, it is shown that neither of these methods is sensitive to reasonable levels of random variation in beta. Thus, the nominal level of random coefficients observed could suggest a much more predominant problem than previously realized.

This paper considers the identification process for stationary betas. A separate issue not addressed here is the efficacy of using a stationary stochastic model in estimating beta (see Bey 1983, McDonald 1983, or Hsu 1984). This question, along with issues raised in this paper, suggests that future applications of the random coefficient model in estimating beta must be careful to insure that the tests are appropriate and capable of identifying the assumed relationship.[5]

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