Permanent and Transitory Movements in Output and Unemployment: Okun's Law Persists

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Abstract

This paper develops a bivariate correlated unobserved components model to investigate the interaction between output and unemployment. The model separates these two key macroeconomic variables into permanent and transitory components and provides estimates of the correlations among these components. The results for the US indicate that fluctuations in both output and unemployment are largely permanent and there exists a negative relationship between these permanent components similar to the Okun's Law relationship between the transitory components. These results call into question macroeconomic theories that imply zero correlation between the different components, as well as theories that consider recessions as purely transitory movements in either output or unemployment.

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Section 1: Introduction

Many macroeconomic models and theories separate the study of economic growth from that of fluctuations. They also often separate the study of permanent movements in the unemployment rate (the natural rate of unemployment or the NAIRU—Non-Accelerating Inflation Rate of Unemployment) from the study of transitory unemployment.[1] The connection between output and unemployment comes through Okun's Law which suggests that an increase in transitory output is accompanied by a decrease in transitory unemployment. Thinking of the economy in this manner implicitly assumes that the components of output and unemployment are uncorrelated except for a negative correlation between the two transitory components.

Theories do exist, however, which suggest the existence of additional nonnegative correlations between the components of output and unemployment. For example, some real business cycle theories, such as the one presented by Kydland and Prescott (1982), imply a negative correlation between the permanent and transitory components of output. In these theories, transitory movements in the series arise primarily from adjustment to permanent changes. Other theories suggest a positive correlation between permanent and transitory movements. For example, a temporary increase in investment may lead to both transitory and permanent increases in output. Hysteresis may also imply a positive correlation between transitory and permanent movements where, for instance, a temporary increase in unemployment may partially persist and become permanent (e.g. Blanchard and Summers 1986).

Economists thus need empirical evidence to distinguish between these different theories. Until recently, however, time series models of output and unemployment have primarily reflected the thinking that the components of major macroeconomic time series are uncorrelated.[2] Clark (1987), Stock and Watson (1988), and others supposed that

[1] Some researchers (for example Blanchard and Quah 1989) assume that the unemployment rate is stationary and thus does not have a permanent component. This assumption will be considered in Section 4.4.

[2] One major exception to this is the Beveridge and Nelson decomposition (1981), which does not assume anything about the correlation between the components. The innovations in the *estimated* components of the Beveridge and Nelson decomposition are perfectly negatively correlated, however the implied correlation between the true components can take on any value. It is possible to solve for this correlation in the univariate case, as shown by Morley, Nelson, and Zivot (2003). The multivariate case has been examined by Schleicher (2003) and will be discussed here in Section 3.

some assumptions about the correlations were necessary for identification of the model. Morley, Nelson, and Zivot (2003, hereafter MNZ) find, however, that in some cases we can estimate an unobserved components (UC) model allowing for correlation between the components. Their correlated unobserved components (UC-UR) model therefore provides empirical evidence useful for distinguishing between different macroeconomic theories.

This paper extends MNZ to a bivariate framework and develops a correlated unobserved components model for output and unemployment. The use of output and unemployment follows Clark (1989)[3] and Blanchard and Quah (1989). The inclusion of unemployment is based on Okun's Law (Okun 1962), which suggests that unemployment should provide additional information about transitory movements in GDP to better identify them.[4] With this model we can estimate the components of unemployment and output jointly and also estimate all the correlations between the components of the two series.

The results from estimating this new time series model shed light on three important debates. The first is the importance of permanent versus transitory movements in macroeconomic time series. In particular, extending MNZ to a bivariate model addresses the critique, as discussed in Cochrane (1994) and Proietti (2002), that a univariate model of GDP might understate the predictability of GDP. Also, Clark (1987) claims that using unemployment data to help identify the transitory movements in real output should strengthen the case for large transitory movements in GDP. The results from estimating the model presented in this paper suggest, however, that movements in US GDP are largely permanent.

Second, including the unemployment rate not only better identifies the transitory movements in GDP, but jointly estimating the components of unemployment and output also provides new estimates of the relative importance of permanent versus transitory movements in the US unemployment rate. This paper thus also contributes to the debate

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[3] Clark (1989, page 23) writes: "One obvious place to look for additional evidence on the size and shape of business cycles is the data on unemployment rates."

[4] The Congressional Budget Office (2004) suggests that using unemployment as a measurement of capacity results in the estimate of the permanent component of GDP also being an estimate of potential GDP, the level of output consistent with stable inflation.

about the variability in the natural rate of unemployment, or the NAIRU,[5] by finding support for a variable permanent component in unemployment. In addition, this model provides a new way to test if the unemployment rate is stationary, which is rejected for the US in favor of a model with significant permanent movements in unemployment.

Third, the bivariate model presented in this paper also allows us to estimate the different relationships between the components of output and unemployment. Four correlations are of particular interest for the US. The first is the correlation between the permanent and transitory components of output. MNZ found that for US GDP these components are significantly negatively correlated. Adding unemployment lends support to the robustness of their result and shows that this negative correlation is not just a consequence of univariate analysis. I also find that the components of unemployment are negatively correlated, suggesting that unemployment responds to shocks in a similar way to GDP. The third important correlation is between the transitory components of GDP and unemployment. This correlation provides an estimate of the traditional Okun's coefficient that improves upon the literature in this area because it directly estimates the correlation rather than first estimating the components and then estimating the correlation between the estimated components.[6] The final interesting correlation is between the permanent components of GDP and unemployment. I refer to this correlation as Okun's coefficient for permanent movements, because we see a similar relationship between unemployment and output in the long run to what Okun posited for the short run. All of these correlations lend support to a theory of the US economy where permanent shocks move the economy while transitory movements primarily reflect the adjustment of the series to the new steady-state values.

The paper proceeds as follows. Section 2 presents the bivariate UC-UR model for unemployment and output. Section 3 discusses identification of the model. Section 4 presents the results of estimating this model with US data and discusses the theoretical

[5] There has been much discussion about the different nuances of these terms. I will use them interchangeably here to represent the permanent component in unemployment. For the different sides of the debate on the variability of trend unemployment, see Weiner (1993), Gordon (1997), Salemi (1999), Grant (2002), and King and Morley (2003).

[6] Barreto and Howland (1993) write: "With respect to Okun's Law, there is no doubt that the correct model is a simultaneous system in which output and unemployment are endogenous." The belief that the correlation between components could not be identified led to the single equation models used in the literature up to now.

implications of the results of the bivariate UC-UR model for the US. Section 5 provides conclusions and suggestions for possible extensions.

Section 2: The Model

Output and unemployment can each be represented as the sum of a "trend" component and a "cycle" component. The "trend" (τ), also called the permanent component, is the steady-state level after removing all temporary movements. The "cycle" (c), also called the transitory component, embodies all temporary movements and is assumed to be stationary:

$$y_t = \tau_{yt} + c_{yt} \quad (1)$$

$$u_t = \tau_{ut} + c_{ut} . \quad (2)$$

A random walk for each of the trend components allows for permanent movements in the series.[7] For output, I also allow for a drift (μ_y) in the trend:

$$\tau_{yt} = \mu_y + \tau_{yt-1} + \eta_{yt} \quad (3)$$

$$\tau_{ut} = \tau_{ut-1} + \eta_{ut} . \quad (4)$$

Following MNZ, Clark (1987 and 1989), and Watson (1986), each transitory component is modeled as an autoregressive process of order two (AR(2)):[8]

$$c_{yt} = \phi_{1y} c_{yt-1} + \phi_{2y} c_{yt-2} + \varepsilon_{yt} \quad (5)$$

$$c_{ut} = \phi_{1u} c_{ut-1} + \phi_{2u} c_{ut-2} + \varepsilon_{ut} . \quad (6)$$

I assume the innovations (η_{yt}, η_{ut}, ε_{yt}, and ε_{yt}) are normally distributed random variables with mean zero and a general covariance matrix (allowing possible correlation between any of the components). The general covariance matrix is the main difference between the model presented here and that of Clark (1989). Clark imposed a restricted covariance matrix that will be discussed further in Section 4.4.

[7] Standard tests cannot reject a unit root for either of the series used. The unemployment rate is bounded between zero and one, but it can undergo permanent shocks. For example, the random walk will capture frequent structural breaks.

[8] Theoretical justification for the AR(2) cycle for unemployment comes from Alogoskoufis and Manning (1988) who argue that the unemployment rate for all countries should be modeled by an AR(2). The AR(2) is also used by Clark (1989) so that the difference between the results of estimating his model versus this model clearly arises from the difference in assumptions about the covariance matrix. A likelihood ratio test indicates that a third lag is not significant.

Casting the model in state-space form makes it possible to use the Kalman filter for maximum likelihood estimation of the parameters and the permanent and transitory components.[9]

Section 3: Identification of the Bivariate UC-UR Model

It might not be obvious that the model, and in particular the correlations, are identified. As shown below, models with white noise or AR(1) transitory components are not identified, so many economists, including Clark (1987, 1989) and Stock and Watson (1988), assumed that all UC models required some restrictions on the correlations for identification.[10] The purpose of this section is to explain why the model used in this paper is identified and why correlated UC models are identified under fairly general conditions. MNZ showed that the univariate UC-UR model is identified under certain conditions. Previous research, by Morley (2004), Engel and Morley (2001), and Scheicher (2003), have used multivariate correlated UC models to study cointegrated series. Schleicher (2003) and this paper show that a multivariate UC-UR model will be identified under certain conditions, even if there is no cointegration.[11] Previous papers, such as Clark (1989) imposed unnecessary restrictions on the magnitude of the correlations.

In order to verify that the parameters of the bivariate unobserved components model are identified we can compare the parameters of the unobserved components model with those of its reduced-form. In this case the UC-UR model can be thought of as "structural" time series model in the sense that Harvey (1993) uses the term where the model includes stochastic components. The corresponding "reduced-form" has a VARIMA (p, q) representation.

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[9]See chapter 3 of Kim and Nelson (1999) or chapter 4 of Harvey (1993) for a discussion of the implementation of the Kalman filter. The estimation was done in GAUSS. See the appendix for the state-space representation.

[10] Stock and Watson write on page 153, "this correlation cannot be estimated directly from a single time series; that is, this correlation is not identified in the usual econometric sense."

[11] Schleicher also estimates his model of consumption and income without imposing cointegration, but he cannot reject cointegration.

For the specific bivariate UC-UR model of equations (1) through (6), the reduced-form VARIMA (2, 2) representation is obtained as follows. Taking first differences:

$$\begin{bmatrix}\Delta y_t\\ \Delta u_t\end{bmatrix} = (1-L)\begin{bmatrix}\tau_{y_t}\\ \tau_{u_t}\end{bmatrix} + (1-L)\begin{bmatrix}c_{y_t}\\ c_{u_t}\end{bmatrix}$$

$$= \begin{bmatrix}\mu\\ 0\end{bmatrix} + \begin{bmatrix}\eta_{y_t}\\ \eta_{u_t}\end{bmatrix} + (1-L)\begin{bmatrix}1-\phi_{1y}L-\phi_{2y}L^2 & 0\\ 0 & 1-\phi_{1u}L-\phi_{2u}L^2\end{bmatrix}^{-1}\begin{bmatrix}\varepsilon_{y_t}\\ \varepsilon_{u_t}\end{bmatrix}$$

Define a vector $\mathbf{x_t}$ as follows:

$$\begin{bmatrix}x_{1t}\\ x_{2t}\end{bmatrix} \equiv \begin{bmatrix}1-\phi_{1y}L-\phi_{2y}L^2 & 0\\ 0 & 1-\phi_{1u}L-\phi_{2u}L^2\end{bmatrix}\begin{bmatrix}\Delta y_t\\ \Delta u_t\end{bmatrix}$$

$$= \begin{bmatrix}\mu^*\\ 0\end{bmatrix} + \begin{bmatrix}\eta_{y_t}\\ \eta_{u_t}\end{bmatrix} - \begin{bmatrix}\phi_{1y}\eta_{y_{t-1}}\\ \phi_{1u}\eta_{u_{t-1}}\end{bmatrix} - \begin{bmatrix}\phi_{2y}\eta_{y_{t-2}}\\ \phi_{2u}\eta_{u_{t-2}}\end{bmatrix} + \begin{bmatrix}\varepsilon_{y_t}\\ \varepsilon_{u_t}\end{bmatrix} - \begin{bmatrix}\varepsilon_{y_{t-1}}\\ \varepsilon_{u_{t-1}}\end{bmatrix}$$

$$= \begin{bmatrix}\mu^*\\ 0\end{bmatrix} + \begin{bmatrix}\nu_{y_t}\\ \nu_{u_t}\end{bmatrix} + \Theta_1^*\begin{bmatrix}\nu_{y_{t-1}}\\ \nu_{u_{t-1}}\end{bmatrix} + \Theta_2^*\begin{bmatrix}\nu_{y_{t-2}}\\ \nu_{u_{t-2}}\end{bmatrix},$$

where $\mu^* = (1-\phi_{1y}L-\phi_{2y}L^2)\,\mu$ and innovations ν_y and ν_u are jointly normally distributed.

The final expression for $\begin{bmatrix}x_{1t}\\ x_{2t}\end{bmatrix}$ is a vector MA(2) using a multivariate version of Granger's lemma (Granger and Newbold 1986).[12]

The AR parameters of the UC-UR model (ϕ_{1u}, ϕ_{2u}, ϕ_{1y}, ϕ_{2y}) are identified, since the reduced-form has the same parameters. The mean of the reduced-form (μ^*) for output combined with the AR parameters for output identify the drift term (μ) from the UC-UR model. The difficulty in confirming identification arises from the parameters in the covariance matrix.

Consider the autocovariance matrices for the vector MA(2) process $\mathbf{x_t}$:

$$\Gamma_j = E\left[\left(\begin{bmatrix}x_{1t}\\ x_{2t}\end{bmatrix} - \begin{bmatrix}\mu^*\\ 0\end{bmatrix}\right)\left(\begin{bmatrix}x_{1t-j}\\ x_{2t-j}\end{bmatrix} - \begin{bmatrix}\mu^*\\ 0\end{bmatrix}\right)'\right] \quad for\ j \le 2,\ \ \Gamma_j = 0 \quad for\ j > 2.$$

The first autocovariance (Γ_0) matrix provides 3 parameters: the variances of x_1 and x_2 and the covariance between the two. The other two remaining autocovariance matrices provide 4 parameters, since the off-diagonal elements are no longer identical.

[12] Note that the theta matrices are 2x2 matrices which we assume are of full rank for identification. This will be true if all of the autocovariances are nonzero.

Generalizing for a structural UC-UR model of n series, each with an AR(p) transitory component, there will always be p + 1 non-zero autocovariance matrices for $\mathbf{x_t}$.[13] Of them, p will provide n^2 parameters. The remaining autocovariance matrix, Γ_0, will always be symmetric, so it will only provide n + n(n-1)/2 parameters. Thus, the reduced-form will provide $\frac{n}{2}(2np + n + 1)$ parameters (in addition to the AR parameters and the mean). In the case studied here, the reduced form provides eleven parameters.

If there is no common permanent component (cointegration) or common transitory component between the series, then the covariance matrix of the bivariate UC-UR model has ten parameters.[14] Any common component merely serves to reduce the number of parameters in the structural UC-UR model. Thus, if the series were to share a common component, the model would remain identified. This is the case both for the specific bivariate UC-UR model considered here as well as the more general case.

In the case of the bivariate UC-UR model considered in this paper, the autocovariance matrices for the process $\mathbf{x_t}$ provide 11 parameters. The UC-UR covariance matrix, however, has only 10 unknowns, so the model is over-identified in terms of the autocovariance parameters.[15] In general, the requirement for identification of a structural UC-UR model where the transitory components are modeled as AR(p) cycles is: $p \geq \frac{3}{2} + \frac{1}{2n}$. An AR(2) will thus always be sufficient for identification, no matter how many series are included. In the univariate case, as was discussed in MNZ, the model is exactly identified with an AR(2) cycle. For the multivariate case, AR(2) cycles will result in an implicit over-identification restriction.

For the univariate case, MNZ showed that it is possible to estimate the reduced-form model and then solve for the covariances since there is a one-to-one mapping

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[13]The discussion in this section assumes there are no moving average terms in the transitory components For the more general case, MNZ show that for the univariate case the requirement for identification for an ARMA(p, q) cycle is that $p \geq q + 2$. An interesting extension, not pursued here, would be to find the equivalent requirement for the multivariate case with $q > 0$.

[14] See the covariance matrix in the state-space form in the appendix. In general, the UC-UR model has $2n^2 + n$ unknowns in the variance-covariance matrix if there are no common trends or common cycles.

[15] We can also compare the MA version to the expression before invoking Granger's lemma to confirm that the model is over-identified. Further exposition on the autocovariances is available in a technical appendix available from the author upon request. Also note that this over-identification restriction is simply an implication of the UC-UR model structure.

between the parameters of the UC-UR model and the reduced-form. Thus in the univariate case, it does not matter whether you estimate the UC-UR model or the ARIMA model, hence the UC-UR model in the univariate case is equivalent to the Beveridge-Nelson decomposition (1981) using the same ARIMA representation as the forecasting model. For the multivariate case, however, the implicit over-identification restriction implies that the multivariate UC-UR model will not necessarily be equivalent to the multivariate Beveridge-Nelson decomposition.

Since the *model* is identified we can estimate it. However, weak identification may still be an issue. The identification of the covariances depends on sufficiently complicated autoregressive dynamics, so the covariances will be weakly identified if estimates of the autoregressive parameters are close to zero. In the case of weak identification, Nelson and Startz (2002) show that as the true variance goes to infinity in the limit of no identification, but the sample variance remains finite. Hence, inference using standard errors may be distorted in the presence of weak identification. Nelson and Startz suggest that likelihood ratio statistics perform better than Wald statistics when weak identification may be present. Thus likelihood ratio statistics will be used in hypothesis testing throughout this paper.[16]

Section 4: The Results

The two variables are the US civilian unemployment rate (u) and the natural log of US real GDP multiplied by 100 (y). The data are quarterly, from 1948:1 – 2003:4.[17] Jointly estimating the permanent and transitory components of the two series should provide better estimates of the components than estimating the components of the two series separately because Okun's Law suggests that the transitory components of the two series are correlated. Thus these results provide new estimates of the components of the series. This model also provides estimates of all the correlations within and across series. From these correlations we have an estimate of Okun's coefficient, as well as any other

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[16] MNZ and Schleicher (2003) both find that the size is approximately correct for smaller samples.

[17] Civilian unemployment rate data obtained from FRED II (Federal Reserve Economic Data) from the Federal Reserve Bank of St. Louis (http://research.stlouisfed.org/fred2), using end-of quarter rate to match with GDP. Real GDP data from the US Department of Commerce, Bureau of Economic Analysis (BEA) (http://www.bea.doc.gov/bea/dn/home/gdp.htm, based on the March 25, 2004 release. The estimated components begin in 1949:1 in the figures below because the program used a four-quarter training sample to start up the Kalman filter.

relationship between the two series. The estimates presented in this section come from joint estimation, but I will present them for each variable separately.

Figures 1 and 3 present the permanent components of GDP and unemployment respectively along with the observed series. They are produced using the smoothed Kalman filter, which uses all information available in the sample so it provides for a better in-sample fit as compared to the basic Kalman filter which only uses information available at time t.[18] The availability of smoothed estimates is one benefit of using the unobserved components model as compared to the Beveridge-Nelson decomposition. Another benefit of using the UC-UR model instead of the Beveridge-Nelson decomposition is that we can create standard error bands from the smoothed Kalman filter..[19] In the case of both GDP and unemployment, using the additional information results in a smoother trend and a more variable transitory component than using the basic filter.[20]

The following subsections present the results of estimating the UC-UR model with the US data. The first two subsections address the estimates of the components of output and unemployment and the relationship between the permanent and transitory components within each series. The third subsection extends the discussion to the across-series relationships and Okun's Law. The fourth subsection compares the results of the UC-UR model with results of other well-known models of output and unemployment and

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[18] Proietti (2002, page 15) claims that smoothing is particularly important in the case of a negative correlation between trend and cycle because "essential information for assessing the cyclical pattern lies in future observations." See chapter 3 of Kim and Nelson (1999) or chapter 4 of Harvey (1993) for discussion of the implementation of the smoothing algorithm. Alternatively, the basic Kalman filter bases the estimates on information available at each time period. The filtered estimates are available from the author.

[19] The estimates of the components with standard error bands are available in an appendix by request from the author. The Beveridge-Nelson decomposition is equivalent to the basic Kalman filter, as shown by MNZ, for the univariate reduced-form ARIMA model. As discussed in Section 3, the multivariate UC-UR is no longer equivalent to the multivariate Beveridge and Nelson decomposition due to the implicit over-identifying restriction(s) in the multivariate UC-UR.

[20] One may observe from Figures 1 and 4 that innovations in the *estimated* permanent and transitory components are perfectly negatively correlated. This is similar to the observation commonly made about the Beveridge-Nelson decomposition (1981). The perfect negative correlation is a property of the estimated components, but not an assumption of the model. The estimates of the components presented in Figures 1 and 4 are, however, optimal given our assumptions (in particular, linearity). The UC-UR model presented here also provides estimates of the correlations between the components, which is not necessarily the same as the perfect negative correlation of the estimated components. Finding the estimate of the correlation from the UC-UR model is another advantage over the Beveridge-Nelson decomposition when the correlation is of interest. This is one reason why it is important to use the estimate of the correlation between the components rather than the correlation of the estimates of the components, as will be discussed further in the Section 4.3 regarding the estimate of Okun's coefficient.

shows that the differences arise from restrictions imposed in those models which are rejected by US data according to the UC-UR model. Finally, the last subsection presents interpretations of the results in light of existing macroeconomic theories.

Section 4.1: The Components of GDP

From the estimates in Table 1 and the estimated permanent component of GDP presented in Figure 1 we can see that this is not the textbook smooth trend. The estimate of the permanent component looks very similar to the GDP series. In fact, the estimated permanent component is slightly more variable than the series itself with the standard deviation of the permanent innovation to GDP (1.5689) being greater than the standard deviation of the first difference of the GDP series (1.005). This result is common to the Beveridge-Nelson decomposition of US GDP, and the findings of MNZ and Morley (2003). The shading represents the NBER-dated recessions. We can see that these correspond to significant negative permanent movements according to this model. The transitory movements are the difference between the series and the trend. These movements do not correspond to the traditional view of a "cycle," and in particular do not encompass the NBER business cycle.

Take for example the 2001 recession, as shown in Figure 2. The permanent component dips well below the series in the middle of the recession. This implies a positive transitory component, since the series is above the steady state value. This result is very different from theories which describe recessions as temporary negative movements.[21]

Next compare the results from the bivariate model to the results of previous estimates of univariate models. The ratio of innovations to trend to transitory innovations in the bivariate model (1.3755) is less than that found by MNZ's univariate model (1.65237). This supports the idea that including an additional variable decreases the trend variability (Cochrane 1994). The decrease, however, is small, and does not result in a much smoother trend. In fact, the estimate of the variability in trend (1.5689) is higher than that for the univariate case (1.2368). The estimates of the bivariate UC-UR model

[21] Some have argued that the oil shocks in the 1970s might exhibit more permanent movements (because of the large "real" shock) than other recessions Mankiw, G. (1989). "Real Business Cycles: A New Keynesian Perspective." Journal of Economic Perspectives **3**(3): 79-90.. Looking at Figure 1, however, these periods do not seem to exhibit much more movement in the permanent component than the other recessions.

are strikingly similar to the findings of Morley (2003). In his bivariate model of output and consumption, Morley found the ratio of innovations to trend to transitory innovations in output to be 1.36 and the standard deviation of trend output to be 1.5.

Similar to MNZ and Morley (2003), I find that the two components of GDP are negatively correlated. If we interpret the negative correlation between the components of GDP to imply a causal relationship between permanent shocks and transitory adjustments of the series to those shocks, then Figure 5 represents the simulated time path of the adjustment of the series to a one-time permanent shock.[22] For example, a positive permanent shock results in an increase in the permanent component today, however the series does not fully adjust on impact. Instead, the series adjusts over time and the transitory component is negative for approximately five quarters. The persistence of the transitory component ($\phi_{1y} + \phi_{2y} = 0.5529$) suggests that the adjustment will be relatively quick with a half-life of less than two quarters.[23]

Following the suggestion of Dupasquier et al. (1999), if we include the dynamics of permanent shocks in potential output, then the measurement of the output gap would be extremely small. The model here assigns all temporary movements to the transitory component, even those movements resulting from the series adjusting to permanent shocks. The size of the correlation between the series suggests that a substantial amount of the transitory component arises from adjustment to permanent shocks, with a very small amount coming from independent, temporary shocks. As discussed in MNZ, the transitory movements here should be the upper bound for the output gap or the business cycle, if we assume that business cycle shocks have no permanent effects. The business cycle could, however, involve permanent shocks. For an example of a time-series model of the business cycle which implies permanent effects of recessions, see Hamilton (1989).

Section 4.2: The Components of Unemployment

Figure 3 presents the estimate of the permanent component of the US unemployment rate along with the unemployment series. Similar to GDP, a sizable amount of the movement in the US unemployment rate appears to arise from permanent

[22] One may also think of one shock which has both temporary and permanent effects.

[23] Note that the model assumes the same adjustment dynamics for all transitory shocks. If different underlying structural shocks produce different adjustment dynamics, then Figure 3 captures the average response, not the response to a specific structural shock.

shocks. Figure 4 focuses on the 2001 recession and shows a positive permanent movement in unemployment, rather than the temporary movement that many theories of recessions would predict.

The estimates for unemployment presented in Table 1 support the impression from Figures 3 and 4 that the permanent component of unemployment is not at all a smooth trend. In particular, we see that the standard deviation of the permanent component (0.7109) is larger than the standard deviation of the first difference of the series (0.445). We also see that the ratio of the standard deviation of permanent innovations to that of temporary innovations ($\sigma_{\eta u}/\sigma_{\varepsilon u}$) is 1.0363. Finally, the estimates suggest that the correlation between permanent and temporary innovations for unemployment is not zero but is instead significantly negative with a point estimate of -0.9286.[24]

Similar to GDP, the negative correlation between the components of unemployment may be interpreted as a causal relationship where the series adjusts over time to a permanent shock. The components are not perfectly negatively correlated, so shocks can occur which have no permanent effects. An estimated correlation of -0.9286 between permanent and temporary innovations for unemployment, however, suggests that most shocks which have temporary effects also have permanent effects. These dynamics can be seen in Figure 6. For example, at the beginning of an expansion, unemployment starts to fall, but the permanent level of the unemployment series falls faster in anticipation of future decreases in unemployment. When these future decreases occur, then the series rejoins the permanent component.

According to the estimates presented in Table 1, the transitory components of GDP and unemployment have very similar impulse-response functions. The transitory component is not very persistent for either series, with each one having a half-life of less than two quarters. Given the assumption of many economists that transitory unemployment is more persistent than transitory GDP, these results may appear surprising. In this model, most of the persistence is captured in the permanent component of unemployment. Many models have previously treated unemployment as stationary,

[24] Restricting this correlation to be zero results in a likelihood value of -367.7987. This implies a likelihood ratio test statistic of 79.982. With one restriction, this has a p-value of 0.0000 so we can clearly reject this restriction.

which explains the finding of greater persistence in unemployment for these models. A test for the significance of the permanent component in unemployment will be presented in Section 4.4.

Section 4.3: Correlations Between the Two Series and Okun's Law

The value of the traditional Okun's coefficient is important for a number of reasons. First, if the unemployment rate is a policy variable, then Okun's coefficient may be interpreted as the size of the economic reward for unemployment reduction. Second, forecasts for output are often taken to imply forecasts for unemployment, but only if the correct coefficient is used. Third, it is useful to know when GDP is above potential and there should be concerns about inflation or when it is below potential and policy action may be considered.

A simple version of the empirical relationship known as Okun's Law can be written in the notation used in this paper as:

$y_t - \tau_{yt} = \lambda(u_t - \tau_{ut}) + v_t$.

This relationship can equivalently be written as:

$c_{yt} = \lambda c_{ut} + v_t$.

where v is another Normal random variable which captures the imperfect correlation between the two transitory components and λ represents the traditional Okun's coefficient for temporary movements. Thus Okun's Law suggests that the transitory components of output and unemployment should be correlated.

To compare the results of this paper with more traditional estimates of Okun's coefficient, we must first consider the effect of the potentially different adjustment processes of GDP and unemployment to the same temporary shock. In the case of the US, however, estimates suggest that both GDP and unemployment adjust very similarly to temporary shocks. Testing the restriction that $\phi_{1y} = \phi_{1u}$ and $\phi_{2y} = \phi_{2u}$ results in a log likelihood value of -327.8347. Comparing this to the unrestricted version, the likelihood ratio test statistic is 0.1140. With two restrictions, the p-value is 0.9446, thus we would not reject the hypothesis that $\phi_{1y} = \phi_{1u}$ and $\phi_{2y} = \phi_{2u}$.

Since we cannot reject the hypothesis that the autoregressive coefficients for GDP and unemployment are the same, we can directly use the correlation between the innovations to transitory GDP and transitory unemployment (which are denoted ε_{yt} and

ε_{ut} respectively) in order to establish the size of Okun's coefficient. Replacing the transitory components with their underlying innovations, we have:

$\varepsilon_{yt} = \lambda\varepsilon_{ut} + \nu_t$.[25]

This specification assumes that at least some of the temporary shocks to GDP are shared with unemployment and that all temporary shocks to unemployment also affect GDP.

To determine the value of λ, recall that the model assumes that ε_{yt} and ε_{ut} are jointly normally distributed and ν_t is an independent normal random variable. First observe that we are considering the distribution of $\varepsilon_{yt}|\varepsilon_{ut}$. We can therefore multiply both sides by ε_{ut}:

$\varepsilon_{ut}\varepsilon_{yt} = \lambda\varepsilon_{ut}\varepsilon_{ut} + \varepsilon_{ut}\nu_t$,

Then take expectations:

$E(\varepsilon_{ut}\varepsilon_{yt}) = \lambda\varepsilon_{ut}\varepsilon_{ut} + \varepsilon_{ut}\nu_t$,

which results in:

$\sigma_{\varepsilon y\varepsilon u} = \lambda\sigma_{\varepsilon u}^2$,

which thus implies:

$\lambda=\sigma_{\varepsilon y\varepsilon u}/\sigma_{\varepsilon u}^2 = -1.3989$. This estimate implies that a 1% decrease in transitory unemployment corresponds to a 1.4% increase in transitory GDP,[26] which is considerably lower than the 3% which Okun (1962) originally suggested, and less than the 2% modern consensus estimate (Grant 2002). Estimates of Okun's coefficient have varied significantly over time and method used, with estimates as small as 0.67% (Prachowny 1993). These estimates, however, have estimated the cyclical components separately and then regressed one on the other in order to estimate Okun's coefficient. Estimating the components first introduces two forms of error. First, since the two components are correlated, it is more efficient to jointly estimate the cyclical components. Second, if the measurement error in the independent variable is correlated with the error term, which

[25] For the US we can work directly with the underlying temporary innovations and compare the results to other estimates of the traditional Okun's coefficient derived from regressions because the adjustment process (as evidenced by the estimates of the autoregressive coefficients) appears to be the same for both GDP and unemployment.

[26] It is important to note that if the two cycles are not perfectly negatively correlated, that the inverse of lambda is not Okun's coefficient in terms of a shock to cyclical GDP affecting unemployment. This is discussed in Barreto and Howland (1993). In our case, the other Okun's coefficient would be $\sigma_{\varepsilon y\varepsilon u}/\sigma_{\varepsilon y}^2$ = -0.5060. This is smaller in absolute value than $1/\lambda$ = -0.7149. Studies often regress the transitory component of unemployment on GDP and then take the inverse of the coefficient as the traditional Okun's coefficient, which would not be correct if the two were not perfectly negatively correlated.

would occur if it is correlated with the measurement error in the dependent variable, then OLS is biased and inconsistent. Therefore, we should use the estimate of the correlation instead of the correlation of the estimates.

The bivariate UC-UR model estimated here does not restrict unemployment and output to have the same temporary innovation. Okun himself posited, and other empirical work has found, that the transitory components of GDP and unemployment may not be perfectly negatively correlated.[27] In the case of the model used here for the US, however, the likelihood ratio test statistic for the hypothesis that the temporary innovations in GDP and unemployment are perfectly negatively correlated is 0.6231. The p-value is 0.43, so we cannot reject this restriction. In fact, the other parameter estimates are extremely robust to this restriction.

The smaller estimate of Okun's coefficient based on this approach versus other approaches might also arise because this approach allows correlation between the two permanent components. If the permanent components are also correlated but all correlation between the two series is forced onto the cyclical components, then the estimate of Okun's coefficient might be biased upward.

Thus, we should also consider the innovations to the permanent components of output and unemployment, which are denoted η_{yt} and η_{ut} respectively.[28] Suppose we have:

$\eta_{yt} = \beta\eta_{ut} + \nu_t$,

where ν is another normal random variable and β is a parameter I refer to as the Okun's coefficient for the permanent components.

We are able to identify β the same way as we identified λ above, which implies $\beta = -2.0733$.[29] If there were no correlation between trend unemployment and output,

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[27] It is, however, common in the literature to interpret Okun's Law as perfect negative correlation between the transitory components of GDP and unemployment, however even Okun himself suggested that there is no reason to expect perfect negative correlation. The inability to reject the perfect negative correlation restriction between GDP cycle and unemployment cycle does somewhat justify the interchanging of these two components in the Phillips Curve.

[28] There are two additional potential Okun's Law-type relationships between the permanent GDP and transitory unemployment and between permanent unemployment and transitory GDP. Since we cannot reject perfect negative correlation between transitory GDP and transitory unemployment, however, these relationships can equivalently be represented by same-series trend and cycle correlation, thus they are not discussed separately here.

which is often assumed in bivariate UC models, then β would be zero. Instead we have a negative relationship, similar to that of transitory unemployment and output, except approximately 1.5 times larger in absolute value. This is not completely surprising considering the estimates suggest more movement in the permanent components of these two series than in the transitory components. This also somewhat explains the larger Okun's coefficients found in studies that employ trend-cycle decomposition that generate smooth trends. Perhaps these studies mistakenly include permanent movements in the transitory component, which would increase their estimate of Okun's coefficient for the temporary movements.

Since we cannot reject perfect negative correlation between temporary movements in GDP and unemployment, the analysis in this paper has focused on the correlation between permanent and temporary movements within a series. There are theories, however, that emphasize the importance of the off-diagonal correlations. In particular, Dreze and Bean (1989) suggest that a temporary shock to unemployment may result in a negative effect on capital accumulation, resulting in a permanent decrease in real output. Although this hypothesis is inconsistent with the correlation found in this study (the estimate is positive), it is still important to consider these correlations as well.

Section 4.4: Comparing the Bivariate UC-UR Model to Other Models

Previous empirical estimates of the permanent and transitory components of output and unemployment can be generally separated into two categories. There are the unobserved components models, based on Clark (1989) and there are the vector autoregression (VAR) models, based on Blanchard and Quah (1989). The substantive difference between the model presented here and the model of Clark (1989) is that Clark restricted the correlations between the components whereas in this paper all components are estimated and the data are allowed to "speak." The substantive difference between the model presented here and the model of Blanchard and Quah (1989) is that Blanchard and Quah assume that the unemployment rate is stationary (and thus enters the VAR in levels), whereas in the model presented here, unemployment includes a permanent component. The results of both Clark and Blanchard and Quah suggest a much larger

[29] If we instead want to consider the left-hand variable to be the permanent innovation to unemployment, then the coefficient would be -0.4257, as compared to $1/\beta = -0.4823$.

transitory component for both GDP and unemployment than what is found using the approach in this paper. This section thus presents evidence that the restrictions employed by both Clark and Blanchard and Quah are rejected by the US data.

Section 4.4.1: Comparing the Model to Clark (1987, 1989)

To test whether the results presented in this paper differ significantly from the findings of Clark (1987, 1989), the model was estimated imposing several different zero-correlation restrictions. The first restriction imposes zero correlation between the permanent and transitory components of GDP while allowing all other correlations to be estimated within the model. Since many models in the past have restricted the correlation between GDP components to equal zero, it is important to test this restriction. The likelihood ratio test statistic is 5.6343. With one restriction, this implies a p-value of 0.0176. Thus we can reject the restriction. With this restriction the results for GDP appear much more similar to those of Clark (1987), but the restriction is rejected by the data. The transitory component of GDP becomes much more persistent ($\phi_{1y} + \phi_{2y} = 0.98$ versus 0.55 without the constraint), and the ratio of permanent innovations to temporary innovations falls, although it is still slightly greater than one. Most noticeable, however, is the impact this assumption has on the estimate of the correlation between the transitory components of GDP and unemployment. The restricted estimate implies that these two are positively correlated, which contradicts Okun's Law.

Clark (1989)[30] imposes perfect negative correlation between the transitory components of GDP and unemployment in order to estimate the correlation between permanent and transitory GDP. As discussed above, however, imposing perfect negative correlation between the two transitory components results in robust estimates of the correlation between the components of GDP. Clark's results differ from the ones presented here because he also imposes zero correlation between all other components. In particular, the difference between Clark's estimates and the ones from the UC-UR model appears to arise from the zero-correlation restrictions between trend GDP and trend unemployment and between the components of unemployment. These zero-

[30] Another UC model of unemployment and output is Runstler (1998). He also uses a common cycle between GDP and unemployment and orthogonality restrictions. Neither of these are imposed in my model.

correlation restrictions are not appropriate for US GDP and unemployment based on the UC-UR results.

Clark (1989) also separately decomposes output and unemployment into nonstationary (trend) and stationary components for several countries. He uses the covariance between the stationary components of the two series to support his case for large temporary movements in real output. He finds evidence that the correlation between the permanent and transitory components of GDP is statistically insignificant. When the components of US GDP and unemployment are jointly estimated with no restriction on the covariances, however, the results are significantly different from those of Clark, and provide further support of a small cyclical component for US GDP.

A final test of potential restrictions of the UC model is to allow correlation between the transitory components (for Okun's Law), but impose zero-correlation restrictions between all other components. The likelihood value of this restricted model was -344.4046, which results in a likelihood ratio test statistic of 33.2539 with 5 restrictions implying a p-value of 0.0000, so we can clearly reject this restriction. Other correlations besides the one implied by Okun's Law are clearly important for the relationship between output and unemployment.

Clark's results are appealing because the estimated transitory component in GDP appears to follow the NBER business cycle dates. His transitory component is similar to the cycle found when using a deterministic linear trend to represent the permanent component of GDP. Although both of these transitory components match what is presented in textbooks, they are rejected by the data when less restrictive models are used. As discussed by Nelson (1988), it appears that these estimates of the transitory component are spurious cycles, not the proper estimate of the transitory component.

Section 4.4.2: Comparing the Model to Blanchard and Quah (1989)

Next, compare the results of the bivariate UC-UR model to the permanent and transitory decomposition of GDP using unemployment by Blanchard and Quah (1989). [31] They assume unemployment is stationary, thus they do not decompose unemployment into two components as appears here. They identify supply disturbances as those which

[31] Other recent papers developing permanent and transitory decompositions of output include Gali (1999), Francis and Ramey (2002), and Rotemberg (2003).

have a permanent effect on output, and demand disturbances those that have no permanent effect on output. These disturbances are assumed to be uncorrelated and not to have a long-run effect on unemployment. Identifying similar shocks in the UC-UR model would be difficult since we would first need to find the part of the innovations to GDP which are uncorrelated with the permanent innovation to unemployment. Next we would need to decompose these parts of innovations into two underlying, uncorrelated shocks. Blanchard and Quah do note, however, that their decomposition is not appropriate as a trend-cycle decomposition because cyclical movements may occur due to both supply and demand disturbances, for example if prices are imperfectly flexible. They further point out that the trend and cycle will be correlated in this case. Thus, rather than trying to decompose the UC-UR innovations into supply and demand disturbances as defined by Blanchard and Quah, I reinterpret their results in light of the UC-UR model.

Blanchard and Quah find that a positive permanent innovation in output has a temporary positive effect on unemployment. This is consistent with the estimates of the UC-UR model, as presented in Table 1. Blanchard and Quah interpret this positive correlation as arising from nominal rigidities or real wage rigidities where aggregate demand does not initially increase enough to prevent unemployment from temporarily falling due to an increase in productivity.

Blanchard and Quah further find a negative correlation for the temporary innovations which also matches the UC-UR model results, and the predictions of Okun's Law. Combining the positive correlation between permanent GDP and temporary unemployment with a negative correlation between the temporary components of GDP and unemployment connects the relationship between permanent GDP and transitory unemployment to the relationship between permanent and transitory GDP. It is particularly clear in the case where the transitory components are perfectly negatively correlated. In this case, the relationship between permanent GDP and transitory unemployment is identical, up to a constant, to the relationship between permanent and transitory GDP. Thus a finding of a positive correlation between permanent GDP and transitory unemployment, combined with a negative correlation between transitory GDP and transitory unemployment, is further support of a negative correlation between permanent and transitory GDP.

Blanchard and Quah also find that Okun's coefficient is "a mongrel coefficient" because it depends on the type of disturbance. This is even more evident in the UC-UR case since there is also a relationship between permanent GDP and permanent unemployment. Blanchard and Quah find a tight relation for demand disturbances, but supply disturbances present a confusing picture. This may arise from the restriction that there is no permanent stochastic component to unemployment. At fairly long horizons, however, Blanchard and Quah find that the implied coefficient slightly exceeds 4 for supply disturbances, which is much higher in absolute value than their estimate of Okun's coefficient. This fits with the finding from the UC-UR model that the Okun's coefficient for the permanent components is larger in absolute value than the traditional Okun's coefficient for the transitory components. They explain this as arising from supply disturbances increasing output with little or no change in employment. Furthermore, Blanchard and Quah's assumption that all movements in unemployment are temporary drives the finding of a much larger transitory component.

When the UC-UR model includes the restriction that all movements in unemployment are stationary, the results are substantially different, which implies that one of the major drivers for the results presented for the UC-UR model is that unemployment experiences permanent movements. Without this assumption, unemployment embodies the transitory component of GDP (the model suggests they are almost perfectly negatively correlated), which results in an insignificant (although still negative) correlation between the permanent component and the transitory component of GDP. Another interesting result is that the correlation between the trend of GDP with unemployment implied by the stationary unemployment model is positive, thus if unemployment is stationary, then we see that a positive shock to trend GDP (for example a technology shock) will be accompanied by a temporary increase in unemployment. When a permanent component in unemployment is allowed, however, then there is a negative relationship between the permanent components as seen in Table 1.

The distribution of the likelihood ratio test statistic is nonstandard, but a Monte Carlo simulation can be used to establish appropriate confidence bands. The data was generated with the results from the partial UC-UR model used as the values for the null. The largest likelihood ratio test statistic generated from 1000 draws was 39.8. The likelihood ratio test statistic for the restriction in the UC-UR model is 67.2, which implies that the null of stationary unemployment can clearly be rejected in favor of the alternative of permanent movements in unemployment. In addition, recall that the UC-UR model finds a large variance for the permanent component of unemployment, when it is not constrained to zero.[32] These results suggest that unemployment should be modeled with a permanent component.

Section 4.5: Interpreting the Results

Accepting the results of this model calls into question a number of macroeconomic theories for the post-war US. In particular, separating the study of permanent movements from that of transitory movements in GDP or unemployment implicitly requires the assumption that they are uncorrelated. The results presented here, however, add to the growing literature that suggests an important link between growth and fluctuations. In addition, since a significant portion of the NBER business cycle movements appear in the permanent component as estimated by this model, the results presented here also call into question theories which explain the business cycle with temporary shocks.

These results also cast doubt on theories which suggest positive correlation between permanent and transitory components in GDP or unemployment. These are often called models of hysteresis where part of a temporary shock persists and becomes permanent. Blanchard and Summers (1986) suggest three possible reasons for hysteresis in the unemployment rate. First, firms may reduce capital stock along with employment after a negative shock which could reduce the subsequent demand for labor. Second, long periods of unemployment may cause workers to lose skills.[33] Third, after a negative shock, insiders who managed to remain employed could push up wages due to increased marginal productivity, and this increase in wages may permanently raise the unemployment rate. Similar stories have also been suggested for output. For example,

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[32] Unemployment may experience a few large permanent movements which might be difficult to capture with the random walk and might better be modeled as structural breaks. Another possibility is that unemployment responds asymmetrically to shocks. Caner and Hansen (2001) find that allowing asymmetric responses in unemployment results in the rejection of a unit root.

[33] This is also an argument for hysteresis suggested by Pissarides (1992).

Okun (1962) suggested that the components of output should be positively correlated because a large output gap may also result in a permanent decrease in GDP due to an increase in the average age of the nation's capital stock. Although hysteresis may still occur for individual shocks, the dominating effect results in a negative correlation between permanent and transitory innovations on average.[34] Proietti (2002), however, suggests that the negative correlation would be consistent with a certain class of hysteresis theories where a positive temporary shock leads to a permanent reduction in trend GDP. An example of this might come from Clark (1987), where he suggests that fiscal policy may increase production today but the resulting higher taxes or interest rates might decrease capital stock and thus lower permanent output.[35]

Based on the estimates presented here, supporting those of MNZ and Morley (2004), there appear to be important interactions between permanent and transitory movements in GDP. As Stock and Watson (1988, page 148) point out, "theories explaining only growth (permanent movements) or only transitory movements cannot provide adequate macroeconomic insights if there are important interactions between the two." The results presented in this paper cast doubt on models that separate the study of GDP growth and transitory movements, as well as models which treat business cycle movements as exclusively temporary.

Several potential interpretations of the UC-UR results do exist. Stock and Watson (1988) and MNZ both interpret negative correlation between permanent and transitory GDP as arising from real shocks which shift permanent GDP today, but it takes time for the series to adjust. Blanchard and Quah (1989) suggest that this adjustment time arises from supply shocks combined with nominal rigidities, such as imperfectly flexible prices. Real business cycle theories explain fluctuations as arising from real shocks which require more than one period for the construction of new productive capital, such as those of Prescott (1987) and Kydland and Prescott (1982). In either case, real shocks affect both trend and transitory movements because the series takes time to adjust to the

[34] Hysteresis may be more important for European examples. For example, Jaeger and Parkinson (1990) find no evidence of hysteresis for the US, but they find evidence of hysteresis for German data.
[35] Examining the 2001 recession presented in Figure 2 suggests that the effects of higher taxes or interest rates were not immediately incorporated into the permanent component at the time of increased fiscal policy. This may be due to the short amount of data available after the recession, but most of the post-war recessions appear similar.

permanent movement. This is true for both output and unemployment. Plosser (1989) cites Black (1987) to extend multi-sector real business cycle models to incorporate unemployment. In Black's model, permanent shocks may require labor and/or capital adjustment between sectors which results in a similar time-to-build movement in unemployment. Further research is needed, however, to determine what causes these permanent shocks[36] and what prevents the series from adjusting immediately to these shocks.

Another possibility is that since this model requires transitory movements to be symmetric, the model may be primarily picking up what happens in expansions (due to significantly more data on expansions). Recessions may still arise from nominal shocks, but the model may not pick up these shocks due to the dominance of expansions in the data.

More research is needed to further explore the Okun's Law-type relationship found between the permanent components and the finding that NBER-dated recessions appear to be accompanied by sizable permanent movements in both GDP and unemployment. These two results appear to go together in that we see costly changes in the unemployment rate during NBER dated recessions, whether the shocks are permanent or temporary. It is important to determine what types of shocks the economy is experiencing, however, in order to choose a correct policy response. For example, there is a much larger economic cost from a permanent increase in the unemployment rate, and the policies used to combat increasing unemployment may not be effective if the increase in permanent.[37]

Section 5: Conclusions and Extensions

This paper presents a bivariate correlated unobserved components model for output and unemployment which eliminates arbitrary restrictions found in other models previously used in the literature. In particular, it allows correlation between all of the

[36] In particular, many economists question the idea of frequent permanent shocks to unemployment. These shocks may arise from supply shocks (Gordon 1997 attempts to control for supply shocks in estimating the permanent component of unemployment to prevent this estimate from "jumping around."), sectoral shifts (King and Morley 2003), changes in matching technology, etc. Vickrey (1993) declares that the permanent component of unemployment "is of course not a fixed datum, but varies over time and place according to the sociopolitical ambience, the mechanics of the labor market, and the vigor of competition."

[37] An important caveat is that there are so many permanent shocks that might offset each other, that a permanent shock to unemployment in one period may be reversed in the next.

components resulting in substantively different estimates than those of previous UC models. Much of the variance in both GDP and unemployment is found in the permanent components, instead of the smoother trend of zero-covariance UC models. In addition, these correlations indicate that there exists an important relationship between the permanent component and transitory component for each series. This suggests that it is inappropriate to treat these components as wholly separate. Finally, Okun's coefficient for permanent movements indicates that GDP and unemployment are even more strongly linked through their permanent components than through their transitory components.

Future directions include applying the bivariate UC-UR model to other developed countries which will allow cross-country comparisons to establish whether the relationships discussed here are specific to the US or are more generally present in developed countries. Where they differ, it would be interesting to explore possible explanations.[38] As noted in Clark (1987), if these results are a "statistical accident," then they should not show up in the analysis for other countries.

Another further extension would be to add asymmetry to the cyclical component following Friedman's plucking model (1969). Both unemployment and GDP may move differently in recessions as compared to expansions. Caner and Hansen (2001) find that the US unemployment rate experiences different regimes resulting in asymmetry. Asymmetric models have also become popular for GDP (Kim and Nelson 1999; 2001). Using Kim and Nelson's (1999) general econometric time series model which incorporates asymmetric movements in the unobserved components, it is possible to incorporate asymmetry in the transitory component of each of the series of the baseline model. The state variable may also be correlated with the error term, thus the approach of Kim, Piger, and Startz (2003) make it possible to instrument for the state variable. This model can then be used to test the robustness of the results from the baseline model in the face of asymmetry. If the transitory component is asymmetric, then using the plucking model should improve the fit not only of the transitory component, but the trend may also become smoother in recessions.

[38] For example, Moosa (1997) shows that the traditional Okun's coefficient varies significantly across countries. It will be interesting to see if this is also true for the coefficient for permanent movements and if this is perhaps related to different countries' social insurance programs.

Figure 1: GDP and the Estimate of the Permanent Component of GDP

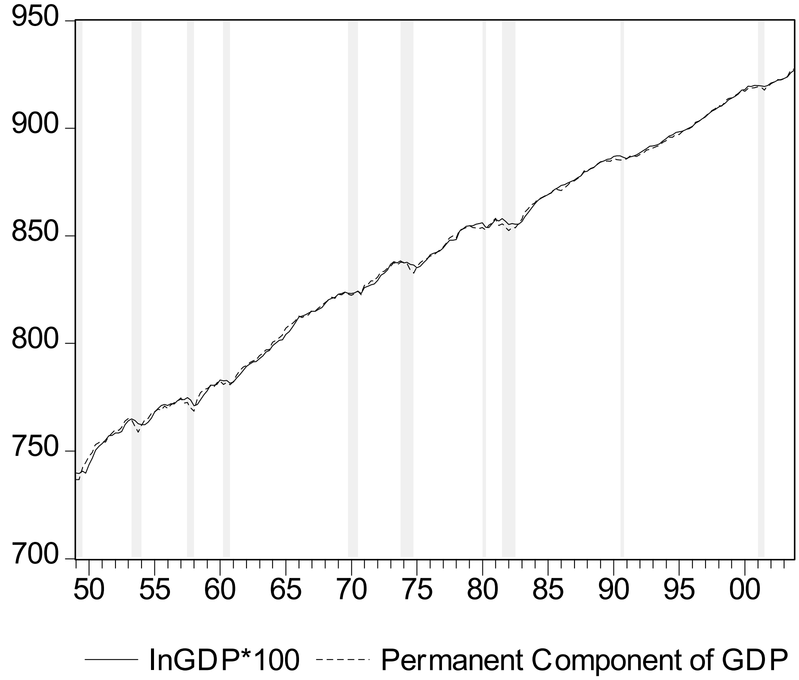

Figure 2: Close-Up of 2001 Recession

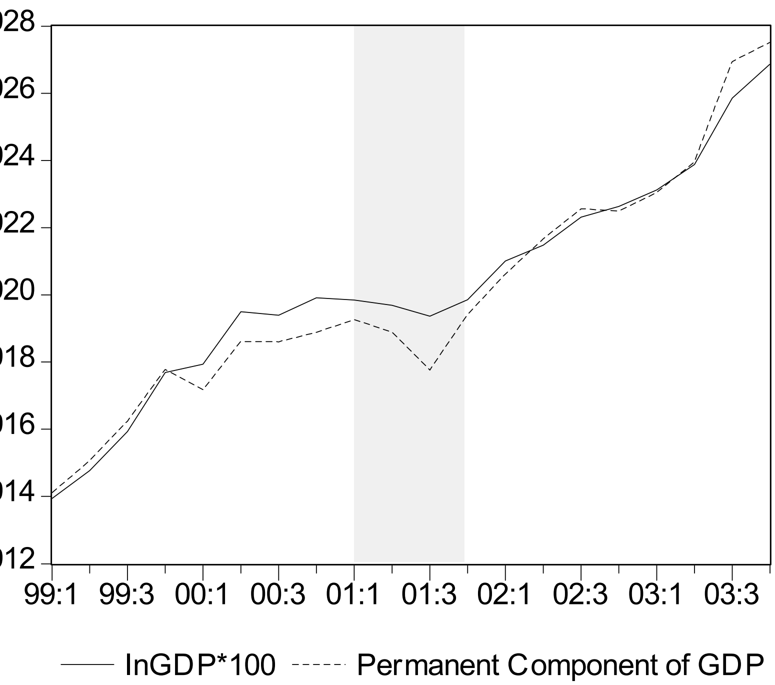

Figure 3: Unemployment: Series and Components

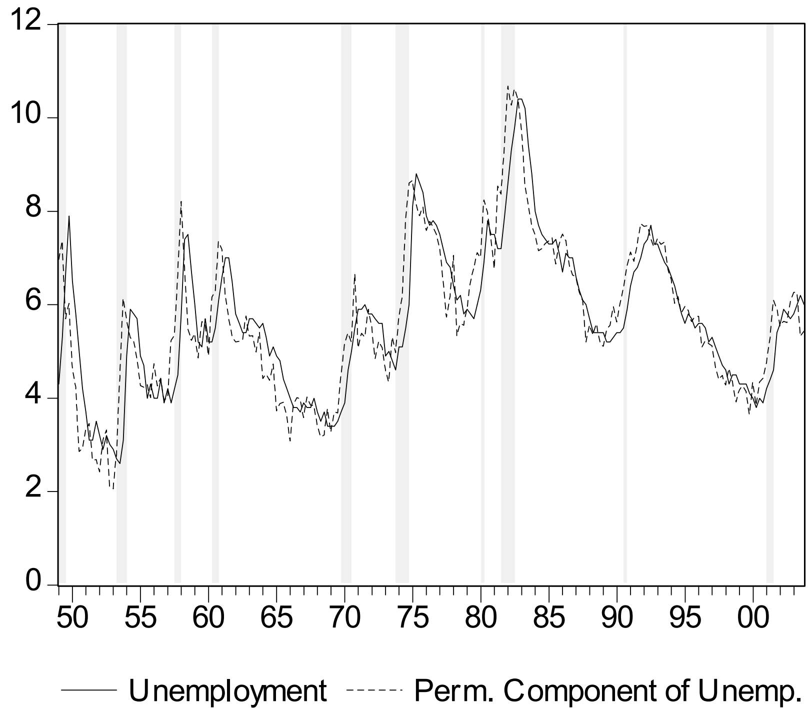

Figure 4: Close-Up of 2001 Recession

Figure 5: Simulated Response of GDP Trend and Series to a Permanent Shock

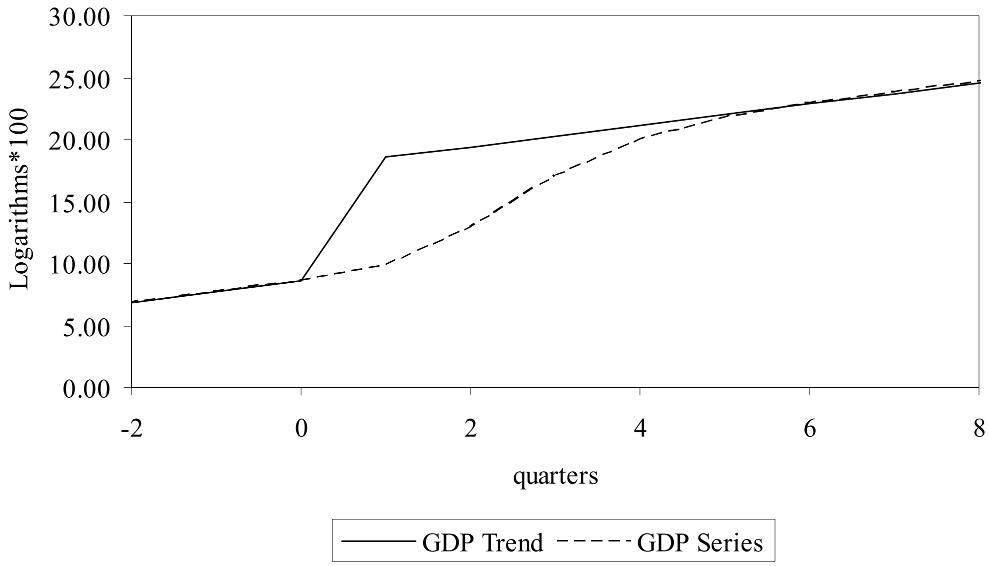

Figure 6: Simulated Response of Unemployment Trend and Series to a Permanent Shock

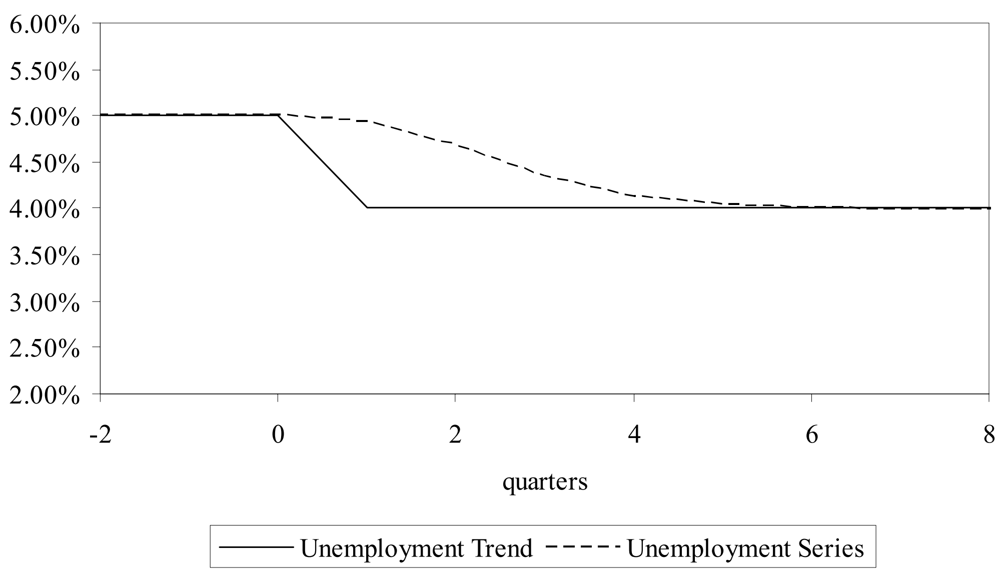

Table 1: Maximum Likelihood Estimates of the Bivariate UC-UR for GDP[39]

Description	Parameter	Estimate (Standard Error)
Output		
S.D. of Permanent Innovation to GDP	$\sigma_{\eta y}$	1.5689 (0.2334)
S.D. of Temporary Innovation to GDP	$\sigma_{\varepsilon y}$	1.1406 (0.3407)
Covariance between GDP Components	$\sigma_{\eta y \varepsilon y}$	-1.5508 (0.7163)
Correlation between GDP Components	$\rho_{\eta y \varepsilon y}$	-0.8666 (0.0500)
GDP Drift	μ	0.8570 (0.0364)
GDP 1st AR parameter	ϕ_{1y}	0.7576 (0.0698)
GDP 2nd AR parameter	ϕ_{2y}	-0.2047 (0.1008)
GDP Roots of AR process		0.3788± 0.2473i
Unemployment		
S.D. of Permanent Innovation to Unemployment	$\sigma_{\eta u}$	0.7109 (0.1037)
S.D. of Temporary Innovation to Unemployment	$\sigma_{\varepsilon u}$	0.6860 (0.1512)
Covariance between Unemployment Components	$\sigma_{\eta u \varepsilon u}$	-0.4528 (0.1693)
Correlation between Unemployment Components	$\rho_{\eta u \varepsilon u}$	-0.9286 (0.0371)
Unemployment 1st AR parameter	ϕ_{1u}	0.7416 (0.0628)
Unemployment 2nd AR parameter	ϕ_{2u}	-0.1789 (0.0621)
Unemployment Roots of AR process		0.3708 ± 0.2033
Cross Series Covariances and Correlations		
Covariance: Permanent Unemp./Permanent GDP	$\sigma_{\eta y \eta u}$	-1.0478 (0.3155)
Correlation: Permanent Unemp./Permanent GDP	$\rho_{\eta y \eta u}$	-0.9395 (0.0249)
Covariance: Permanent GDP/Transitory Unemp.	$\sigma_{\eta y \varepsilon u}$	1.0696 (0.3822)
Correlation: Permanent GDP/Transitory Unemp.	$\rho_{\eta y \varepsilon u}$	0.9939 (0.0206)
Covariance: Permanent Unemp./Transitory GDP	$\sigma_{\eta u \varepsilon y}$	0.5397 (0.2837)
Correlation: Permanent Unemp./Transitory GDP	$\rho_{\eta u \varepsilon y}$	0.6644 (0.1289)
Covariance: Transitory GDP/Transitory Unemp.	$\sigma_{\varepsilon y \varepsilon u}$	-0.6583 (0.3688)
Correlation: Transitory GDP/Transitory Unemp.	$\rho_{\varepsilon y \varepsilon u}$	-0.8414 (0.0903)

[39] The log likelihood value is -327.7777.

Appendix 1: State Space Form

Assuming unemployment and output each have a permanent trend component implies that we cannot estimate them as stationary. Conditional on initial values, however, a random walk is stationary, thus we require an initial value for each tau. It is possible to concentrate the initial value for tau out of the likelihood function, so it will be consistent no matter which approach we choose. This model therefore uses a diffuse prior for the initial value of each tau, but the results are robust to instead estimating them as parameters. Casting the model in state-space form makes it possible to use the Kalman filter for maximum likelihood estimation of the parameters.

Observation Equation: $$\begin{bmatrix} y_t \\ u_t \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_{yt} \\ \tau_{ut} \\ c_{yt} \\ c_{yt-1} \\ c_{ut} \\ c_{ut-1} \end{bmatrix}$$

State Equation: $$\begin{bmatrix} \tau_{yt} \\ \tau_{ut} \\ c_{yt} \\ c_{yt-1} \\ c_{ut} \\ c_{ut-1} \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_{1y} & \phi_{2y} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_{1u} & \phi_{2u} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_{yt-1} \\ \tau_{ut-1} \\ c_{yt-1} \\ c_{yt-2} \\ c_{ut-1} \\ c_{ut-2} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_{yt} \\ \eta_{ut} \\ \varepsilon_{yt} \\ \varepsilon_{ut} \end{bmatrix}$$

Variance-Covariance Matrix:

$$E\left[\begin{bmatrix} \eta_{yt} \\ \eta_{ut} \\ \varepsilon_{yt} \\ \varepsilon_{ut} \end{bmatrix} \begin{bmatrix} \eta_{yt} & \eta_{ut} & \varepsilon_{yt} & \varepsilon_{ut} \end{bmatrix} \right] = \begin{bmatrix} \sigma^2_{\eta_y} & \sigma_{\eta_y \eta_u} & \sigma_{\eta_y \varepsilon_y} & \sigma_{\eta_y \varepsilon_u} \\ \sigma_{\eta_y \eta_u} & \sigma^2_{\eta_u} & \sigma_{\eta_u \varepsilon_y} & \sigma_{\eta_u \varepsilon_u} \\ \sigma_{\eta_y \varepsilon_y} & \sigma_{\eta_u \varepsilon_y} & \sigma^2_{\varepsilon_y} & \sigma_{\varepsilon_y \varepsilon_u} \\ \sigma_{\eta_y \varepsilon_u} & \sigma_{\eta_u \varepsilon_u} & \sigma_{\varepsilon_y \varepsilon_u} & \sigma^2_{\varepsilon_u} \end{bmatrix}$$

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