Solutions: HW1.

1.48. We have

$$z_0 = r_0 e^{j\theta_0} = r_0 \cos\theta_0 + j r_0 \sin\theta_0 = x_0 + j y_0$$

(a) $z_1 = x_0 - j y_0$

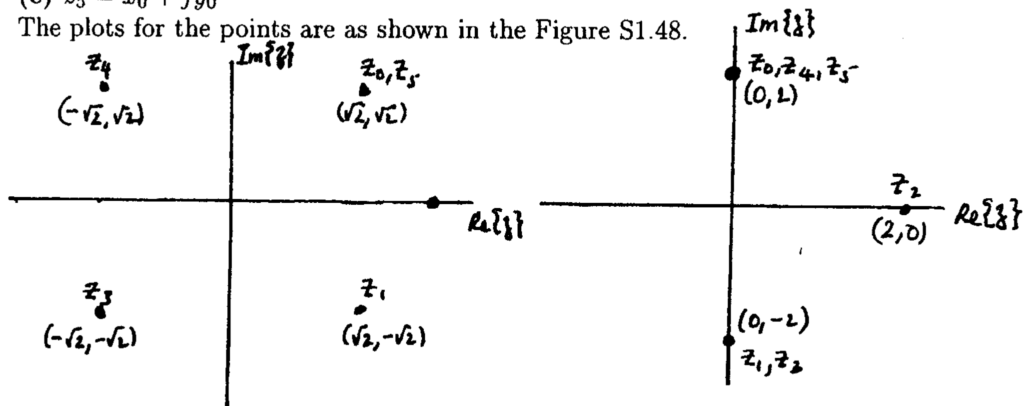
(b) $z_2 = \sqrt{x_0^2 + y_0^2}$

(c) $z_3 = -x_0 - j y_0 = -z_0$

✓ (d) $z_4 = -x_0 + j y_0$

✓ (e) $z_5 = x_0 + j y_0$

The plots for the points are as shown in the Figure S1.48.



$r_0 = 2,\ \theta_0 = \pi/4$ **Figure S1.48** $r_0 = 2,\ \theta_0 = \pi/2$

1.49. (e) $8e^{-j\pi}$

(i) $e^{j\pi/6}$

(j) $\sqrt{2}e^{j11\pi/12}$

Plot depicting these points is as shown in Figure S1.49.

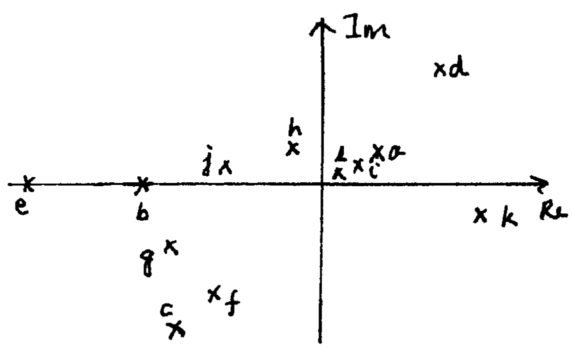


Figure S1.49

1.51. (a) We have

$$e^{j\theta} = \cos\theta + j \sin\theta. \qquad \text{(S1.51–1)}$$

and

$$e^{-j\theta} = \cos\theta - j \sin\theta. \qquad \text{(S1.51–2)}$$

Summing eqs. (S1.51-1) and (S1.51-2) we get

$$\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta}).$$

HW1.

1.52. **(a)** $zz^* = re^{j\theta}re^{-j\theta} = r^2$

(c) $z + z^* = x + jy + x - jy = 2x = 2\mathcal{R}e\{z\}$

1.53. **(a)** $(e^z)^* = (e^x e^{jy})^* = e^x e^{-jy} = e^{x-jy} = e^{z^*}$.

1.54. **(a)** For $\alpha = 1$, it is fairly obvious that

$$\sum_{n=0}^{N-1} \alpha^n = N.$$

For $\alpha \neq 1$, we may write

$$(1-\alpha)\sum_{n=0}^{N-1} \alpha^n = \sum_{n=0}^{N-1} \alpha^n - \sum_{n=0}^{N-1} \alpha^{n+1} = 1 - \alpha^N.$$

Therefore,

$$\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}.$$

(b) For $|\alpha| < 1$,

$$\lim_{N\to\infty} \alpha^N = 0.$$

Therefore, from the result of the previous part,

$$\lim_{N\to\infty} \sum_{n=0}^{N-1} \alpha^n = \sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}.$$

(c) Differentiating both sides of the result of part (b) wrt α, we get

$$\begin{aligned} \frac{d}{d\alpha}\left(\sum_{n=0}^{\infty} \alpha^n\right) &= \frac{d}{d\alpha}\left(\frac{1}{1-\alpha}\right) \\ \sum_{n=0}^{\infty} n\alpha^{n-1} &= \frac{1}{(1-\alpha)^2} \end{aligned}$$

(d) We may write

$$\sum_{n=k}^{\infty} \alpha^n = \alpha^k \sum_{n=0}^{\infty} \alpha^n = \frac{\alpha^k}{1-\alpha} \text{ for } |\alpha| < 1.$$

1.55. **(a)** The desired sum is

$$\sum_{n=0}^{9} e^{j\pi n/2} = \frac{1 - e^{j\pi 10/2}}{1 - e^{j\pi/2}} = 1 + j.$$

(d) The desired sum is

$$\sum_{n=2}^{\infty} (1/2)^n e^{j\pi n/2} = (1/2)^2 e^{j\pi 2/2} \sum_{n=0}^{\infty} (1/2)^n e^{j\pi n/2} = -\frac{1}{4}\left[\frac{4}{5} + j\frac{2}{5}\right].$$

HW1: Solutions

B-1: Solve the equation $z^4 = 2$ $(z \in \mathbb{C})$.

Ans: $z^2 = \pm\sqrt{2}$

$z^2 = \sqrt{2} \Rightarrow z = \pm\sqrt[4]{2}$

$z^2 = -\sqrt{2} \Rightarrow z = \pm\sqrt[4]{2}\,j$.

B-2: $e^{j\pi} + \cos^2\phi + \sin^2\phi$ (since $\cos^2\phi + \sin^2\phi = 1$)

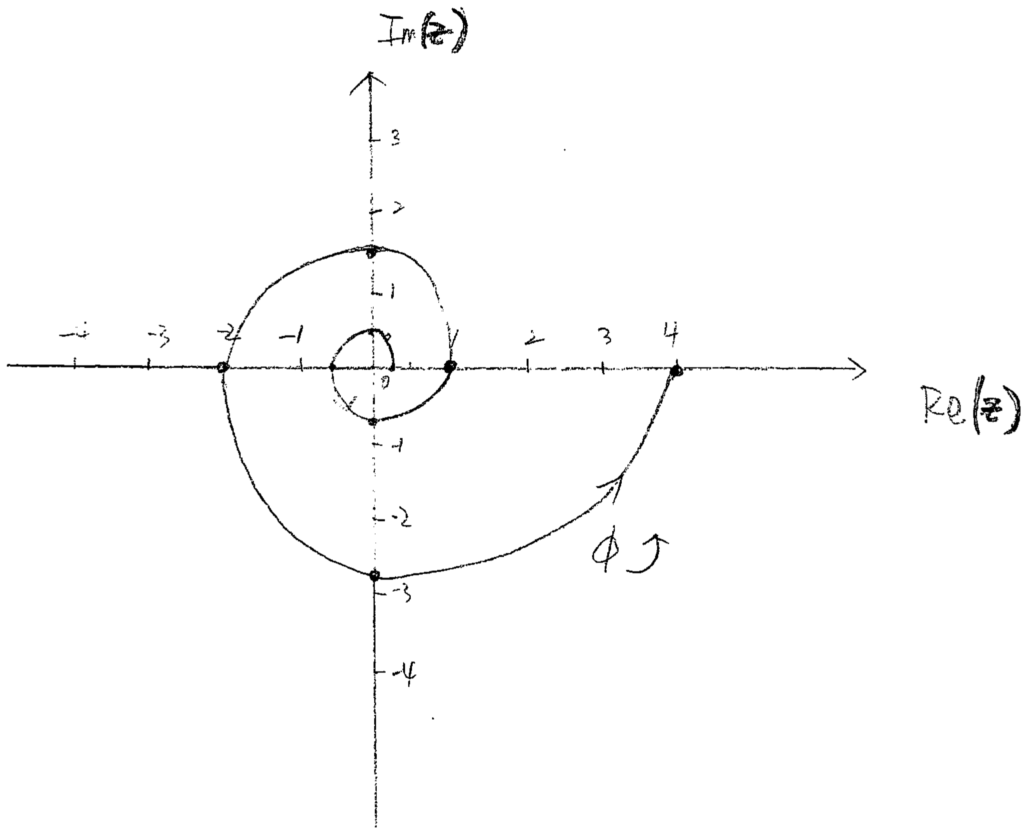
$= e^{j\pi} + 1 = \cos\pi + j\sin\pi + 1$

$= -1 + 0 + 1 = 0$

B-3: sketch the curve $z(\phi) = 2^{\phi/\pi} e^{j\phi}$.

Ans:

ϕ	-2π	$-\frac{3}{2}\pi$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π
z	$\frac{1}{4}$	$\sqrt{\frac{1}{8}}$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}j$	1	$\sqrt{2}j$	-2	$-\sqrt{8}j$	4



Form 1: Some Common Sinusoid Values

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\operatorname{ctan}\theta$
0	0	1	0	N/A
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{2}$	1	0	N/A	0
$\frac{2}{3}\pi$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$
$\frac{3}{4}\pi$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1
$\frac{5}{6}\pi$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$
π	0	-1	0	N/A

Form 2: Some Sinusoid Relations

Angle \ function	Sin	Cos	tan
$-\theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$
$\frac{\pi}{2}\pm\theta$	$\cos\theta$	$\mp\sin\theta$	$\mp\operatorname{ctan}\theta$
$\pi\pm\theta$	$\mp\sin\theta$	$-\cos\theta$	$\pm\tan\theta$
$\frac{3}{2}\pi\pm\theta$	$-\cos\theta$	$\pm\sin\theta$	$\mp\operatorname{ctan}\theta$
$2\pi\pm\theta$	$\pm\sin\theta$	$\cos\theta$	$\pm\tan\theta$
$n\pi\pm\theta$	$\pm(-1)^n\sin\theta$	$(-1)^n\cos\theta$	$\pm\tan\theta$