HW2: Solutions

1.21. The signals are sketched in Figure S1.21.

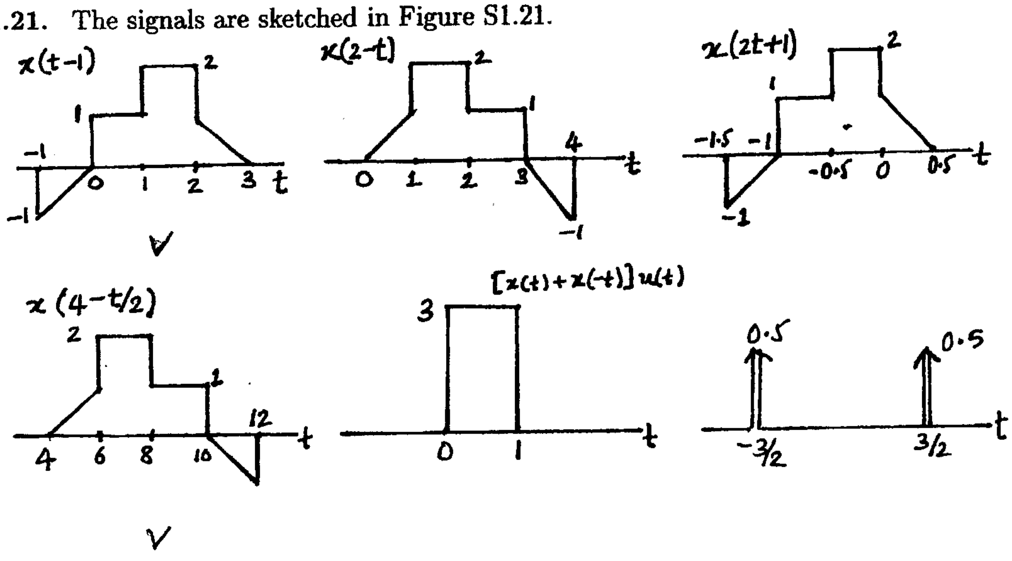

Figure S1.21

1.22. The signals are sketched in Figure S1.22.

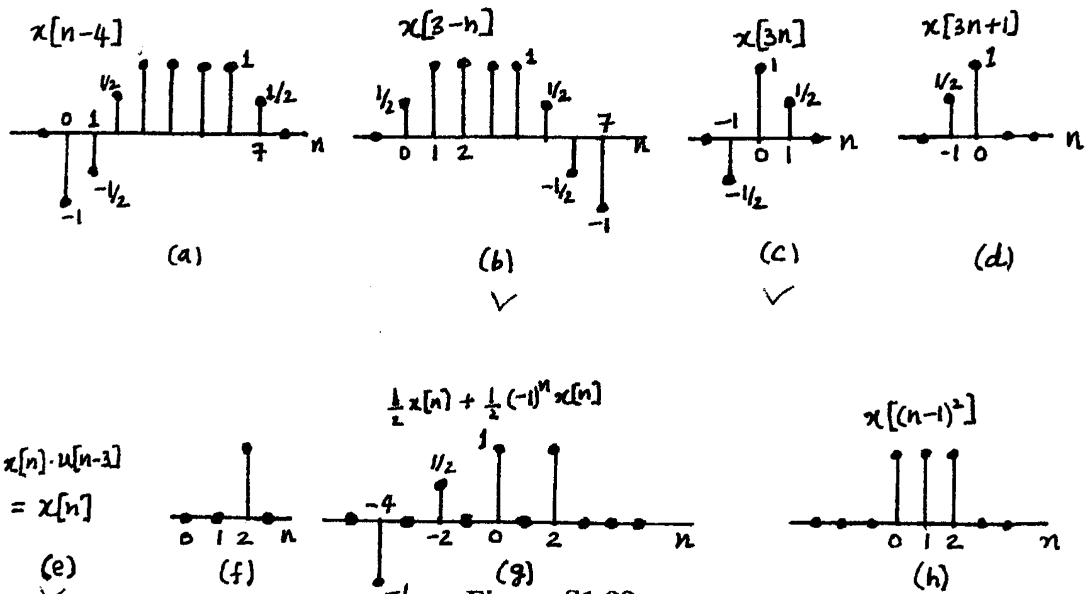

Figure S1.22

1.32. All statements are true.

(1) $x(t)$ periodic with period T; $y_1(t)$ periodic, period $T/2$.

(2) $y_1(t)$ periodic, period T; $x(t)$ periodic, period $2T$.

(3) $x(t)$ periodic, period T; $y_2(t)$ periodic, period $2T$.

(4) $y_2(t)$ periodic, period T; $x(t)$ periodic, period $T/2$.

1.56. **(a)** The desired integral is

$$\int_0^4 e^{j\pi t/2}dt = \left.\frac{e^{\pi t/2}}{j\pi/2}\right|_0^4 = 0.$$

HW2 : Solutions

(d) The desired integral is

$$\int_0^\infty e^{-(1+j)t}dt = \left.\frac{e^{-(1+j)t}}{-(1+j)}\right|_0^\infty = \frac{1}{1+j} = \frac{1-j}{2}.$$

(f) The desired integral is

$$\int_0^\infty e^{-2t}\sin(3t)dt = \int_0^\infty \left[\frac{e^{-(2-3j)t} - e^{-(2+3j)t}}{2j}\right]dt = \frac{1/2j}{2-3j} + \frac{1/2j}{2+3j} = \frac{3}{13}.$$

Problem B:

1.2a. Since $\omega = \frac{2\pi M}{N}$, the fundamental period $T_0 = m\frac{2\pi}{\omega} = m\frac{N}{M}$. Assume $N = 12$,

- if $M = 4$, $T_0 = 3$;
- if $M = 5$, $T_0 = 12$;
- if $M = 7$, $T_0 = 12$;
- if $M = 4$, $T_0 = 6$.

For arbitrary M and N, if M and N are coprime, then the fundamental period $T_0 = N$. If M and N has a common divisor k, i.e. $M = k \cdot M_1$ and $N = k \cdot N_1$, M_1 an N_1 are coprime, then the fundamental period $T_0 = N_1$. If $M = N/2$, the signal $x(t)$ is zero for all n. The fundamental period is 1.

The Matlab code for $M = 4$ is the following. For other cases, you just need to replay M by the different values.

```
clear all;
clf;
N=12;
n=[0:2*N-1];
M=4;
x=sin(2*pi*M*n/N);
subplot(211)
stem(n,x);
xlabel('n');
ylabel('x_M[n]');
title('M=4');
```

1.2d. The fundamental period of x_1 is 4. The fundamental period of x_2 is 4 and the fundamental period of x_3 is 16. The code for x_1 is shown as follows.

```
clear all;
clf;
n=[0:31];
x_1=sin(pi*n/4).*cos(pi*n/4);
%( or x_2=cos(pi*n/4).^2;  x_3=sin(pi*n/4).*cos(pi*n/8))
subplot(311)
stem(n,x_1);
xlabel('n');
ylabel('x_1[n]');
```

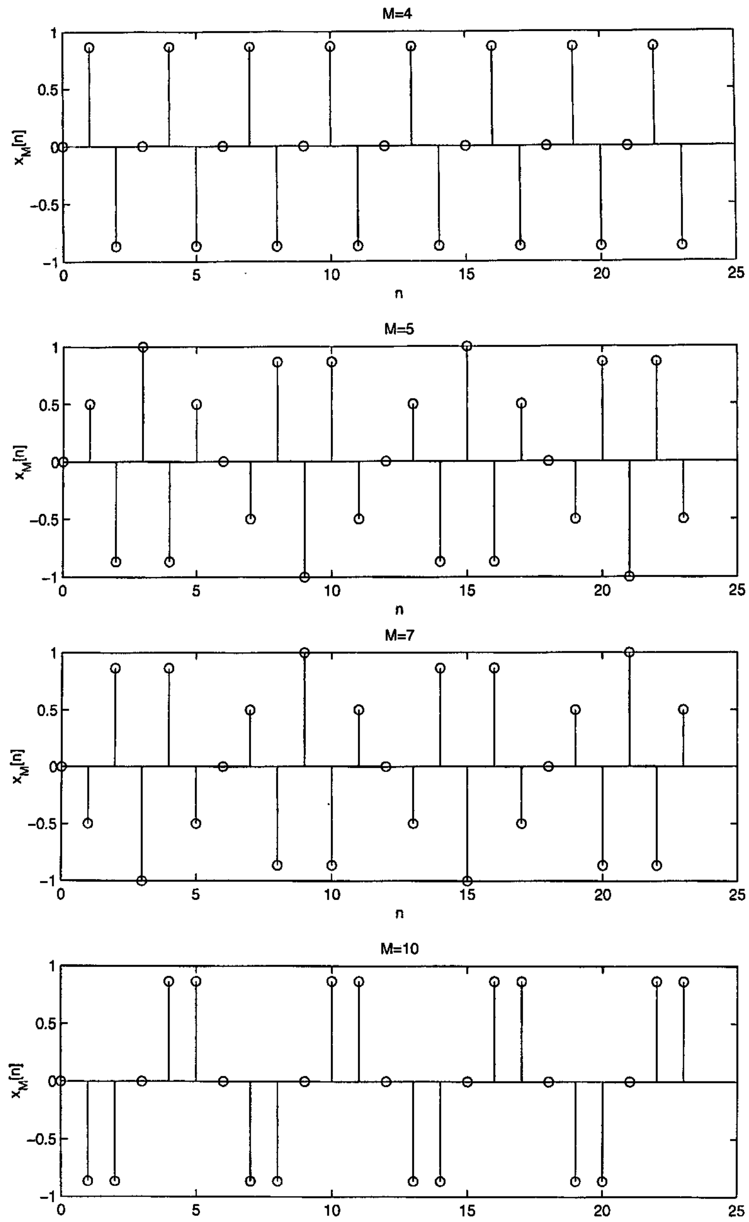

Figure 1: Homework 2:1.2(a)

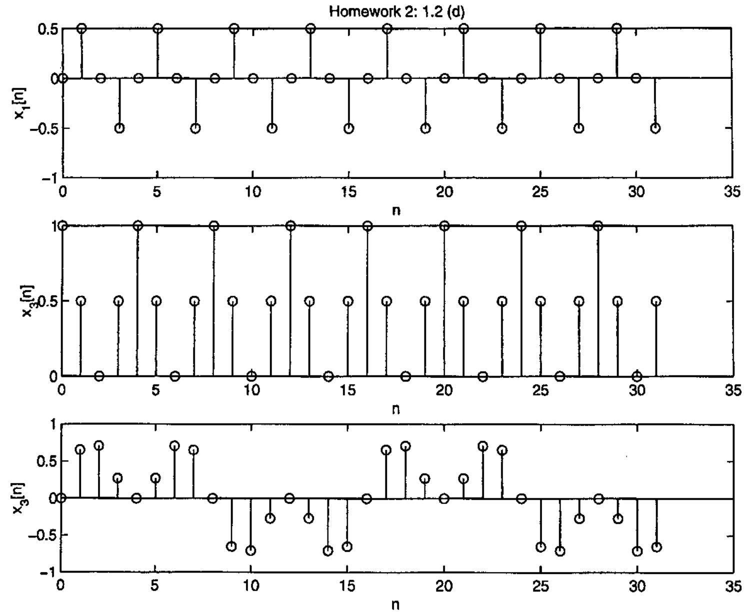

Figure 2: Homework 2:1.2(d)

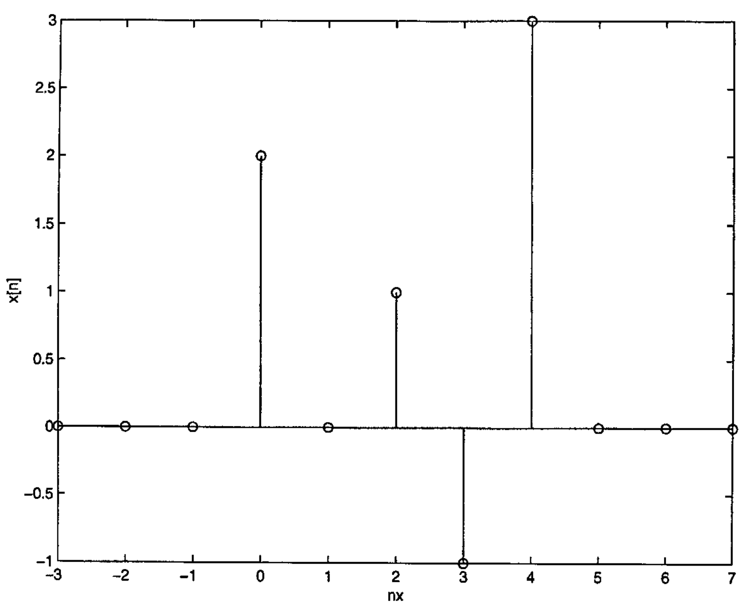

Figure 3: Homework 2:1.3(a)

1.3.(a). The codes for (a) and (c) are the following:

```
clear all;
clf;
nx=[-3:7]
x=zeros(1,length(nx));
for i=1:1:length(nx)
    if nx(i)==0
        x(i)=2;
        elseif nx(i)==2
        x(i)=1;
    elseif nx(i)==3
        x(i)=-1;
    elseif nx(i)==4
        x(i)=3;
    end
end

figure(1)
stem(nx,x);
xlabel('nx');
ylabel('x[n]');

ny1 = nx+2;
ny2 = nx-1;
ny3 = -nx;
ny4 = -nx+1;
y1=x;
y2=x;
y3=x;
y4=x;
subplot(4,1,1);
stem(ny1,y1);
subplot(4,1,2);
stem(ny2,y2);
subplot(4,1,3);
stem(ny3,y3);
subplot(4,1,4);
stem(ny4,y4);
```

1.3(c). y_1 is delayed by 2, y_2 is advanced by 1, y_3 is flipped, y_4 is flipped and then delayed by 1.

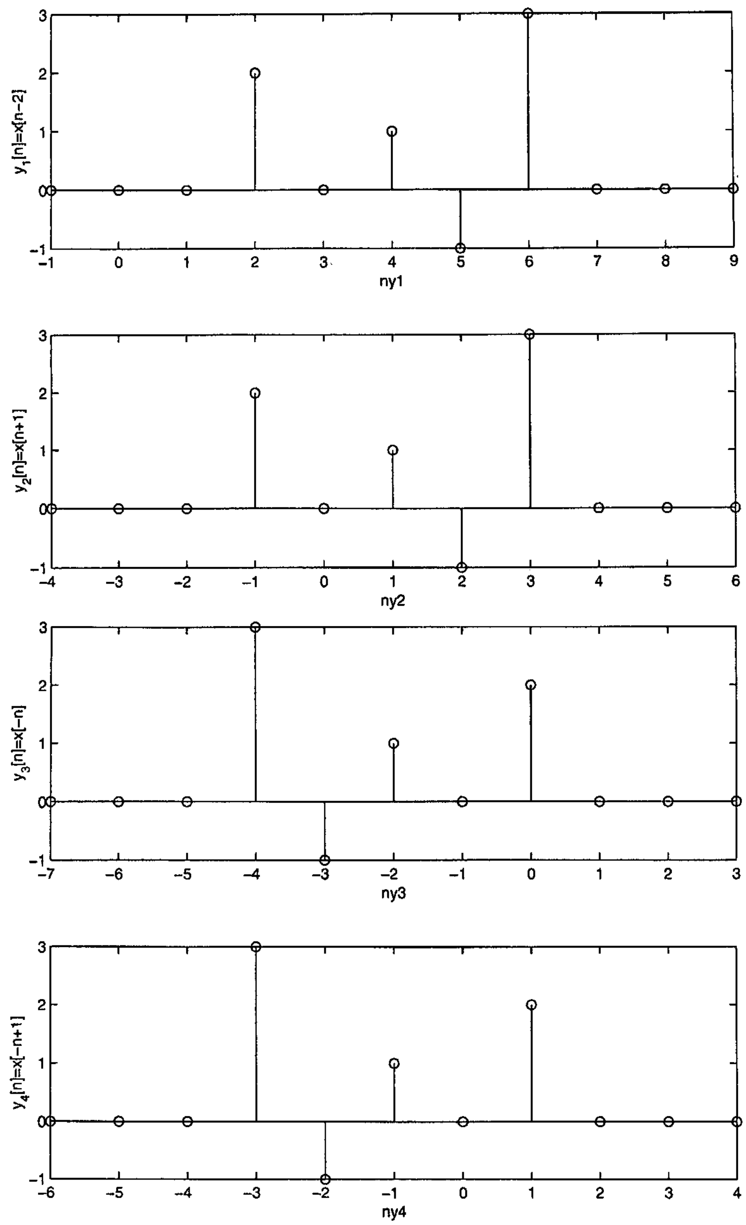

Figure 4: Homework 2:1.3(c)

Problem C-1:

(a) $\ln(1/(1+j)) = \ln\left(\frac{1-j}{2}\right) = \ln\left(\frac{1}{\sqrt{2}}e^{-\frac{\pi}{4}j}\right) = -\frac{1}{2}\ln 2 - \frac{\pi}{4}j.$

(b)

$$\begin{aligned}\cos(1+j) &= \frac{e^{j(1+j)}+e^{-j(1+j)}}{2} = \frac{e^{-1}\cdot e^{j}+e\cdot e^{-j}}{2} = \frac{(e^{-1}+e)\cos(1)}{2} + j\frac{(e^{-1}-e)\sin(1)}{2} \\ &= \cos(1)\cdot\cosh(1) - j\sin(1)\cdot\sinh(1).\end{aligned}$$

(c) $(1-j)^j = \left(\sqrt{2}\cdot e^{-j\frac{\pi}{4}}\right)^j = (\sqrt{2})^j\cdot e^{\frac{\pi}{4}} = e^{j\ln\sqrt{2}}\cdot e^{\frac{\pi}{4}} = e^{\frac{\pi}{4}}\cdot\cos(\ln\sqrt{2}) + je^{\frac{\pi}{4}}\cdot\sin(\ln\sqrt{2}).$

Problem C-2:

(a) Let $x(t) = 5 + 10\cos(100t + \frac{\pi}{3})$, the power of $x(t)$ is

$$\begin{aligned}P_\infty &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2dt \\ &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}(25 + 100\cos(100t+\frac{\pi}{3}) + 100\cos^2(100t+\frac{\pi}{3}))dt \\ &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}(25 + 100\cos(100t+\frac{\pi}{3}) + 100\frac{1+\cos(200t+\frac{2\pi}{3})}{2}dt \\ &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}(75 + 100\cos(100t+\frac{\pi}{3}) + 50\cos(200t+\frac{2\pi}{3})dt \\ &= 75.\end{aligned}$$

(b) Let $x(t) = 10\cos(5t)\cos(10t)$, which is equal to $5\cos(15t) + 5\cos(5t)$ by the formula $\cos(\alpha)\cos(\beta) = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta)$. The power of $x(t)$ is

$$\begin{aligned}P_\infty &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2dt \\ &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}25(\cos^2(15t) + 2\cos(15t)\cos(5t) + \cos^2(5t))dt \\ &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}25(\frac{1+\cos(30t)}{2} + \cos(20t) + \cos(10t) + \frac{1+\cos 10t}{2})dt \\ &= 25.\end{aligned}$$

(c) Let $x(t) = e^{j\alpha t}\cos(\omega_0 t)$, the power of $x(t)$ is

$$\begin{aligned}P_\infty &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2dt = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|e^{j\alpha t}|^2|\cos(\omega_0 t)|^2dt \\ &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\frac{1+\cos(2\omega_0 t)}{2}dt \\ &= \frac{1}{2}.\end{aligned}$$