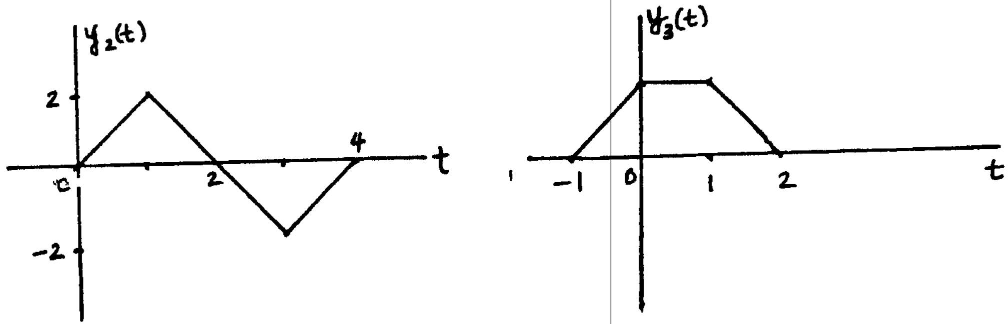
**1.27.** (a) Linear, stable.
(b) Memoryless, linear, causal, stable.
(c) Linear
(d) Linear, causal, stable.
(e) Time invariant, linear, causal, stable.
(f) Linear, stable.
(g) Time invariant, linear, causal.

**1.31.** (a) Note that $x_2(t) = x_1(t) - x_1(t-2)$. Therefore, using linearity we get $y_2(t) = y_1(t) - y_1(t-2)$. This is as shown in Figure S1.31.
(b) Note that $x_3(t) = x_1(t) + x_1(t+1)$. Therefore, using linearity we get $y_3(t) = y_1(t) + y_1(t+1)$. This is as shown in Figure S1,31.




**1.34.** (a) Consider

$$\sum_{n=-\infty}^{\infty} x[n] = x[0] + \sum_{n=1}^{\infty}\{x[n] + x[-n]\}.$$

If $x[n]$ is odd, $x[n] + x[-n] = 0$. Therefore, the given summation evaluates to zero.
(b) Let $y[n] = x_1[n]x_2[n]$. Then

$$y[-n] = x_1[-n]x_2[-n] = -x_1[n]x_2[n] = -y[n].$$

This implies that $y[n]$ is odd.
(c) Consider

$$\begin{aligned}\sum_{n=-\infty}^{\infty} x^2[n] &= \sum_{n=-\infty}^{\infty}\{x_e[n] + x_o[n]\}^2 \\ &= \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n] + 2\sum_{n=-\infty}^{\infty} x_e[n]x_o[n].\end{aligned}$$

Using the result of part (b), we know that $x_e[n]x_o[n]$ is an odd signal. Therefore, using the result of part (a) we may conclude that

$$2\sum_{n=-\infty}^{\infty} x_e[n]x_o[n] = 0.$$

Therefore,

$$\sum_{n=-\infty}^{\infty} x^2[n] == \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n].$$

(d) Consider

$$\int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} x_e^2(t)dt + \int_{-\infty}^{\infty} x_o^2(t)dt + 2\int_{-\infty}^{\infty} x_e(t)x_o(t)dt.$$

Again, since $x_e(t)x_o(t)$ is odd,

Therefore,

$$\int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} x_e^2(t)dt + \int_{-\infty}^{\infty} x_o^2(t)dt.$$

**1.41.** **(a)** $y[n] = 2x[n]$. Therefore, the system is time invariant.

**(b)** $y[n] = (2n-1)x[n]$. This is not time-invariant because $y[n-N_0] \neq (2n-1)x[n-N_0]$.

**(c)** $y[n] = x[n]\{1+(-1)^n+1+(-1)^{n-1}\} = 2x[n]$. Therefore, the system is time invariant.

**1.42.** **(a)** Consider two systems $S_1$ and $S_2$ connected in series. Assume that if $x_1(t)$ and $x_2(t)$ are the inputs to $S_1$, then $y_1(t)$ and $y_2(t)$ are the outputs, respectively. Also, assume that if $y_1(t)$ and $y_2(t)$ are the inputs to $S_2$, then $z_1(t)$ and $z_2(t)$ are the outputs, respectively. Since $S_1$ is linear, we may write

$$ax_1(t) + bx_2(t) \xrightarrow{S_1} ay_1(t) + by_2(t),$$

where $a$ and $b$ are constants. Since $S_2$ is also linear, we may write

$$ay_1(t) + by_2(t) \xrightarrow{S_2} az_1(t) + bz_2(t),$$

We may therefore conclude that

$$ax_1(t) + bx_2(t) \xrightarrow{S_1,S_2} az_1(t) + bz_2(t).$$

Therefore, the series combination of $S_1$ and $S_2$ is linear.

Since $S_1$ is time invariant, we may write

$$x_1(t-T_0) \xrightarrow{S_1} y_1(t-T_0)$$

and

$$y_1(t-T_0) \xrightarrow{S_2} z_1(t-T_0).$$

Therefore,

$$x_1(t-T_0) \xrightarrow{S_1,S_2} z_1(t-T_0).$$

Therefore, the series combination of $S_1$ and $S_2$ is time invariant.

**(b)** False. Let $y(t) = x(t)+1$ and $z(t) = y(t)-1$. These correspond to two nonlinear systems. If these systems are connected in series, then $z(t) = x(t)$ which is a linear system.

**(c)** Let us name the output of system 1 as $w[n]$ and the output of system 2 as $z[n]$. Then,

$$\begin{aligned} y[n] &= z[2n] = w[2n] + \frac{1}{2}w[2n-1] + \frac{1}{4}w[2n-2] \\ &= x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2] \end{aligned}$$

The overall system is linear and time-invariant.

**1.43.** **(a)** We have

$$x(t) \xrightarrow{S} y(t).$$

Since $S$ is time-invariant,

$$x(t-T) \xrightarrow{S} y(t-T).$$

Now, if $x(t)$ is periodic with period $T$, $x(t) = x(t-T)$. Therefore, we may conclude that $y(t) = y(t-T)$. This implies that $y(t)$ is also periodic with period $T$. A similar argument may be made in discrete time.

**1.42.** **(a)** Consider two systems $S_1$ and $S_2$ connected in series. Assume that if $x_1(t)$ and $x_2(t)$ are the inputs to $S_1$, then $y_1(t)$ and $y_2(t)$ are the outputs, respectively. Also, assume that if $y_1(t)$ and $y_2(t)$ are the inputs to $S_2$, then $z_1(t)$ and $z_2(t)$ are the outputs, respectively. Since $S_1$ is linear, we may write

$$ax_1(t) + bx_2(t) \xrightarrow{S_1} ay_1(t) + by_2(t),$$

where $a$ and $b$ are constants. Since $S_2$ is also linear, we may write

$$ay_1(t) + by_2(t) \xrightarrow{S_2} az_1(t) + bz_2(t),$$

We may therefore conclude that

$$ax_1(t) + bx_2(t) \xrightarrow{S_1,S_2} az_1(t) + bz_2(t).$$

Therefore, the series combination of $S_1$ and $S_2$ is linear.

Since $S_1$ is time invariant, we may write

$$x_1(t - T_0) \xrightarrow{S_1} y_1(t - T_0)$$

and

$$y_1(t - T_0) \xrightarrow{S_2} z_1(t - T_0).$$

Therefore,

$$x_1(t - T_0) \xrightarrow{S_1,S_2} z_1(t - T_0).$$

Therefore, the series combination of $S_1$ and $S_2$ is time invariant.

**(b)** False. Let $y(t) = x(t) + 1$ and $z(t) = y(t) - 1$. These correspond to two nonlinear systems. If these systems are connected in series, then $z(t) = x(t)$ which is a linear system.

**(c)** Let us name the output of system 1 as $w[n]$ and the output of system 2 as $z[n]$. Then,

$$\begin{aligned} y[n] &= z[2n] = w[2n] + \frac{1}{2}w[2n-1] + \frac{1}{4}w[2n-2] \\ &= x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2] \end{aligned}$$

The overall system is linear and time-invariant.

## Problem from exercise book:

1.4 a. $y[n] = \sin((\pi/2)x[n])$. You can obtain the output signals $y_1[n]$ and $y_2[n]$ with respect to the input signals $x_1[n] = \delta[n]$ and $x[n] = 2\delta[n]$. Notice that $y[n] = \sin((\pi/2)(x_1[n] + x_2[n]))$ is not equal to $y_1[n] + y_2[n]$. Therefore, the system is not linear.

```
clear all;
clf;
n=[-5:5]
delta=zeros(1,length(n))
delta((length(n)+1)/2)=1;
```

```
x1=delta;
x2=2*delta;
y1=HW3_1_4a_fun(x1)
y2=HW3_1_4a_fun(x2)
y3=HW3_1_4a_fun(x1+x2)
y4=y1+y2;

subplot(221)
stem(n,y1,'g')
xlabel('n');
ylabel('y_1[n]');
axis([-5 5 0 1.5])

subplot(222)
stem(n,y2,'r')
axis([-5 5 0 1.5])
xlabel('n');
ylabel('y_2[n]');

subplot(223)
stem(n,y3)
xlabel('n');
ylabel('sin(\pi/2(x_1+x_2))');
axis([-5 5 -1.5 0])

subplot(224)
stem(n,y4)
axis([-5 5 0 1.5])
xlabel('n');
ylabel('y_1[n]+y_2[n]');

%%%%function in a different file name as HW3_1_4a_fun.m

function y=HW3_1_4a_fun(x)
y=sin(pi/2*x);
```

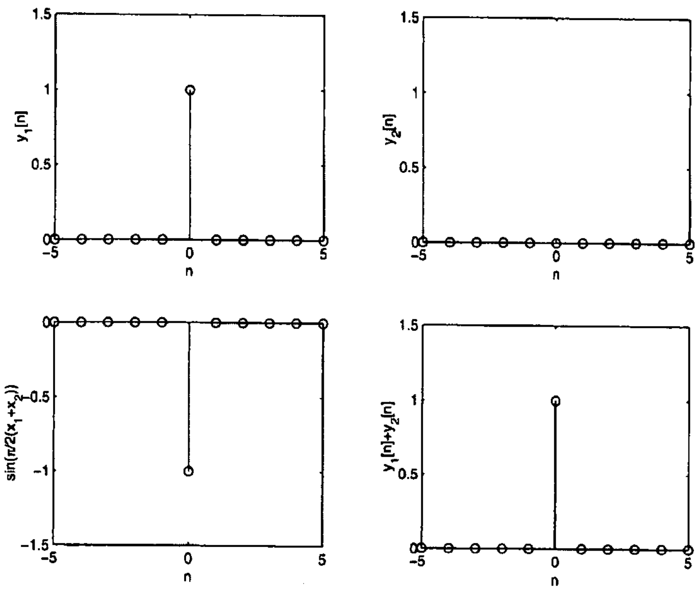



Figure 1: Homework 3:1.4(a)

1.4 b. $y[n] = x[n] + x[n+1]$. After you obtain the plots for $y[n]$, you may find out it is nonzero when $x[n]$ is zero. Hence it is not causal.

```
clear all;
clf;
n=[-6:9];
for i=1:length(n)
    if n(i)>=0
    u_n(i)=1;
    end
    if  n(i)>=-1
    u_n1(i)=1;
```

```
    end
end

y=u_n+u_n1;

subplot(311)
stem(n,u_n)
xlabel('n');
ylabel('x[n]');

subplot(312)
stem(n,u_n1)
xlabel('n');
ylabel('x[n+1]');

subplot(313)
stem(n,y)
xlabel('n');
ylabel('y[n]=x[n]+x[n+1]');
```

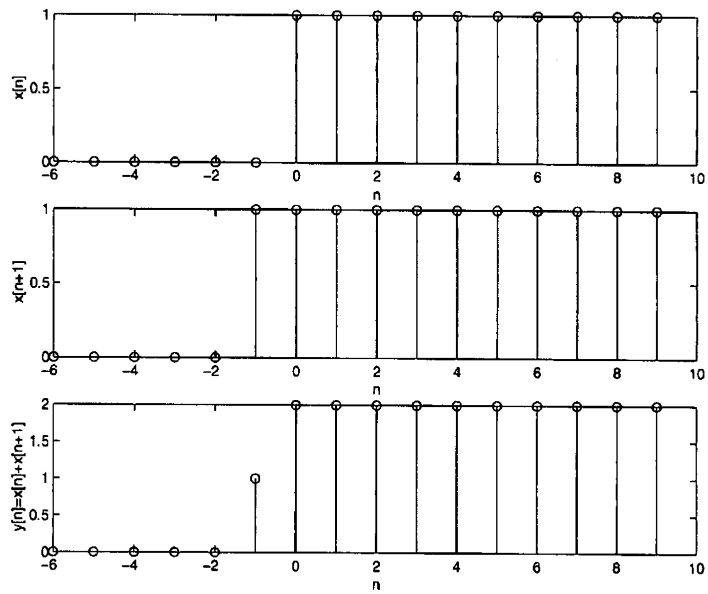



Figure 2: Homework 3:1.4(b)

1.4. c. $y[n] = \log(x[n])$. You may pick the signal like $x[n] = e^{-n}$ or $1/n$. Here we pick $x[n] = 1/n$ for $n \geq 1$ and $x[n]$ is 1 for $n \leq 0$. This is because $x[n]$ need to be greater than 0 for the natural logarithm. You may find that $|x[n]| < 1$ but $|y[n]|$ is unbounded.

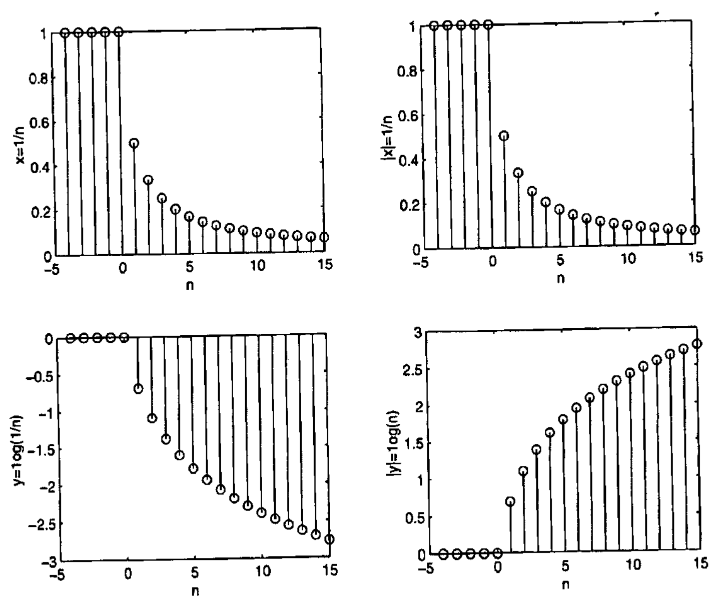



Figure 3: Homework 3:1.4(c)

```
clear all;
clf;
n=[1:20];
x=zeros(1,length(n));
for n=1:20
    if n<=5;
    x(n)=1;
    else
    x(n)=1./(n-5+1);
   end
end
m=[-4:15];

subplot(221)
stem(m,x)
xlabel('n');
ylabel('x=1/n');

subplot(222)
stem(m,abs(x))
xlabel('n');
ylabel('|x|=1/n');
subplot(223)
stem(m,log(x),'r')
xlabel('n');
ylabel('y=log(1/n)');

subplot(224)
stem(m,abs(log(x)),'r');
xlabel('n');
ylabel('|y|=log(n)');
```

1.4. e. $y[n] = x^3[n]$. The system is time-invariant, causal, stable and invertible, but it is not linear. We can choose $x_1$ and $x_2$ as the same signals in problem 1.4 a. It can be shown that $y_1 + y_2$ is not equal to $y = (x_1 + x_2)^3$, therefore, it is not linear.

```
clear all;
clf;
n=[-5:5]
delta=zeros(1,length(n))
delta((length(n)+1)/2)=1;

x1=delta;
x2=2*delta;
y1=x1.^3+x2.^3;
y2=(x1+x2).^3
```

```
subplot(221)
stem(n,x1,'r')
xlabel('n');
ylabel('x_1[n]');
axis([-5 5 0 2])

subplot(222)
stem(n,x2,'r')
xlabel('n');
ylabel('x_2[n]');

subplot(223)
stem(n,y1,'r')
xlabel('n');
ylabel('y=(x_1[n]+x_2[n])^2');
axis([-5 5 0 30])

subplot(224)
stem(n,y2,'g')
xlabel('n');
ylabel('y=x_1^3[n]+x_2^3[n]');
```

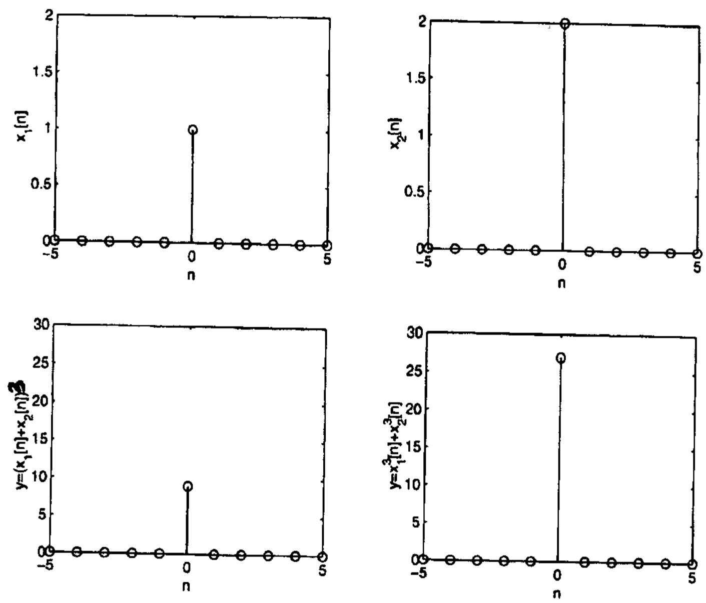



Figure 4: Homework 3:1.4(e)

1.4. f. $y[n] = nx[n]$. The system is causal, linear, invertible, but it is not time-invariant and not stable. First, we pick the input signal $x_1[n] = \delta(n)$ and $x_2 = \delta(n-1)$. If the system is time-invariant, then the output $y_2[n]$ can be obtained by shifting the $y_1[n]$ to the right. The plots shows that it is not the case, so the system is not time invariant.

```
clear all;
clf;

% it is causal, linear, invertible.
% it is not time-invariant, not stable.

%1. Time varying
n=[-5:5]
x1=zeros(1,length(n))
x1((length(n)+1)/2)=1;
y1=n.*x1;

m=[-5:5]
x2=zeros(1,length(m))
x2((length(m)+1)/2+1)=1;
y2=n.*x2;

figure(1)
subplot(221)
stem(n,x1,'r')
```

```
xlabel('n');
ylabel('x_1[n]=\delta[n]');


subplot(222)
stem(n,x2,'r')
xlabel('n');
ylabel('x_2[n]=\delta[n-1]')

subplot(223)
stem(n,y1,'r')
xlabel('n');
ylabel('y_1[n]=nx_1[n]');
axis([-5 5 0 1]);

subplot(224)
stem(n,y2,'r')
xlabel('n');
ylabel('y_2[n]=nx_1[n-1]');
```

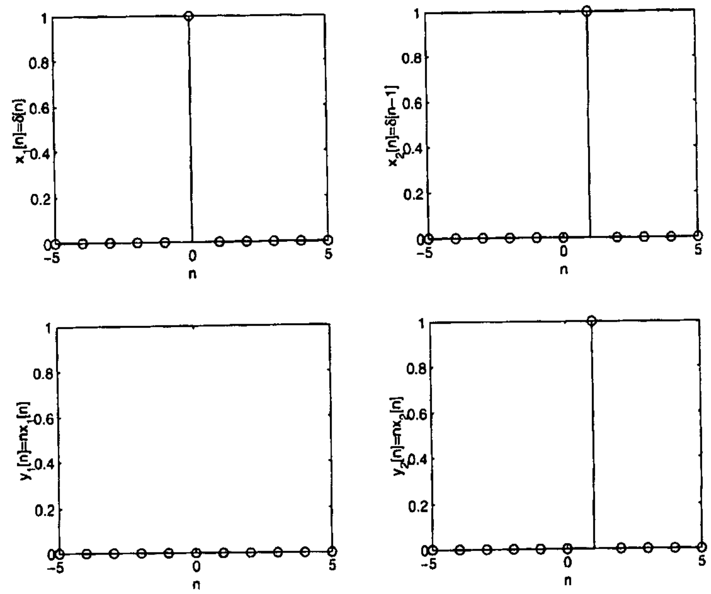



Figure 5: Homework 3:1.4(f)-time varying

Continue with the program, we can pick $x[n]$ as the step function, which is bounded. The output signal is unbounded, so the system is not stable.

```
%2:instability
u1=ones(1,length(n));
for i=1:length(n)
   if n(i)<0
       u1(i)=0;
   end
end
y=n.*u1;

figure(2)

subplot(211)
stem(n,u1)
xlabel('n');
ylabel('x[n]=u[n]');


subplot(212)
stem(n,y)
xlabel('n');
ylabel('y[n]=nu[n]');
```

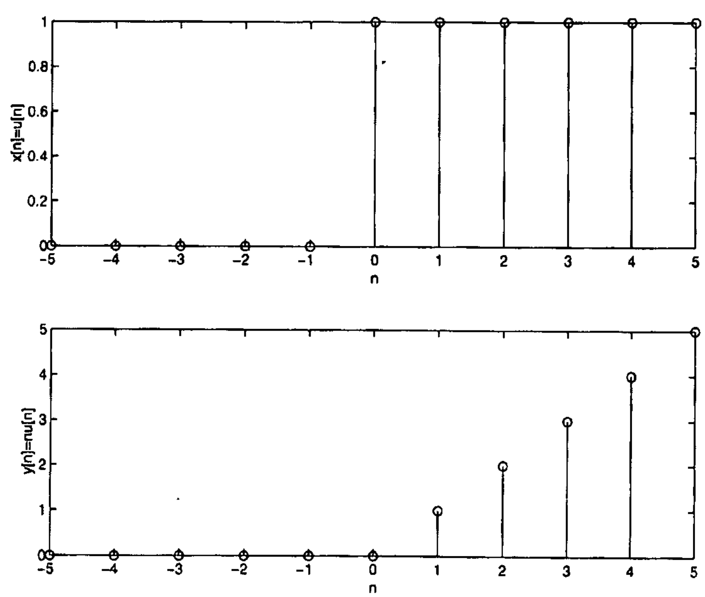



Figure 6: Homework 3:1.4(f)-instability