4.19. We know that

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}.$$

Since it is given that $y(t) = e^{-3t}u(t) - e^{-4t}u(t)$, we can compute $Y(j\omega)$ to be

$$Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega} = \frac{1}{(3+j\omega)(4+j\omega)}.$$

Since, $H(j\omega) = 1/(3+j\omega)$, we have

$$X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = 1/(4+j\omega)$$

Taking the inverse Fourier transform of $X(j\omega)$, we have

$$x(t) = e^{-4t}u(t).$$

4.23. For the given signal $x_0(t)$, we use the Fourier transform analysis eq. (4.8) to evaluate the corresponding Fourier transform

$$X_0(j\omega) = \frac{1-e^{-(1+j\omega)}}{1+j\omega}.$$

(i) We know that

$$x_1(t) = x_0(t) + x_0(-t).$$

Using the linearity and time reversal properties of the Fourier transform we have

$$X_1(j\omega) = X_0(j\omega) + X_0(-j\omega) = \frac{2 - 2e^{-1}\cos\omega - 2\omega e^{-1}\sin\omega}{1+\omega^2}.$$

(ii) We know that

$$x_2(t) = x_0(t) - x_0(-t).$$

Using the linearity and time reversal properties of the Fourier transform we have

$$X_2(j\omega) = X_0(j\omega) - X_0(-j\omega) = j\left[\frac{-2\omega + 2e^{-1}\sin\omega + 2\omega e^{-1}\cos\omega}{1+\omega^2}\right].$$

(iii) We know that

$$x_3(t) = x_0(t) + x_0(t+1).$$

Using the linearity and time shifting properties of the Fourier transform we have

$$X_3(j\omega) = X_0(j\omega) + e^{j\omega}X_0(-j\omega) = \frac{1 + e^{j\omega} - e^{-1}(1+e^{-j\omega})}{1+j\omega}.$$

4.25. (a) Note that $y(t) = x(t+1)$ is a real and even signal. Therefore, $Y(j\omega)$ is also real and even. This implies that $\sphericalangle Y(j\omega) = 0$. Also, since $Y(j\omega) = e^{j\omega}X(j\omega)$, we know that $\sphericalangle X(j\omega) = -\omega$.

(b) We have

$$X(j0) = \int_{-\infty}^{\infty} x(t)dt = 7.$$

(c) We have

$$\int_{-\infty}^{\infty} X(j\omega)d\omega = 2\pi x(0) = 4\pi.$$

4.26. **(a)** (iii) We have

$$
\begin{aligned}
Y(j\omega) &= X(j\omega)H(j\omega) \\
&= \left[\frac{1}{1+j\omega}\right]\left[\frac{1}{1-j\omega}\right] \\
&= \frac{1/2}{1+j\omega} + \frac{1/2}{1-j\omega}
\end{aligned}
$$

Taking the inverse Fourier transform, we obtain

$$y(t) = \frac{1}{2}e^{-|t|}.$$

4.33. **(a)** Taking the Fourier transform of both sides of the given differential equation, we obtain

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2}{-\omega^2 + 6j\omega + 8}.$$

Using partial fraction expansion, we obtain

$$H(j\omega) = \frac{1}{j\omega + 2} - \frac{1}{j\omega + 4}.$$

Taking the inverse Fourier transform,

$$h(t) = e^{-2t}u(t) - e^{-4t}u(t).$$

(c) Taking the Fourier transform of both sides of the given differential equation, we obtain

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2(-\omega^2 - 1)}{-\omega^2 + \sqrt{2}j\omega + 1}.$$

Using partial fraction expansion, we obtain

$$H(j\omega) = 2 + \frac{-\sqrt{2} - 2\sqrt{2}j}{j\omega - \frac{-\sqrt{2}+j\sqrt{2}}{2}} + \frac{-\sqrt{2} + 2\sqrt{2}j}{j\omega - \frac{-\sqrt{2}-j\sqrt{2}}{2}}.$$

Taking the inverse Fourier transform,

$$h(t) = 2\delta(t) - \sqrt{2}(1+2j)e^{-(1+j)t/\sqrt{2}}u(t) - \sqrt{2}(1-2j)e^{-(1-j)t/\sqrt{2}}u(t).$$

4.34. **(a)** We have

$$\frac{Y(j\omega}{X(j\omega)} = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}.$$

Cross-multiplying and taking the inverse Fourier transform, we obtain

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 4x(t).$$

(b) We have

$$H(j\omega) = \frac{2}{2 + j\omega} - \frac{1}{3 + j\omega}.$$

Taking the inverse Fourier transform we obtain,

$$h(t) = 2e^{-2t}u(t) - e^{-3t}u(t).$$

4.36. **(a)** The frequency response is

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{3(3+j\omega)}{(4+j\omega)(2+j\omega)}.$$

(b) Finding the partial fraction expansion of answer in part (a) and taking its inverse Fourier transform, we obtain

$$h(t) = \frac{3}{2}\left[e^{-4t} + e^{-2t}\right] u(t).$$

(c) We have

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{(9+3j\omega)}{8+6j\omega-\omega^2}.$$

Cross-multiplying and taking the inverse Fourier transform, we obtain

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 3\frac{dx(t)}{dt} + 9x(t).$$

Matlab homework: The program is the following. The two filters are both low pass filters. The advantages of logarithmic plots include

(a) The magnitude plot of $H(jw)$ is approximately as piecewise linear plots, and the slope changes according to the locations of the poles and zeros(which will be discussed later in this course).

(b) Lin/lin plot has trouble to cover both lower and higher frequency values when jw has higher degrees. The log/log plot allows the frequency and the magnitude to be over a wider range.

(c) The log/log changes the multiplication operation into the addition operation, and it make the computation simpler.

```
%homework 8 4.33(a) and 4.34
clear all;
b=2;
a=[1 6 8];
figure(1)
freqs(b,a);
title('the frequency response of Problem 4.33(a)');
% low pass filter

print -dpsc hw8_3_33.ps

b=[1 4];
a=[1 5 6];
figure(2)
freqs(b,a);
title('The frequency response of Problem 4.34');
% low pass filter

print -dpsc hw8_3_34.ps
```

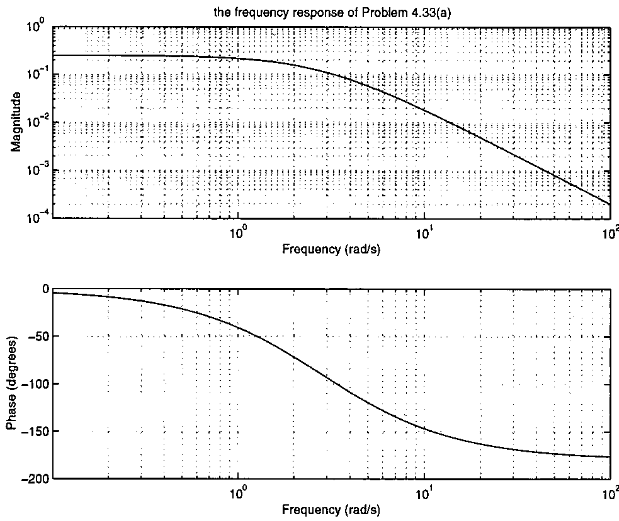

Figure 1: Homework 8:3.33(a)

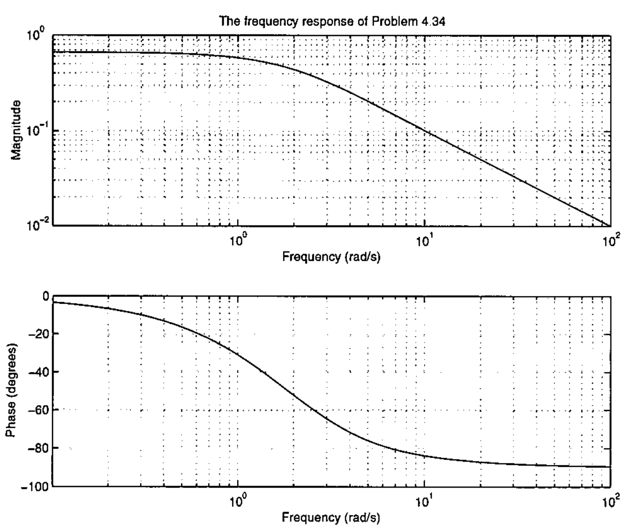

Figure 2: Homework 8:3.34