Practice Exam 2

11/14/05 MH

Problem 3

$$y[n] = h[n] \quad \circ\!\!-\!\!\bullet \quad H(e^{j\omega}); \qquad h[n] = \underbrace{\tfrac{1}{2}\delta[n]}_{(a)} + \underbrace{\left(\tfrac{1}{2}\right)^n \cos\tfrac{\pi n}{3}\, u[n]}_{(b)}$$

(a) $\frac{1}{2}\delta[n] \quad \circ\!\!-\!\!\bullet \quad \frac{1}{2}$

(b) $\frac{1}{2}\left[\left(\frac{1}{2}\right)^n e^{j\frac{\pi n}{3}} + \left(\frac{1}{2}\right)^n e^{-j\frac{\pi n}{3}}\right] u[n]$

$$\frac{1}{2}\left[\frac{1}{1-\frac{1}{2}e^{j\pi/3}e^{-j\omega}} + \frac{1}{1-\frac{1}{2}e^{-j\pi/3}e^{-j\omega}}\right] =$$

$$\frac{1}{2}\left[\frac{2-\frac{1}{2}\left(e^{j\pi/3}+e^{-j\pi/3}\right)e^{-j\omega}}{1-\frac{1}{2}\left(e^{j\pi/3}+e^{-j\pi/3}\right)e^{-j\omega}+\frac{1}{4}e^{-2j\omega}}\right]$$

(a)+(b):

$$\frac{1}{2}\left[\frac{2-\cos\pi/3\, e^{-j\omega}}{1-\cos\pi/3\, e^{-j\omega}+\frac{1}{4}e^{-2j\omega}} + 1\right]$$

$$= \frac{1}{2}\left[\frac{3-2\cos\pi/3\, e^{-j\omega}+\frac{1}{4}e^{-2j\omega}}{1-\cos\pi/3\, e^{-j\omega}+\frac{1}{4}e^{-2j\omega}}\right] = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

→ Difference equation:

$$y[n] - \cos\pi/3\, y[n-1] + \tfrac{1}{4}y[n-2] = \tfrac{3}{2}x[n] - \cos\pi/3\, x[n-1] + \tfrac{1}{8}x[n-2]$$

Note that $\cos\pi/3 = \frac{1}{2}$ → further simplification.

Multiplication property could be used, too.