Path Loss Exponent Estimation in Large Wireless Networks

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***Abstract*—In wireless channels, the path loss exponent (PLE) has a strong impact on the quality of the links, and hence, it needs to be accurately estimated for the efficient design and operation of wireless networks. This paper addresses the problem of PLE estimation in large wireless networks, which is relevant to several important issues in communications such as localization, energy-efficient routing, and channel access. We consider a large ad hoc network where nodes are distributed as a homogeneous planar Poisson point process and the channels are subject to Nakagami-m fading. Under these settings, we propose and study three distributed algorithms for estimating the PLE at each node, which explicitly take into account the interference in the network. Additionally, we provide simulation results to demonstrate the performance of the algorithms and quantify the estimation errors.**

I. Introduction

A. Motivation

In wireless networks, the path loss over a link is commonly modeled by the product of a distance component (commonly known as large-scale path loss) and a small-scale fading component [1]. The large-scale path loss model assumes that the received signal strength attenuates with the transmitter-receiver distance d as d^{γ}, where γ is the path loss exponent (PLE) of the channel. Fading represents the deviation of the signal strength from the power-law decay. While the large-scale path loss part is assumed to be deterministic, fading is often modeled as a stochastic process. This distinction, however, does not hold in scenarios where the nodes themselves are randomly arranged, and thus the precise distances between them are subject to uncertainty. A critical requirement for the efficient design and operation of such networks is characterizing the large-scale behavior of the channel and accurately estimating the PLE, amidst the uncertainty in the locations of the nodes.

This problem is non-trivial even for a single link due to the existence of multipath propagation and thermal noise. For large ad hoc networks with multiple transmitter-receiver pairs, the problem is further complicated due to the following reasons: First, the achievable performance of a typical ad hoc or sensor network is not only susceptible to noise and fading, but also interference-limited due to the presence of simultaneous transmitters. Dealing with fading and interference simultaneously is a major challenge in the estimation problem. Moreover, in view of the distance uncertainties, we will need to consider the small-scale fading and distance ambiguities jointly, i.e., define a spatial point process that incorporates both. In this paper, we consider a large planar wireless network where nodes are arranged as a Poisson point process and present three distributed algorithms to accurately estimate the channel's PLE in the presence of fading, noise and interference. We also provide simulation results to illustrate the performance of the algorithms and study the estimation error.

B. Related Work

In most of the prior literature on PLE estimation algorithms, authors have assumed a simplified channel model consisting only of a large-scale path loss component and a shadowing counterpart, and therefore, their methods have focused mainly on RSS-based localization techniques. We are however not aware of any related work that has considered fading, and most importantly, interference in the system model.

Estimation based on a known internode distance probability distribution is discussed in [2]. The authors assume that the distance distribution between two neighboring nodes i and j is known or can be determined easily. With the transmit power at node i equal to P_0[dBm] (assume this is a constant for all nodes), the RSS at node j is modeled by a log-normal distribution as

$$P_{ij}[\text{dBm}] \sim \mathcal{N}(\overline{P}_{ij}[\text{dBm}], \sigma^2_{\text{dB}}),$$

where σ denotes the log-normal spread and $\overline{P}_{ij}[\text{dBm}] = P_0[\text{dBm}] - 10\gamma \log_{10} d_{ij}$. Now, if the neighbor's distance distribution is given by $p_R(r)$, then

$$\overline{P}_{ij} = P_0 \mathbb{E}_R \left[R^{-\gamma} \right]. \tag{1}$$

The value of γ is estimated by equating $\overline{P}_{ij}$ to the empirical mean value of the received powers taken over several node pairs i and j.

If the nearest neighbor distribution is in a complicated form that is not integrable, an idea similar to the quantile-quantile plot can be used [2]. For cases where it might not be possible to obtain the neighbor distance distribution, the idea of estimating γ using the concept of the Cayley-Menger determinant or the pattern matching technique [2] is useful.

In [3], the authors consider a network where the path loss between a few low-cost sensors is measured and stored for future use. They propose an algorithm that employs interpolation techniques to estimate the path loss between a sensor and any arbitrary point in the network. In [4], a PLE estimator based on the method of least squares is discussed and used in the design of an efficient handover algorithm. However, as described earlier, the situation is completely different when interference and fading are considered and we cannot use these purely RSS-based estimators.

II. System Model

We consider an infinite ad hoc network on $\mathbb{R}^2$, where nodes are distributed as a homogeneous Poisson point process (PPP) Φ of density λ. Accordingly, the number of points lying in a compact set B, denoted by $\Phi(B)$, is Poisson-distributed with mean $\lambda\nu_2(B)$, where $\nu_2(\cdot)$ is the two-dimensional Lebesgue measure (area). Also, the number of points in disjoint sets are independent random variables.

The attenuation in the channel is modeled as the product of the large-scale path loss with exponent γ and a flat block-fading component. To obtain a concrete set of results, the fading amplitude H is taken to be Nakagami-m distributed. Letting $m = 1$ results in the well-known case of Rayleigh fading, while lower and higher values of m signify stronger and weaker fading scenarios respectively. The case of no fading is modeled by setting $m \to \infty$. When dealing with received signal powers, we use the power fading variable denoted by $G = H^2$. Since G captures the random deviation from the large-scale path loss, $\mathbb{E}_G[G] = 1$. The moments of G are [5, Eqn. 17]

$$\mathbb{E}_G[G^n] = \frac{\Gamma(m+n)}{m^n\Gamma(m)}, \quad n \in \mathbb{R}^+. \tag{2}$$

and the variance is $\sigma_G^2(G) = 1/m$. We take the noise to be AWGN with mean power N_0.

Since the PLE estimation is usually performed during network initialization, it is reasonable to assume that the transmissions in the system during this phase are uncoordinated. Therefore, we take the channel access scheme to be ALOHA. We denote the ALOHA contention probability by a constant p meaning that nodes independently decide to transmit with probability p or stay idle with probability $1 - p$ in any time slot[1]. Consequently, the set of transmitters at any given moment forms a PPP Φ' of density λp. Also, since there is no information available for power control, we assume that all the transmit powers are equal to unity. Then, the interference at node y is given by

$$I_\Phi(y) = \sum_{z\in\Phi'} G_{zy}\|z-y\|^{-\gamma},$$

where G_{zy} is the fading gain of the channel and $\|\cdot\|$ denotes the Euclidean distance.

We define the communication from the transmitter at x to the receiver at y to be successful if the signal-to-noise and interference ratio (SINR) at y is larger than a threshold Θ. Mathematically speaking, an outage occurs if and only if

$$\frac{G_{xy}\|x-y\|^{-\gamma}}{N_0 + I_{\Phi'\setminus\{x\}}(y)} \le \Theta, \tag{3}$$

where $I_{\Phi'\setminus\{x\}}(y)$ denotes the interference in the network at y due to all the transmitters, except the desired one at x.

III. Path Loss Exponent Estimation

This section describes three completely distributed algorithms for PLE estimation, each based on a certain network characteristic, and provides simulation results on the estimation errors. The first algorithm uses the mean value of the interference and assumes that the network density is known beforehand. Algorithms 2 and 3 are based on outage probabilities and the network's connectivity properties respectively, and do not require knowledge of the density λ or the Nakagami parameter m.

The PLE estimation problem is essentially tackled by equating the empirically (observed) measured values of the aforementioned network characteristics to the theoretically established ones to obtain $\hat{\gamma}$. In each time slot, nodes either transmit (w.p. p) or listen to record measurements (w.p. $1-p$). Upon obtaining the required measurement values over several time slots, the estimation process can be performed at each node in the network in a distributed fashion.

The simulation results are obtained using MATLAB. The PPP is generated by distributing a Poisson number of points uniformly randomly in a 50×50 square with density 1. To avoid border effects, we use the measurements recorded at the node lying closest to the center of the network. To analyze the mean error performance of the algorithms, we consider the estimates resulting from $50,000$ different realizations of the PPP. The contention probability is taken to be $p = 0.05$ in each case[2], and $N_0 = -25$ dBm.

A. Algorithm 1: Estimation Using the Mean Interference

In many situations, the network density is a design parameter and known. In other cases, it is possible to estimate the density (see [6, Sec. 2.7] and the references therein for a survey of the estimation methods for a PPP). A simple technique to infer the PLE γ when the nodal density is known is based on the mean interference in the network.

According to this method, nodes simply need to record the strength of the received power that they observe and use it to estimate γ. For $\gamma > 2$ (a fair assumption in a wireless scenario), the mean interference is theoretically equal to [7]

$$\mu_I = C_1 = 2\pi\lambda p\frac{A_0^{2-\gamma}}{\gamma-2}, \tag{4}$$

where A_0 is the near-field radius of the antenna. Consequently, the mean received power is $\mu_R = \mu_I + N_0$, and is independent of m.

[1] The beginning and ending times of a slot is based on the individual node's clock cycle. Thus, time slots across different nodes need not (and in general, will not) be synchronized. We will only assume that the duration of the slots are the same.

[2] This value of p was found to be suitable to obtain several quasi-different realizations of the PPP Φ' and helped obtain accurate estimates in a reasonable number of time slots.

The algorithm based on the mean interference matches the sample and theoretic values of the mean received power and is described as follows.

- Record the strengths of the received powers $R_1, \dots, R_N$ at any arbitrarily chosen node during N time slots. Eventually, the empirical mean received power $(1/N)\sum_{i=1}^{N} R_i$ converges.
- Equating the observed mean value to the theoretical value of μ_R, γ can be estimated by using a look-up table and the known values of p, N_0, A_0 and estimated (or known) density $\hat{\lambda}$.

Fig. 1 depicts the mean squared error (MSE) values of the estimated PLE $\hat{\gamma}$ using the above algorithm for different γ and N values. The estimates are seen to be fairly accurate over a wide range of PLE values.

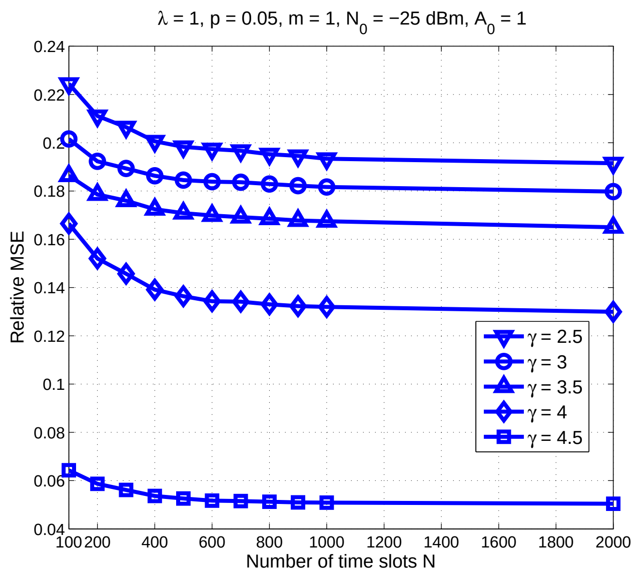

Fig. 1. MSE of $\hat{\gamma}$ versus the number of time slots for different PLE values, for the estimation method based on the mean interference. The error is small when the path loss exponent is small.

B. Algorithm 2: Estimation Based on Virtual Outage Probabilities

We now describe an estimation method based on outage probabilities that does not require the knowledge of the network density or the Nakagami fading parameter. We first overview some theoretical results and then present a practical scheme to estimate γ.

In [8], it is shown that when the signal power is exponential distributed, the success probability p_s across any link in the Poisson network is equal to the product of the Laplace transforms of noise and interference. In particular, when the transceiver pair separation is unity[3], we have [7]

$$p_s = c_1 \exp(-c_2 \Theta^{2/\gamma}), \tag{5}$$

[3]When the transmitter node is unit distance away from the receiver node, the PLE will not affect the received power strength. This case is particularly helpful for implementation of the PLE estimation algorithm.

where $c_1 = \exp(-N_0\Theta)$ and

$$c_2 = \lambda p \pi \mathbb{E}[G^{2/\gamma}]\Gamma\left(1-\frac{2}{\gamma}\right) = \frac{\lambda p \pi \Gamma\left(m+\frac{2}{\gamma}\right)\Gamma\left(1-\frac{2}{\gamma}\right)}{\Gamma(m) m^{2/\gamma}}.$$

To estimate γ, the nodes are required to measure the SINR values during several time slots and use it to compute the empirical success probability, which matches the theoretical value (5). However, note that it is impractical to place transmitters for each receiver node where a SINR measurement is taken. Instead, nodes can can simply measure the received powers, and compute the (virtual) SINRs taking the signal powers to be independent realizations of an exponential random variable. This algorithm is implemented at each node as follows.

- Record the values of the received powers $R_1, \dots, R_N$ at the node during N time slots. Take the signal powers S_i, $1 \le i \le N$, to be N independent realizations of an exponential random variable with unit mean. Using the values S_i/R_i $\forall i$, a histogram of the observed SINR values is obtained.
- Evaluate the empirical success probabilities at two different thresholds, i.e., compute $p_{s,j} = (1/N)\sum_{i=1}^{N} \mathbf{1}_{\{S_i/R_i > \Theta_j\}}$, $j = 1, 2$. Eventually, the success probability values converge.
- Assuming that the empirically observed values match the theoretical values, we have from (5),

$$\frac{\ln p_{s,1} + N_0\Theta_1}{\ln p_{s,2} + N_0\Theta_2} = \left(\frac{\Theta_1}{\Theta_2}\right)^{2/\gamma}.$$

An estimate of γ is obtained as

$$\hat{\gamma} = \frac{2\ln(\Theta_1/\Theta_2)}{\ln\left((\ln p_{s,1} + N_0\Theta_2) / (\ln p_{s,2} + N_0\Theta_2)\right)}, \tag{6}$$

which is independent of both λ and m.

Fig. 2 plots the MSE of $\hat{\gamma}$ for $\Theta_1 = 10$ dB and $\Theta_2 = 0$ dB for different γ and N values. We observe that the error is small when the PLE is small, but increases with larger values of γ. Also, Fig. 3 plots the MSE of $\hat{\gamma}$ versus m at various PLE values for the estimation method based on outage probabilities. We observe that the algorithm performs more accurately at lower values of m. We provide an intuitive explanation for this behavior in III-D.

C. Algorithm 3: Estimation Based on the Cardinality of the Transmitting Set

When the network density λ and the Nakagami parameter m are unknown, the PLE can be also be accurately estimated based on the connectivity properties of the network. In this subsection, we derive the average number of nodes that are connected to any arbitrary node in the network, and describe a PLE estimation algorithm based on our analysis.

For any node, define its *transmitting set* as the group of transmitting nodes whom it receives a packet from, in a given time slot. More formally, for receiver y, transmitter node x is in its transmitting set, T_y if they are connected, i.e., the SINR at y is greater than a certain threshold Θ. Note that this set

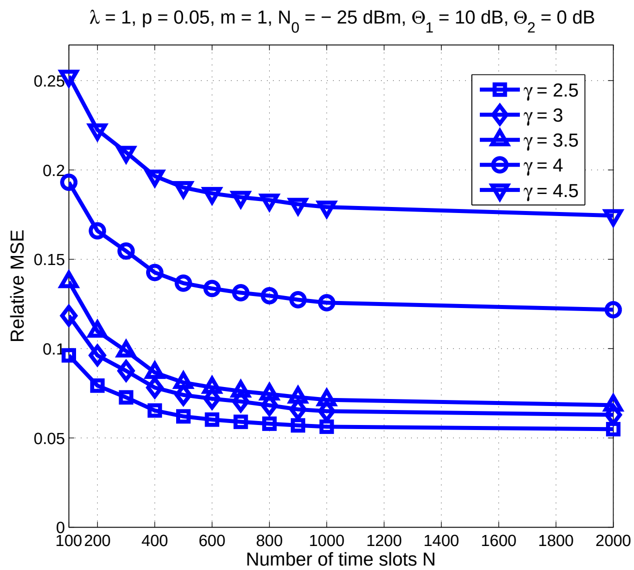

Fig. 2. MSE of $\hat{\gamma}$ versus the number of time slots for the estimation method based on virtual outage probabilities.

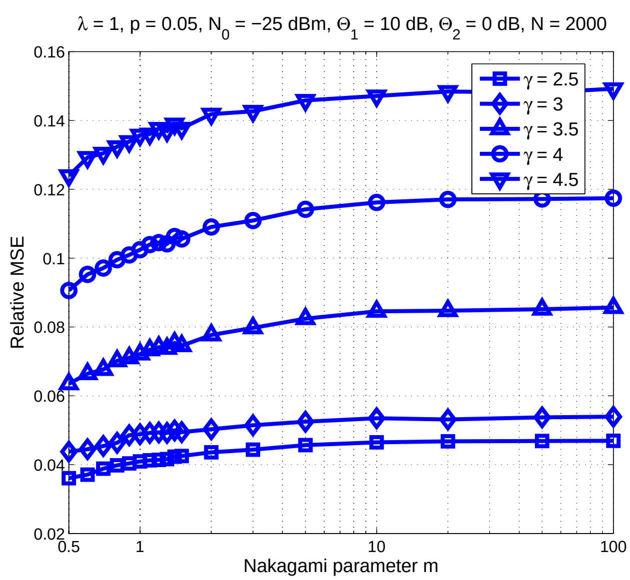

Fig. 3. MSE of $\hat{\gamma}$ versus the Nakagami parameter m for different PLE values, for the estimation method based on virtual outage probabilities.

changes from time slot to time slot. Also note that for $\Theta \geq 1$, the cardinality of the transmitting set can at most be one, and that transmitter is the one with the best channel to the receiver. The estimation algorithm is based on matching the theoretical and empirical values of the mean number of elements in the transmitting set. The following proposition forms the basis of this estimation scheme.

Proposition 3.1: Under the conditions of Nakagami-m fading for $m \in \mathbb{N}$ and $N_0 \ll I$, for any arbitrary node, the mean cardinality of the transmitting set, $\bar{N}_T$, is proportional to $\Theta^{-2/\gamma}$.

Proof: The pdf of the the Nakagami-m power fading variable is given by [5]

$$p_G(x) = \frac{m^m}{\Gamma(m)} x^{m-1} \exp(-mx), \quad m \geq 1/2. \tag{7}$$

For $N_0 \ll I$, the success probability for a transceiver pair at an arbitrary distance R units apart can be expressed as

$$\begin{aligned} p_s(R) &= \mathbb{E}_I \left[\Pr(GR^{-\gamma} > I\Theta \mid I)\right] \\ &= \mathbb{E}_I \left[\int_{I\Theta R^\gamma}^{\infty} \frac{m^m}{\Gamma(m)} x^{m-1} \exp(-mx) dx\right] \\ &= \frac{1}{\Gamma(m)} \int_0^\infty \Gamma(m, x\Theta R^\gamma m) p_I(x) dx, \end{aligned} \tag{8}$$

where $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function[4] and $p_I(x)$ denotes the pdf of the interference.

The expressions can be further simplified when m is an integer. For $m \in \mathbb{N}$, we have

$$\begin{aligned} p_s(R) &\overset{(a)}{=} \sum_{k=0}^{m-1} \frac{1}{k!} \int_0^\infty (x\Theta R^\gamma m)^k \exp(-x\Theta R^\gamma m) P_I(x) dx \\ &\overset{(b)}{=} \sum_{k=0}^{m-1} \frac{(-\Theta R^\gamma m)^k}{k!} \frac{d^k}{ds^k} M_I(s)|_{s=\Theta R^\gamma m}, \end{aligned} \tag{9}$$

where $M_I(s)$ is the moment generating function (MGF) of I. Here, (a) is obtained from the series expansion of the upper incomplete gamma function and (b) using the definition of the MGF. When the node distribution is Poisson, we have the following closed-form expression for the MGF [7, Eqn. 20]:

$$M_I(s) = \exp(-\lambda p \pi \mathbb{E}_G[G^{2/\gamma}]\Gamma(1-2/\gamma)s^{2/\gamma}), \quad \gamma > 2.$$

Using this, we get

$$p_s(R) = \exp(-c_3 R^2) \sum_{k=0}^{m-1} \frac{\left(c_3 R^2\right)^k}{k!} \left(\frac{2}{\gamma}\right)^k, \quad m \in \mathbb{N} \tag{10}$$

where $c_3 = \lambda p \pi \mathbb{E}_G(G^{2/\gamma})\Gamma(1-2/\gamma)(\Theta m)^{2/\gamma} = c_2 m^{2/\gamma}$.

Now, we consider a receiver node O, shift it to the origin and analyze the transmitting set for this "typical" node. Consider a disc of radius a centered at the origin. Let E denote the event that an arbitrarily chosen transmitter inside this disc is in O's transmitting set. Since the nodes in the disc are uniformly randomly distributed, we have

$$\begin{aligned} \Pr(E) &= \mathbb{E}_R[p_s(R) \mid R] \\ &= \frac{2\pi}{\pi a^2} \int_0^a \sum_{k=0}^{m-1} \frac{\exp(-c_3 r^2) r^{2k}}{k!} \left(\frac{2c_3}{\gamma}\right)^k r dr \\ &= \frac{1}{a^2} \sum_{k=0}^{m-1} \left(\frac{2c_3}{\gamma}\right)^k \int_0^a \frac{\exp(-c_3 r^2)}{k!} r^{2k} 2r dr \\ &\overset{(a)}{=} \frac{1}{a^2 c_3} \sum_{k=0}^{m-1} \left(\frac{2}{\gamma}\right)^k \frac{1}{k!} \int_0^{c_3 a^2} t^k \exp(-t) dt \\ &\overset{(b)}{=} \frac{1}{a^2 c_3} \sum_{k=0}^{m-1} \left(\frac{2}{\gamma}\right)^k \frac{1}{k!} \left(1 - \Gamma(k+1, c_3 a^2)\right), \end{aligned}$$

[4]Mathematica: Gamma[a,z]

where (a) is obtained by a simple change of variables ($c_3 r^2 = t$) and (b) using the definition of the incomplete gamma function.

Thus, $\bar{N}_T = c_4$, where

$$\begin{aligned} c_4 &= \lim_{a\to\infty} N_a \Pr(E) \\ &\overset{(a)}{=} \frac{\lambda p\pi}{c_3}\sum_{k=0}^{m-1}\left(\frac{2}{\gamma}\right)^k = \frac{\lambda p\pi}{c_3}\frac{1-\left(\frac{2}{\gamma}\right)^m}{1-\frac{2}{\gamma}} \\ &\overset{(b)}{=} \frac{\Gamma(m)\left(1-\left(\frac{2}{\gamma}\right)^m\right)}{\Gamma(m+\frac{2}{\gamma})\Gamma(2-\frac{2}{\gamma})\Theta^{2/\gamma}}. \end{aligned} \tag{11}$$

Here, (a) is obtained using the fact that $\lim_{z\to\infty}\Gamma(a,z) = \Gamma(a)$ and (b) using the definition of c_3 and (2). ■

From (11), we see that $\bar{N}_T$ is inversely proportional to $\Theta^{2/\gamma}$. Therefore, when m is a positive integer, the ratio of the mean cardinalities of the transmitting set at two different values of Θ is independent of m. This forms the main idea behind the estimation algorithm, and we surmise that this behavior holds at arbitrary $m \in \mathbb{R}^+$.

For the rest of the subsection, we assume that the system is interference-limited, i.e, $N_0 \ll I$. The algorithm based on the cardinality of the transmitting set works as follows.

- For a known threshold value $\Theta_1 \geq 1$, set $N_{T,1}(i) = 1$ at time slot i, $1 \leq i \leq N$, if the node can decode a packet and $N_{T,1}(i) = 0$ otherwise. Eventually, the empirical mean observed at any node over several time slots, $\bar{N}_{T,1} = (1/N)\sum_{i=1}^{N} N_{T,1}(i)$ converges.
- Likewise, evaluate $\bar{N}_{T,2} = (1/N)\sum_{i=1}^{N} N_{T,2}(i)$ for another threshold value, $\Theta_2 \geq 1$.
- Equating the mean cardinalities of the transmitting set for the two different threshold values, we obtain

$$\bar{N}_{T,1}/\bar{N}_{T,2} = (\Theta_2/\Theta_1)^{2/\gamma}.$$

- Following this, γ is estimated as

$$\hat{\gamma} = (2\ln(\Theta_2/\Theta_1))\,/\,\ln(\bar{N}_{T,1}/\bar{N}_{T,2}). \tag{12}$$

Thus, this algorithm does not require the knowledge of either λ or m.

Fig. 4 plots the empirical MSE of $\hat{\gamma}$ for algorithm 3 versus the number of time slots N for various PLE values, while Fig. 5 shows the MSE of $\hat{\gamma}$ versus m. Again, we see that the MSE is low at lower values of m and increases with m.

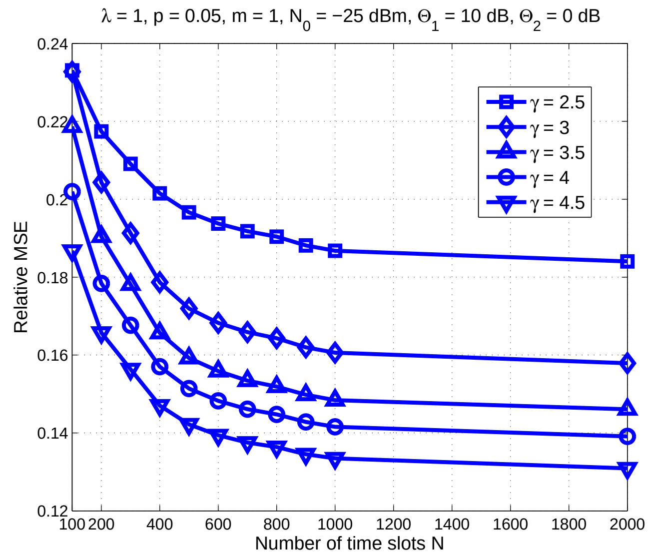

Fig. 4. MSE of $\hat{\gamma}$ versus the number of time slots for different PLE values, for the estimation method based on the mean cardinality of the transmitting set.

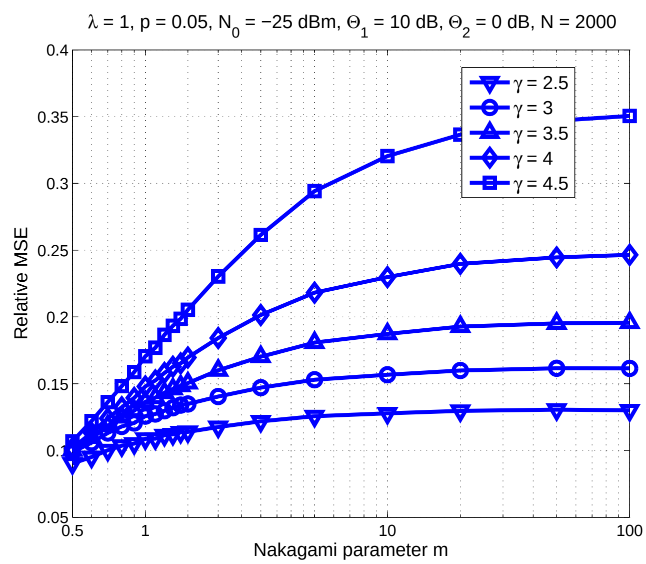

Fig. 5. MSE of $\hat{\gamma}$ versus the Nakagami parameter m for the estimation algorithm based on the mean cardinality of the transmitting set.

D. Discussion

The issue of PLE estimation is a challenging problem, yet needs to be accurately performed for the efficient design and operation of wireless networks. We have proposed three algorithms for this purpose that are fully distributed and can be employed at each node in the network. Furthermore, they do not require any information on the locations of other nodes in the network or the Nakagami parameter m. Simulation results validate that the estimates are quite accurate over a large range of the system parameters γ and m. Also, the convergence of the MSE is seen to occur occur within about 2000 time slots for each of the algorithms. For time slots of the order of milliseconds, it takes only a few seconds to estimate the PLE in practice.

There is a caveat though, that we wish to address here. Recall that each of the estimation algorithms works by equating empirically measured values of certain network characteristics with their corresponding theoretical values. While in theory, we usually assume that we have access to a large number of independent network realizations and derive results for an "average network", the problem in practice is that we have only a single realization of the node distribution at hand. Thus, even though the set of transmitters and the fading component of the channel change independently in different slots, the node locations remain the same. Thus, in general, the estimates are biased. We remark that the bias (and the MSE) can be significantly lowered if the nodes that record measurements have access to several independent realizations of the PPP.

This also intuitively explains the fact that for Algorithms 2 and 3, the MSE decreases with decreasing m. Indeed, from (2), the variance of G is given by $1 + 1/m$, which increases with decreasing m. Considering the fading and link distance ambiguities jointly, a lower value of m is equivalent to having greater randomness in the location of the nodes (upon taking the fading component to be a constant). Thus, the nodes are able to see several diverse realizations of the process over different time slots, and can estimate the PLE more accurately.

IV. Summary

We are the first ones to address the PLE estimation problem in large wireless networks in the presence of node location uncertainties, m-Nakagami fading and most importantly, interference. We assume that nodes are arranged as a homogeneous PPP on the plane and the channel access scheme is slotted ALOHA (at least during the PLE estimation phase). Under these settings, we present three distributed algorithms for PLE estimation, and provide simulation results to demonstrate their performances. This work is easily extensible to one- or three-dimensional networks as well.

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