Spatiotemporal Cooperation in Heterogeneous Cellular Networks

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Abstract

This paper studies downlink communication in a heterogeneous cellular network where a set of geographically separated base stations (BSs) cooperates in transmitting data to a common receiver. If a decoding error occurs, data is cooperatively retransmitted by a possibly different set of BSs, such that the receiver can benefit from spatiotemporal BS cooperation. Specific cooperation techniques studied in this paper include joint transmission, base station silencing, and the Alamouti space-time code. Using tools from stochastic geometry, the coverage probability at the typical user is characterized as an integral function of the network parameters and the sets of cooperating BSs. The expressions derived reveal the existence of two qualitatively different operating regimes. In the high-coverage regime, the typical user is diversity-limited, so cooperation techniques exploiting spatiotemporal diversity are highly effective in increasing coverage. It is shown that retransmissions always yield time diversity, while channel state information at the transmitters is required to harvest spatial diversity via joint transmission. In the low-coverage regime, on the other hand, the typical user is interference-limited, so cooperation techniques such as joint transmission and base station silencing are effective in increasing coverage as they suppress part of the interference power.

Index Terms

Alamouti code, base station cooperation, base station silencing, diversity gain, heterogeneous networks, joint transmission.

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I. INTRODUCTION

A. Motivation and Contributions

This paper studies spatiotemporal cooperation in heterogeneous networks where BSs selected from multiple network tiers cooperate in transmitting data to a common user. In the event a decoding error, cooperative retransmissions of erroneous data take place, such that the common user can benefit from spatiotemporal BS cooperation. This setup may arise, for instance, in next-generation heterogeneous cellular networks employing inter-cell interference coordination or other BS cooperation techniques. Assuming that all transmitters and receivers in the network are equipped with a single antenna, the channel between the cooperating BSs and the common user can be modeled as a distributed MISO system. As a consequence, distributed implementation of space-time coding can lead to increased network coverage, due to diversity gain and reduced inter/intra cell interference.

In this general framework, our contributions are as follows. First, we present a tractable model for studying cooperation in heterogeneous networks that can in principle be used to analyze arbitrary spatiotemporal cooperation strategies. We then specialize the model to the case where the BSs in each network tier form a spatially homogeneous point process and the set of cooperating BSs are selected based on the average received power at the typical user in the network (strongest-BS association model). We assume that a subset of BSs cooperatively retransmits data if the signal-to-interference (SIR) ratio at the typical user is lower then a given threshold value θ, which we refer to as outage event. In this setting, we define the coverage probability at the typical user as the probability that no outage event occurs after the first retransmission.

We consider two cooperation strategies:

- Joint transmission (JT): The BSs cooperate by jointly transmitting the same data to the typical user,
- BS silencing (BSS): The BSs cooperate by silencing the strongest interfering BSs at the typical user, hence reducing the interference power.

The motivation for studying JT comes from the homonymous strategy in the most recent LTE standard [2], while BSS is a lower complexity technique that overcomes the biggest limitation of JT, i.e., the significant backhaul network control traffic needed to establish cooperation.

Similarly, we consider two decoding methods:

- Independent attempts: As in a simple type-I HARQ scheme, the typical user discards the received data in the first erroneous transmission and attempts decoding ex novo from the retransmission.
- Chase combining: As in a type-II HARQ scheme, if a retransmission takes place, then the typical user uses maximum-ratio-combining (MRC) to combine the received data in the first transmission and in the retransmission. The outage event in this case is defined as the event that the sum of the SIRs after two transmissions is lower than the threshold θ.

For each combination of cooperation/decoding techniques, we characterize the coverage probability at the typical user as an integral function of the network parameters and the number of cooperating BSs.

The derived expressions are then used to analytically characterize the rate of exponential increase of the coverage probability as $\theta \to 0$, which we refer to as diversity gain. We prove that in all the cases under consideration the diversity gain increases with the number of retransmissions, but is independent of the number of cooperating BSs, unless the cooperating BSs have knowledge of their respective small-scale fading gain to the receiver. Channel state information (CSI), in fact, allows the BSs to jointly transmit phase-shifted copies of the same signal that add coherently at the receiver. We then show that by distributed implementation of space-time coding techniques at the cooperating BSs, spatial diversity can be achieved even if CSI is not available at the transmitters. We illustrate this point by analyzing a specific scheme where two BSs apply Alamouti's scheme to cooperatively transmit to the typical user. We prove that in this case a diversity gain of two is achievable even though the BSs do not have CSI, i.e., spatial diversity can be fully exploited.

Another noteworthy observation that emerges from numerical evaluations of our derived expressions is that joint transmission and BSS achieve comparable coverage probability performance for large values of θ. This result can be understood by noticing that, in the large θ regime, the coverage probability is determined mainly by the probability that the interference power at the typical user is large. It follows that the cooperation schemes that reduce the amount of interference at the typical user are highly effective in increasing coverage. Moreover, the power gain provided by joint transmission has a negligible impact on the coverage probability compared to the gain due to interference suppression attained by both joint transmission and

BSS.

B. Related Work and Paper Organization

Finally, we wish to mention a few related works in the literature. Cooperative retransmission has been mostly studied in the context of relay-assisted communication. In the information-theoretic literature, [3] analyzed the performance of a decode–forward relay scheme where relay nodes decode the data transmitted by the source and cooperatively forward it toward the destination. [4] compared the outage probability and the diversity gain that can be achieved by using various relaying techniques. [5] and [6] studied cooperative retransmission protocols and proposed a decentralized algorithm for relay selection. One limitation of these existing studies, however, is that they do not model the interference caused by other BSs or relay nodes in the wireless networks. This paper aims at overcoming this limitation by accurately modeling the interference using tools from stochastic geometry. Stochastic geometry models for heterogeneous networks have been recently proposed in [7]–[9], where the BSs in different network tiers are assumed to be spatially distributed according to Poisson point processes (PPPs). Following a similar model, [10], [11] studied the advantages of cooperative relaying in a homogeneous network, while we previously studied joint transmission in heterogeneous networks in [12]. Our previous work, however, did not account for cooperative retransmissions. One technical issue that arises when dealing with retransmissions is the fact the interference at the typical user is correlated in time. Interference correlation has been previously studied in [13]–[15], but in the context of single-tier networks. [16] studied the effect of interference correlation on the performance of MRC in a SIMO setting. This paper is not the first to characterize the diversity gain in the high coverage regime. Previously, [17] showed repetition coding does not provide any diversity gain in an ad hoc wireless network, due to the correlation of the interference across retransmissions. [18] analyzed the diversity loss due to interference correlation in a SIMO channel model. It should also be remarked that BSS was first studied in [19] in the case of a single-tier network.

The remainder of the paper is organized as follows. Section II introduces the system model. Section III and Section IV present the main results for joint transmission and BS silencing, while Section V is devoted to Alamouti coding. Section VI presents some numerical results. Section VII concludes the paper.

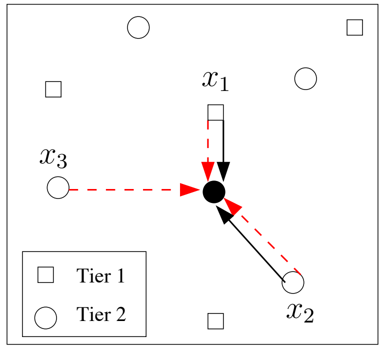

Fig. 1. Example of a two-tier heterogeneous network where the two best BSs, in the sense of average receive power, cooperate in transmitting data to a common receiver located at the origin (solid arrows). Assuming that a decoding error occurs, data is cooperatively retransmitted by the best three BSs (dotted arrows). The model considered in this paper allows the number of cooperating BSs to vary across retransmissions.

II. System Model

A. Heterogeneous Network Model

We consider a heterogeneous wireless network composed of K independent network tiers of BSs with different deployment densities and transmit powers. It is assumed that the BSs belonging to the jth tier have transmit power P_j and are spatially distributed according to a two-dimensional homogeneous PPP Φ_j of density λ_j, $j = 1, \ldots, K$. We denote by $\Phi = \Phi_1 \cup \cdots \cup \Phi_K$ the spatial process obtained by superposing the K network tiers, and we define $\nu : \mathbb{R}^2 \to \{1, \ldots, K\}$ as the function that maps every point $x \in \Phi$ into the index of the network tier to which the BS located at x belongs.

Due to the stationarity of Φ, we assume without loss of generality that the typical user is located at the origin of the coordinate system $(0, 0) \in \mathbb{R}^2$. The path loss of signals transmitted by the BS located at $x_i \in \Phi$ to the typical user is characterized by two different phenomena, the average path loss and the small-scale fading. The average path loss coefficient is denoted in the sequel by l_i and is related to the BS's distance from the origin $\|x_i\|$ and its transmit power $P_{\nu(x_i)}$ as

$$l_i^2 = \frac{P_{\nu(x_i)}}{\|x_i\|^\alpha}, \tag{1}$$

where $\alpha > 2$ denotes the path loss exponent of the propagation environment. The small-scale fading coefficient is denoted by h_k and is modeled as Rayleigh fading, a reasonable model if propagation occurs in a built-up urban scattering environment. Notice that the average path loss coefficients naturally induce an ordering of the BSs in the network in terms of their average signal strength at the typical user. Accordingly, we assume that the coefficients in (1) are ordered in decreasing order of magnitude

$$l_1^2 \geq l_2^2 \geq \cdots , \tag{2}$$

such that x_i denotes hereinafter the location of the BS with the ith largest average path loss coefficient l_i^2 (see Fig. 1).

We assume that the top n_1 BSs in this ordered list cooperate in transmitting data to the typical user. This is a natural assumption motivated by the fact that, in practical systems, users maintain a ranked list of the surrounding cells based on received signal power and may be connected to many of them simultaneously. If the SIR at the typical user after the first transmission is below a certain threshold θ, then we say that an outage event occurs and the transmission is declared unsuccessful. In this case, we assume that the top n_2 BSs cooperate in retransmitting data to the typical user using repetition coding. Notice that we allow the number of cooperating BSs to vary from n_1 to n_2 across retransmissions, a reasonable assumption since the BSs' availability to cooperate may vary in time due to varying load/channel conditions, delays in the backhaul network, and errors in decoding of ACK/NAK messages sent back from the user. As an example, Fig. 1 illustrates a two-tier heterogeneous network where $n_1 = 2$ BSs belonging to different network tiers cooperate in transmitting data, which is then retransmitted by a larger set of $n_2 = 3$ BSs.

In this setup, the received channel output at the typical user in the kth transmission, $k = 1, 2$, can be written as the sum of a desired signal and an interfering signal as follows

$$\sum_{i \in \mathcal{C}_k} l_i h_{ik} s + \sum_{i > n_k} l_i h_{ik} s_{ik}, \tag{3}$$

where $\mathcal{C}_k \equiv \mathcal{C}[k] \subseteq \{1, 2, \ldots, n_k\}$ denotes a subset of the cooperating BSs dependent on the cooperation strategy under consideration, s denotes the channel input symbol of interest, while $s_{ik} \equiv s_i[k]$ denotes the channel input symbol sent by the ith interfering BS. We consider two cooperation strategies:

- Joint transmission (JT): In this case, all n_k cooperating BSs jointly transmit the same symbol s to the typical user. Accordingly,

$$\mathcal{C}_k = \{1, 2, \ldots, n_k\}$$

for every $k = 1, 2$.

- BS silencing (BSS): In this case,

$$\mathcal{C}_k = \{1\}$$

for every k, so BSs cooperate by simply silencing the BSs with ith strongest average path loss, $i = 2, 3, \ldots, n_k$, hence reducing the aggregate interference power.

The system model in (3) embodies the following assumptions:

- Compliantly to the Rayleigh fading assumption, the small-scale fading coefficients $\{h_{ik}\}$ are assumed to be i.i.d. standard complex Gaussian random variables independent of everything else. In particular, the coefficients h_{i1} and h_{i2} in the first transmission and the retransmission are assumed to be independent, a valid assumption if retransmissions take place at a time scale that is larger than the coherence time of the channel.
- The position of the typical user is assumed to remain fixed during the retransmission, a valid assumption in low-Doppler channel models. As a consequence, the average path loss coefficients in (3) are assumed to be time invariant.
- The symbols $\{s_{ik}\}$ and s are assumed to be i.i.d. zero-mean random variables of unit variance. By making such an assumption, we ignore the fact that the BSs which contribute to the total interference power may cooperate to serve other users in the network. In the case of BSS, it is clear that this assumption leads to conservative results because by silencing some of the interferers we can only increase the SIR at the typical user. A similar result, counterintuitive at first glance, was also proved in [12, Proposition 1] for the case of JT, where we showed that cooperation among the interferers can only increase the coverage probability at the typical user.
- The background noise power is assumed to be negligible compared to the total aggregate interference power (interference-limited regime assumption). Accordingly, (3) does not include the contribution of the background thermal noise to the received signal.

It follows from (3) that the SIR at the typical user at the end of the kth transmission is

$$\mathrm{SIR}_k = \frac{|\sum_{i\in\mathcal{C}_k} l_i\, h_{ik}|^2}{I_k} \tag{4}$$

where we define

$$I_k := \sum_{i>n_k} l_i^2 |h_{ik}|^2 \tag{5}$$

as the aggregate interference power. Notice that I_1 and I_2 are correlated random variables, since the average path loss terms are time invariant.

At the receiver side, we consider two decoding methods:

- Independent Attempts: In this case, the typical user makes independent decoding attempts after each transmission. Given a threshold θ, we define the coverage probability as

$$\mathsf{P} = \mathbb{P}(\mathrm{SIR}_1 > \theta) + \mathbb{P}(\mathrm{SIR}_2 > \theta, \mathrm{SIR}_1 < \theta) \tag{6}$$

  where $\mathbb{P}(\mathrm{SIR}_1 < \theta)$ denotes the probability of the outage event after the first transmission.
- Chase Combining: Here, we assume that if a retransmission takes place, the typical user performs MRC of the received signal in two transmissions. In this case, the combined SIR at the output of the MRC receiver is $\mathrm{SIR}_1 + \mathrm{SIR}_2$. For a given threshold θ, the coverage probability is then defined as

$$\mathsf{P}^{\mathrm{MRC}} = \mathbb{P}(\mathrm{SIR}_1 > \theta) + \mathbb{P}(\mathrm{SIR}_1 + \mathrm{SIR}_2 > \theta, \mathrm{SIR}_1 < \theta). \tag{7}$$

In the sequel, we refer to these two decoding methods as retransmission without MRC and with MRC, respectively.

III. Coverage Probabilities

In this section, we present the main results of the paper. For each combination of cooperation/decoding techniques, we characterize the coverage probability at the typical user as an integral function of the network parameters and the number of cooperating BSs n_1 and n_2. We focus our attention on the case $n_1 \leq n_2$, but the same analysis can be repeated verbatim for the case $n_2 \geq n_1$.

A. Joint Transmission without MRC

First, we consider the case of JT with independent decoding attempts. The following key technical lemma characterizes the joint complimentary cumulative distribution function (ccdf) of SIR_1 and SIR_2 as a function of the system parameters.

Lemma 1: For every $\theta_1, \theta_2 \geq 0$

$$\begin{aligned}
&\mathbb{P}\left(\mathrm{SIR}_1 > \theta_1, \mathrm{SIR}_2 > \theta_2\right) \\
&= \int_{\mathcal{A}} \exp\left(-u_{n_2}\big(1 + 2G\big(\theta_1\ u_{n_2}^{-\frac{\alpha}{2}}\ v_1^{-1},\ \theta_2\ u_{n_2}^{-\frac{\alpha}{2}}\ v_2^{-1}\big)\big) - \textstyle\sum_{i=n_1+1}^{n_2} \log\big(1 + \theta_1 u_i^{-\frac{\alpha}{2}} v_1^{-1}\big)\right)\ \mathrm{d}\mathbf{u}, \\
&=: \phi(n_1, n_2, \theta_1, \theta_2),
\end{aligned} \tag{8}$$

where the integral is over the set

$$\mathcal{A} = \left\{\mathbf{u} \in \mathbb{R}_+^{n_2} : u_1 < u_2 < \ldots < u_{n_2}\right\}, \tag{9}$$

and where we define

$$G(x, y) := \int_1^\infty \left(1 - \frac{1}{(1 + xr^{-\alpha})(1 + yr^{-\alpha})}\right) r\ \mathrm{d}r \tag{10}$$

and

$$v_k = u_1^{-\frac{\alpha}{2}} + u_2^{-\frac{\alpha}{2}} + \ldots + u_{n_k}^{-\frac{\alpha}{2}}, \quad k = 1, 2. \tag{11}$$

Proof: See Appendix A. ■

Remark 1: In the special case where $n_1 = n_2 = n$ and $\theta_1 = \theta_2 = \theta$, by performing the change of variable $u_j = u_{n_k} t_j$ for $1 \leq j \leq n_k - 1$ and then integrating over u_{n_k}, it can be easily shown that (8) simplifies to

$$\begin{aligned}
&\phi(n, n, \theta_1, \theta_2) \\
&= \begin{cases} \left(1 + 2G(\theta_1, \theta_2)\right)^{-1}, & n = 1; \\ \displaystyle\int_{\mathcal{D}} \frac{(n-1)!}{\left(1 + 2G(\theta_1\ \tau_n^{-1}, \theta_2\ \tau_n^{-1})\right)^n} \mathrm{d}\mathbf{t}, & n > 1, \end{cases}
\end{aligned} \tag{12}$$

where

$$\mathcal{D} = \left\{\mathbf{t} \in \mathbb{R}_+^{n-1} : 0 < t_1 < \ldots\ldots < t_{n-1} < 1\right\}$$

and

$$\tau_n = 1 + t_1^{-\frac{\alpha}{2}} + \ldots t_{n-1}^{-\frac{\alpha}{2}}.$$

Using (12), the marginal ccdfs $\mathbb{P}(\mathrm{SIR}_k > \theta)$ can be calculated as

$$\mathbb{P}(\mathrm{SIR}_k > \theta) = \phi(n_k, n_k, \theta, 0), \qquad k = 1, 2. \tag{13}$$

Remark 2: The integral function $G(x,y)$ defined in (10) can not be expressed in closed form in general but can be easily evaluated numerically since it is related to the hypergeometric function ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ as follows

$$\begin{cases} \frac{\frac{x^2}{1+x}\,{}_2F_1\left(1,1;2-\frac{2}{\alpha};\frac{x}{1+x}\right)-\frac{y^2}{1+y}\,{}_2F_1\left(1,1;2-\frac{2}{\alpha};\frac{y}{1+y}\right)}{(\alpha-2)(x-y)}, & x \neq y; \\ \frac{x}{\alpha(1+x)}\left(1+\frac{\alpha+2}{\alpha-2}{}_2F_1\left(1,1;2-\frac{2}{\alpha};\frac{x}{1+x}\right)\right), & x = y. \end{cases} \tag{14}$$

Also, closed-form expressions exist for specific values of $\alpha > 2$. For example, it can be easily verified that if $\alpha = 4$, then

$${}_2F_1\left(1,1;2-\frac{2}{\alpha};\frac{x}{1+x}\right) \equiv \frac{1+x}{\sqrt{x}}\tan^{-1}(\sqrt{x}).$$

Using Lemma 1, we prove the following result.

Theorem 1: The coverage probability (6) for JT without MRC is equal to

$$\mathsf{P} = \phi(n_1,n_1,\theta,0) + \phi(n_2,n_2,\theta,0) - \phi(n_1,n_2,\theta,\theta), \tag{15}$$

where $\phi(n_1,n_2,\theta,\theta)$ is defined in (8).

Proof: By (6), the coverage probability can be re-written as

$$\begin{aligned} \mathsf{P} &= \mathbb{P}(\text{SIR}_1 > \theta) + \mathbb{P}(\text{SIR}_2 > \theta, \text{SIR}_1 < \theta) \\ &\overset{(a)}{=} \sum_{k=1}^{2} \mathbb{P}(\text{SIR}_k > \theta) - \mathbb{P}(\text{SIR}_2 < \theta, \text{SIR}_1 < \theta) \\ &= \mathbb{P}\left(\{\text{SIR}_1 > \theta\} \cup \{\text{SIR}_2 > \theta\}\right), \end{aligned} \tag{16}$$

where (a) follows from basic set theory. Now, we can apply the inclusion-exclusion formula with $\mathbb{P}(\text{SIR}_k > \theta)$ in (13) and $\mathbb{P}(\text{SIR}_1 > \theta,\ \text{SIR}_2 > \theta)$ as in Lemma 1 to derive the result. ■

The result in Theorem 1 only depends on the number of cooperating BSs n_1 and n_2, the threshold θ, and the path loss exponent α. Hence, we can draw similar conclusions as in [12, Remark 1] on the fact that (15) is independent of the number of network tiers K, and their respective power levels and deployment densities.

Remark 3: By substituting (12) into (15), it follows that in the special case when $n_1 = n_2 = n$, i.e., if the number of cooperating BSs does not change after the first transmission,

$$\mathsf{P} = \frac{2}{1+2G(\theta,0)} - \frac{1}{1+2G(\theta,\theta)}, \tag{17}$$

if $n = 1$, and

$$\mathsf{P} = 2\phi(n,n,\theta,0) - \int\limits_{D} \frac{(n-1)!}{\left(1+2G(\theta_1\, \tau_n^{-1}, \theta_2\, \tau_n^{-1})\right)^n} \mathrm{d}\mathbf{t}, \tag{18}$$

if $n > 1$. If we further specialize (17) to the case $\alpha = 4$, we obtained the remarkably simple closed-form expression

$$\frac{2}{1+\sqrt{\theta}\tan^{-1}(\sqrt{\theta})} - \frac{1}{1+\frac{3}{2}\sqrt{\theta}\tan^{-1}(\sqrt{\theta}) + \frac{\theta}{2(1+\theta)}}.$$

B. Joint Transmission with MRC

In the case of JT with MRC at the receiver, we have the following result.

Theorem 2: The coverage probability in (7) for JT with MRC is

$$\mathsf{P}^{\mathrm{MRC}} = \int\limits_0^\infty \sigma(n_1, n_2, z, (\theta - z)^+) \mathrm{d}z, \tag{19}$$

where

$$\begin{aligned}
&\sigma(n_1, n_2, z, a) \\
&:= \int_{\mathcal{A}} \exp\Big(-u_{n_2}\big(1+2G\big(z u_{n_2}^{-\frac{\alpha}{2}} v_2^{-1},\ a u_{n_2}^{-\frac{\alpha}{2}} v_1^{-1}\big)\big) - \textstyle\sum_{i=n_1+1}^{n_2} \log\big(1+a u_i^{-\frac{\alpha}{2}} v_1^{-1}\big)\Big) \\
&\quad \times 2u_{n_2}^{1-\frac{\alpha}{2}}\ v_2^{-1}\ H\big(z u_{n_2}^{-\frac{\alpha}{2}} v_2^{-1},\ a u_{n_2}^{-\frac{\alpha}{2}} v_1^{-1}\big)\ \mathrm{d}\mathbf{u},
\end{aligned} \tag{20}$$

with $\mathcal{A}$ as in (9), v_k as in (11), and where we define

$$H(x,y) := \frac{\partial}{\partial x} G(x,y) = \int_1^\infty \frac{r^{1-\alpha}}{(1+xr^{-\alpha})^2\ (1+yr^{-\alpha})} \mathrm{d}r.$$

Proof: See Appendix B. ■

Remark 4: In the special case where $n_1 = n_2 = n$, (19) simplifies to

$$\mathsf{P}^{\mathrm{MRC}} = \phi(n,n,\theta,0) + \int\limits_0^\theta \sigma(n,n,z,\theta-z) \mathrm{d}z \tag{21}$$

and in particular, if $n = 1$,

$$\mathsf{P}^{\mathrm{MRC}} = \frac{1}{1+2G(\theta,0)} + \int_0^\theta \frac{2H(z,\theta-z)}{(1+2G(z,\theta-z))^2} \mathrm{d}z. \tag{22}$$

By comparing (12) and (22), notice that the first term at the right hand side of (22) is equals $\phi(1,1,\theta,0)$, which by (13) denotes the coverage probability after one transmission using a single BS. Therefore, the integral term in (22) represents the gain due to retransmission with MRC.

C. Base Station Silencing

In the case of BSS we prove the following theorem, whose proof is omitted as it follows similar steps as the proofs of Theorems 1 and 2.

Theorem 3: The coverage probability in (6) for BSS without MRC is equal to

$$\mathsf{P} = \varphi(n_1, n_1, \theta, 0) + \varphi(n_2, n_2, \theta, 0) - \varphi(n_1, n_2, \theta, \theta), \tag{23}$$

where

$$\begin{aligned}
&\varphi(n_1, n_2, \theta_1, \theta_2)\\
&:= \int_{\mathcal{A}} \exp\Big(-u_{n_2}\big(1 + 2G\big(\theta_1\, u_{n_2}^{-\frac{\alpha}{2}}\, u_1^{\frac{\alpha}{2}},\, \theta_2\, u_{n_2}^{-\frac{\alpha}{2}}\, u_1^{\frac{\alpha}{2}}\big)\big) - \textstyle\sum_{i=n_1+1}^{n_2} \log\big(1 + \theta_1 u_i^{-\frac{\alpha}{2}} u_1^{\frac{\alpha}{2}}\big)\Big)\, \mathrm{d}\mathbf{u}.
\end{aligned} \tag{24}$$

The coverage probability in (7) for retransmission with MRC is equal to

$$\mathsf{P}^{\mathrm{MRC}} = \int_0^\infty \varsigma(n_1, n_2, z, (\theta - z)^+)\mathrm{d}z, \tag{25}$$

where

$$\begin{aligned}
&\varsigma(n_1, n_2, z, a)\\
&:= \int_{\mathcal{A}} \exp\Big(-u_{n_2}\big(1 + 2G\big(z\, u_{n_2}^{-\frac{\alpha}{2}}\, u_1^{\frac{\alpha}{2}},\, a\, u_{n_2}^{-\frac{\alpha}{2}}\, u_1^{\frac{\alpha}{2}}\big)\big) - \textstyle\sum_{i=n_1+1}^{n_2} \log\big(1 + a u_i^{-\frac{\alpha}{2}} u_1^{\frac{\alpha}{2}}\big)\Big)\\
&\quad \times 2u_{n_2}^{1-\frac{\alpha}{2}}\, u_1^{\frac{\alpha}{2}}\, H\big(z\, u_{n_2}^{-\frac{\alpha}{2}}\, u_1^{\frac{\alpha}{2}},\, a\, u_{n_2}^{-\frac{\alpha}{2}}\, u_1^{\frac{\alpha}{2}}\big)\, \mathrm{d}\mathbf{u},
\end{aligned} \tag{26}$$

with $\mathcal{A}$ in (24) and (26) defined as in (9).

Remark 5: By comparing (8) and (24), notice that the integral function φ is defined exactly as ϕ after replacing v_k in (8) by $u_1^{-\alpha/2}$. Similarly, the integral function ς defined in (26) is equal to σ in (20) after replacing v_k by $u_1^{-\alpha/2}$.

Remark 6: In the special case when $n_1 = n_2 = n > 1$, it can be verified, by making the change of variables $u_i = u_n t_i$, $i = 1, 2, \ldots, n-1$, that the coverage probabilities without MRC and with MRC at the typical user simplify to

$$\mathsf{P} = 2\varphi(n, n, \theta, 0) - \int_0^1 \frac{(n-1)(1-t)^{n-2}}{\big(1 + 2G(\theta t^{\alpha/2}, \theta t^{\alpha/2})\big)^n}\mathrm{d}t, \tag{27}$$

and

$$\mathsf{P}^{\mathrm{MRC}}=\int_0^\infty \mathrm{d}z\int_0^1 \frac{n(n-1)(1-t)^{n-2}H(zt^{\frac{\alpha}{2}},(\theta-z)^{+}t^{\frac{\alpha}{2}})}{\frac{1}{2}t^{-\frac{\alpha}{2}}(1+2G(zt^{\frac{\alpha}{2}},(\theta-z)^{+}t^{\frac{\alpha}{2}}))^{n+1}}\mathrm{d}t, \tag{28}$$

respectively.

Remark 7: It can be easily shown that the marginal ccdf $\mathbb{P}(\mathrm{SIR}_k>\theta)$ obtained by setting in $\varphi(n_k,n_k,\theta,0)$ in Theorem 3 recovers the expression derived in [19, Theorem 1].

IV. High-Coverage Regime: Diversity Gains

In this section, we use the integral expressions derived in Section III to study the qualitative behavior of the coverage probability in the high-coverage regime. As in [19, Definition 3], we define the diversity gain d_{n_1,n_2} at the typical user as the rate of exponential increase of the coverage probability as $\theta\to 0$, i.e.,

$$d_{n_1,n_2}=\lim_{\theta\to 0}\frac{\log(1-\mathsf{P})}{\log\theta}. \tag{29}$$

In (29) the subscripts n_1 and n_2 are adopted to emphasize the dependency of the diversity gain on the number of cooperating BSs. Our main result is an analytical expression for d_{n_1,n_2} for all the cooperation schemes/receiver methods considered in this paper.

As a first step, we state a technical lemma of independent interest, providing asymptotic forms for the outage probability in the small θ regime.

Lemma 2: Let Y_1,Y_2 be i.i.d. chi-squared distributed random variables with m degrees of freedom and let (J_1,J_2) be a pair of arbitrarily distributed positive random variables mutually independent of (Y_1,Y_2) such that

$$\mathsf{E}[(J_1J_2)^{m/2}]<\infty. \tag{30}$$

Then, as $\theta\to 0$,

$$\mathbb{P}\left(\frac{Y_1}{J_1}<\theta,\frac{Y_2}{J_2}<\theta\right)\sim\ \theta^m\,\frac{\mathsf{E}\Big((J_1J_2)^{m/2}\Big)}{2^m\left(\Gamma(m/2+1)\right)^2} \tag{31}$$

and

$$\mathbb{P}\left(\frac{Y_1}{J_1}+\frac{Y_2}{J_2}<\theta\right)\sim\ \theta^m\,\frac{\mathsf{E}\Big((J_1J_2)^{m/2}\Big)}{2^m\Gamma(m+1)}. \tag{32}$$

Proof: See [20]. ■

Notice that in Lemma 2 no assumptions are made on the distribution of the pair (J_1, J_2) modeling the possibly correlated interference powers at the typical receiver in two consecutive transmissions, except for the finiteness of a certain moment of the joint distribution. Therefore, Lemma 2 can be applied in other situations than those treated in this paper. For the specific setting under consideration, the following technical lemma ensures that (30) is satisfied.

Lemma 3: Under the system model in (3), we have

$$\mathsf{E}_{l_j, I_k}\left(\frac{I_1^{n_1}}{l_1^2 l_2^2 \ldots l_{n_1}^2} \, \frac{I_2^{n_2}}{l_1^2 l_2^2 \ldots l_{n_2}^2}\right) < \infty,$$

where I_k is defined in (5).

Proof: See [20]. ■

Using Lemmas 2 and 3, we can derive the following diversity gain result for the case of JT.

Theorem 4: For every $n_1, n_2 \geq 1$, the diversity gain in (29) for JT is

$$d_{n_1,n_2} = 2$$

both with MRC and without MRC.

Proof: First, we write the outage events for retransmission with and without MRC in form of the events considered in Lemma 2. Then, we prove that the condition in Lemma 2 is satisfied in our case.

Focusing on retransmission without MRC, we can write the outage event as

$$\begin{aligned}
&\bigcap_{k=1}^{2}\{\mathrm{SIR}_k < \theta\} \\
&= \bigcap_{k=1}^{2}\left\{\left|\sum_{j=1}^{n_k} l_j \, h_{jk}\right|^2 < \theta I_k\right\} \\
&= \bigcap_{k=1}^{2}\Bigg\{\underbrace{\frac{\left|\sum_{j=1}^{n_k} l_j \, h_{jk}\right|^2}{\sum_{j>n_k} l_j^2/2}}_{Y_k} < \underbrace{\frac{2\theta I_k}{\sum_{j>n_k} l_j^2}}_{\theta J_k}\Bigg\}.
\end{aligned} \tag{33}$$

Here, Y_k is exponentially distributed with mean 2 or, equivalently, chi-squared distributed with $m = 2$ degrees of freedom due to the fact that $|\sum_{j=1}^{n_k} l_j \, h_{jk}|^2$ is exponentially distributed with mean $\sum_{j>n_k} l_j^2$. Y_1 and Y_2 are independent since the fading coefficients h_{jk} are mutually independent.

In case of retransmission with MRC, the coverage probability in (7) can be further expressed as

$$\begin{aligned} \mathsf{P}^{\mathrm{MRC}} &= \mathbb{P}(\mathrm{SIR}_1 > \theta) + \mathbb{P}(\mathrm{SIR}_1 + \mathrm{SIR}_2 > \theta,\ \mathrm{SIR}_1 < \theta) \\ &\overset{(a)}{=} \mathbb{P}(\mathrm{SIR}_1 + \mathrm{SIR}_2 > \theta) \end{aligned} \tag{34}$$

where (a) follows from set theory and the fact that $\mathrm{SIR}_k > 0$. Hence, the outage event can be written as

$$\{\mathrm{SIR}_1 + \mathrm{SIR}_2 < \theta\} \equiv \left\{ \frac{Y_1}{J_1} + \frac{Y_1}{J_1} < \theta \right\} \tag{35}$$

where Y_k and J_k are defined as for the case of retransmission without MRC above. Now, we need to prove that $\mathsf{E}_{J_1,J_2}[J_1 J_2]$ is finite to apply the result in Lemma 2. We have

$$\begin{aligned} \mathsf{E}_{J_1,J_2}[J_1 J_2] &= 4\, \mathsf{E}_{l_j,I_k} \left[\prod_{k=1}^{2} \frac{I_k}{\sum_{j=1}^{n_k} l_j^2} \right] \\ &\overset{(a)}{\leq} 4\, \mathsf{E}_{l_j,I_{k,1}} \left[\prod_{i=1}^{2} \frac{I_{k,1}}{l_1^2} \right] \end{aligned} \tag{36}$$

where $I_{k,1}$ is the aggregate interference power assuming $n_k = 1$; (a) follows due to the fact that we are increasing the numerator inside the product by adding a positive term $\sum_{j=2}^{n_k} l_j^2$ and decreasing the denominator inside the product by subtracting the same positive term. From Lemma 3 with $n_1 = n_2 = 1$, it follows that the above expression is finite. Hence, we get a diversity gain of 2 for retransmission with and without MRC. ■

A similar result holds for the case of BSS.

Theorem 5: For every $n_1, n_2 \geq 1$, the diversity gain in (29) for BSS is

$$d_{n_1,n_2} = 2$$

both with MRC and without MRC.

Proof: By following similar steps as in the proof of Theorem 4, we can derive the outage events for retransmission without MRC and with MRC in the form of (33) and (35), respectively with $Y_k = 2|h_{1k}|^2$ which is chi-squared distributed with 2 degrees of freedom and $J_k = 2I_k/l_1^2$. Now, we need to prove that $\mathsf{E}_{J_1,J_2}[J_1 J_2]$ is finite to apply the result in Lemma 2. We have

$$\begin{aligned} \mathsf{E}_{J_1,J_2}[J_1 J_2] &= 4\, \mathsf{E}_{l_j,I_k}\left[\prod_{k=1}^{2} \frac{I_k}{l_1^2}\right] \\ &\overset{(a)}{\leq} 4\, \mathsf{E}_{l_j,I_{k,1}}\left[\prod_{k=1}^{2} \frac{I_{k,1}}{l_1^2}\right] \end{aligned} \tag{37}$$

where $I_{k,1}$ is the aggregate interference power assuming $n_k = 1$; (a) follows due to the fact that we are increasing the numerator inside the product by adding a positive term $\sum_{j=2}^{n_k} l_j^2$. Using Lemma 3 with $n_k = 1$, we can prove that the above expression is finite. Hence, we get a diversity gain of 2 for retransmission with and without MRC. ■

It follows from Theorems 4 and 5 that in all cooperation/decoding techniques under consideration the diversity gain (29) is two, independently of the number of cooperating BSs. This result can be understood noticing that in our setup the fading coefficients across two transmissions are assumed to be independent, so they provide time diversity that translates in lower outage probability. Had we considered a scenario with N retransmissions, the diversity gain would have been N. At the same time, the result in Theorem 4 shows that despite having multiple BSs simultaneously transmitting, cooperation via joint transmission fails in exploiting the spatial diversity provided by the independent fading coefficients from the spatially separated BSs to the typical user. This result can be explained as follows. Observe from (3) that the signals transmitted by the BSs in $\mathcal{C}_k$ sum non-coherently at the receiver and therefore the effective channel $\sum_{i \in \mathcal{C}_k} l_i\, h_{ik}$ is statistically equivalent to a SISO Rayleigh fading channel, both in the case of JT and BSS.

To contrast this negative result, we show next that if the cooperating BSs have knowledge of their respective small-scale fading gain to the receiver, then spatial diversity can be exploited via joint transmission. CSI, in fact, allows the BSs to jointly transmit phase-shifted copies of the same signal that add coherently at the receiver. Specifically, if h_{ik} is known at the transmitter, then the BS located at x_i can compensate the phase shift caused by the channel by premultiplying the transmitted symbol by $h_{ik}^*/|h_{ik}|$. It follows that in this case the SIR at the end of the k-th transmission can be written as

$$\mathrm{SIR}_k = \frac{\left|\sum_{i=1}^{n_k} l_j\, |h_{ik}|\right|^2}{I_k}, \tag{38}$$

In this setup, we have the following result.

Theorem 6: For every $n_1, n_2 \geq 1$, the diversity gain in (29) for JT with CSI at the transmitters satisfies

$$d_{n_1,n_2} = n_1 + n_2$$

both with MRC and without MRC at the receiver.

Proof: See Appendix C. ■

It follows from Theorem 6 that we get full diversity gain when BSs have CSI. A similar result is obtained in [12, Theorem 5] where a single transmission is considered with coherent JT.

V. Alamouti Coding

In the previous section, we showed that in order to exploit spatial diversity by JT, the cooperating BSs need to have CSI. This limitation of joint transmission can be overcome by distributed implementation of space-time coding techniques at the cooperating BSs. We illustrate this point by analyzing a specific scheme where two BSs apply Alamouti's code to cooperatively transmit to the typical user.

In Alamouti's code, pairs of coded symbols are transmitted to the typical receiver in two channel uses, typically adjacent resource elements in the time/frequency domain, such that the channel gain from the transmitter to the receiver can be assumed to remain constant during the transmission of the pair of symbols.

Following the notations in (3), the received signal in two adjacent resource elements can be written as

$$\begin{aligned} y_1 &= l_1\, h_1\, s_1 + l_2\, h_2\, s_2 + z_1 \\ y_2 &= -l_1\, h_1\, s_2^* + l_2\, h_2\, s_1^* + z_2 \end{aligned} \tag{39}$$

where $z_k = \sum_{j>2} l_j\, h_j\, s_{jk}$ for $k = 1, 2$; h_j denotes the random fading coefficient between BS located at x_j and the user located at origin. h_j's are i.i.d. zero mean complex Gaussian random variables with unit variance. s_1 and s_2 are the two desired symbols and $*$ denotes the Hermitian operator. The above equations can be rewritten as

$$\underbrace{\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} l_1\, h_1 & l_2\, h_2 \\ l_2\, h_2^* & -l_1\, h_1^* \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} s_1 \\ s_2 \end{bmatrix}}_{\mathbf{s}} + \underbrace{\begin{bmatrix} z_1 \\ z_2^* \end{bmatrix}}_{\mathbf{z}} \tag{40}$$

Next, we consider a suboptimal receiver which treats the $\mathbf{z}$ as white Gaussian noise and hence projects the received signal $\mathbf{y}$ on to the columns of matrix H, such that the matched filter output can be expressed as

$$\frac{1}{\sqrt{|\det(H)|}}H^*\mathbf{y} = \sqrt{|\det(H)|}\mathbf{s} + \frac{1}{\sqrt{|\det(H)|}}H^*\mathbf{z}. \tag{41}$$

Assuming that the symbols s_1, s_2 and s_{jk} are independent of each other and have zero mean and unit variance, we can write the SIR as

$$\mathrm{SIR}_1 = \mathrm{SIR}_2 = \frac{|\det(H)|}{I}, \tag{42}$$

where I denotes the aggregate interference power given by

$$I = \sum_{j>2} l_j^2 \, |h_j|^2. \tag{43}$$

A. Coverage Probability

Using the expression for SIR_k in (42), we can define the coverage probability for a given threshold θ as

$$\begin{aligned} \mathsf{P}_\mathrm{c} &= \mathbb{P}\left(\bigcap_{k=1}^{2}\{\mathrm{SIR}_k > \theta\}\right) \\ &= \mathbb{P}\left(\mathrm{SIR}_1 > \theta\right). \end{aligned} \tag{44}$$

We derive the coverage probability result for Alamouti coding in Theorem 7.

Theorem 7: The coverage probability in (44) for Alamouti coding is

$$\int_0^1 \left(\frac{1}{(1+2G(\theta t^{\alpha/2},0))^2} - \frac{t^{\alpha/2}}{(1+2G(\theta,0))^2}\right)\frac{\mathrm{d}t}{1-t^{\alpha/2}}. \tag{45}$$

Proof: See Appendix D ■

Notice that similar to the previous coverage probability results, the coverage probability for Alamouti coding is independent of the number of network tiers, their transmit powers and densities.

B. Diversity Gain

In this subsection, we prove the diversity gain result for Alamouti coding. Similar to the definition in (29), we define the diversity gain for Alamouti coding as

$$d = \lim_{\theta \to 0} \frac{\log(1 - \mathsf{P}_{\mathrm{c}})}{\log \theta}. \tag{46}$$

Theorem 8: The diversity gain in (46) for Alamouti coding is 2.

Proof: As $\theta \to 0$, using the fact that $G(x, 0) \sim \frac{x}{\alpha-2} - \frac{x^2}{2(\alpha-1)}$ and $(1+x)^{-2} \sim 1 - 2x + 3x^2$ as $x \to 0$, the coverage probability in (45) can be written as

$$\mathsf{P}_{\mathrm{c}} \sim \int_0^1 \left(1 - \frac{2\theta^2 t^{\alpha/2}}{\alpha - 1} - \frac{12\theta^2 t^{\alpha/2}}{(\alpha-2)^2}\right) \mathrm{d}t$$

$$\Rightarrow 1 - \mathsf{P}_{\mathrm{c}} \sim \theta^2 \left(\frac{2}{\alpha - 1} + \frac{12}{(\alpha-2)^2}\right) \frac{2}{\alpha + 2}. \tag{47}$$

Hence, we get the diversity gain of 2. ■

VI. Numerical Evaluation

In this section, we present numerical evaluations of the integral expressions for the coverage probabilities derived in this paper. We assume $\alpha = 4$ for all the numerical evaluations.

A. High Coverage Regime

Fig. 2 compares the coverage probability achieved by the Alamouti code (45) with those of JT without MRC (17) and JT with MRC (22) in the special case of retransmission without cooperation, i.e., $n_1 = n_2 = 1$. Notice that all these schemes achieve a diversity gain of two. By contrast, we plot the performance of a scheme studied in [12] based on JT with $n = 2$ cooperating BSs and no retransmission (hereinafter referred to as spatial cooperation), which only achieves a diversity gain of one. Notice the difference in the rate of convergence to 1 of the corresponding curves as $\theta \to 0$. It can also be noticed from the figure that the spatial cooperation outperforms (17) and (22) for $\theta > 5$ dB. In the low-coverage regime, in fact, the typical user is interference-limited, so the spatial cooperation achieves higher coverage by suppressing part of the interference power. Notice that the Alamouti code achieves the best performance in all regimes.

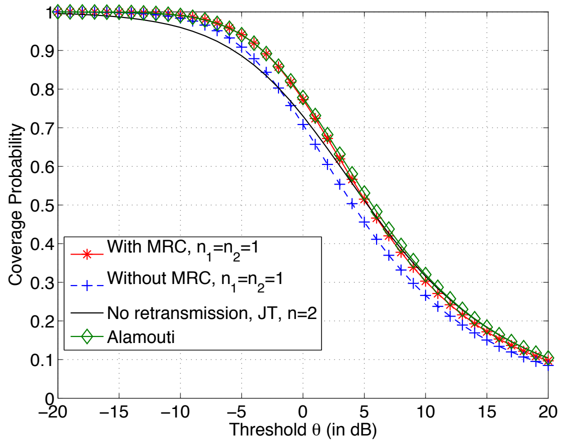

Fig. 2. Coverage probabilities to compare the effect of diversity gain using (13), (17), (22) and (45).

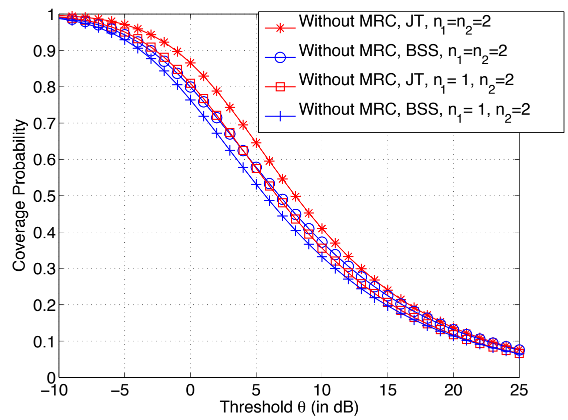

Fig. 3. Coverage probabilities comparing JT and BSS for different number of cooperating BSs using (15), (18), (23) and (27).

In terms of resources, assuming that it takes one resource block for a BS to transmit the message to the receiver, spatial cooperation with $n = 2$ cooperating BSs always uses two resource blocks regardless of whether the user needs it or not. Alamouti code always uses four resource blocks to transmit two messages, i.e., two resource blocks per message. In case of retransmission without cooperation, the message is retransmitted only when the first transmission is unsuccessful. Therefore, we do not necessarily have to use two resource blocks in case of retransmission without cooperation. In fact, the expected number of resource blocks used in case of retransmission without cooperation is $1 + 1 \cdot \mathbb{P}(\text{SIR}_1 < \theta) = 2 - \frac{1}{1+\sqrt{\theta}\tan^{-1}(\sqrt{\theta})}$, which is less than two resource blocks. Retransmission without cooperation also minimizes the backhaul overhead in distributing the desired symbols among cooperating BSs. Therefore, retransmission without cooperation is the most resource efficient way to serve the user for low values of θ.

B. Low Coverage Regime

Fig. 3 compares the performance of JT without MRC and BSS without MRC in two cases, $(n_1, n_2) = (2, 2)$ and $(n_1, n_2) = (1, 2)$ using (15), (18), (23) and (27). As expected, JT outperforms BSS in general. However, the performance gap between these two schemes reduces as θ grows and approaches to zero for $\theta > 20$ dB. This means that in the low-coverage regime the power gain provided by JT has a negligible impact on the coverage probability compared to

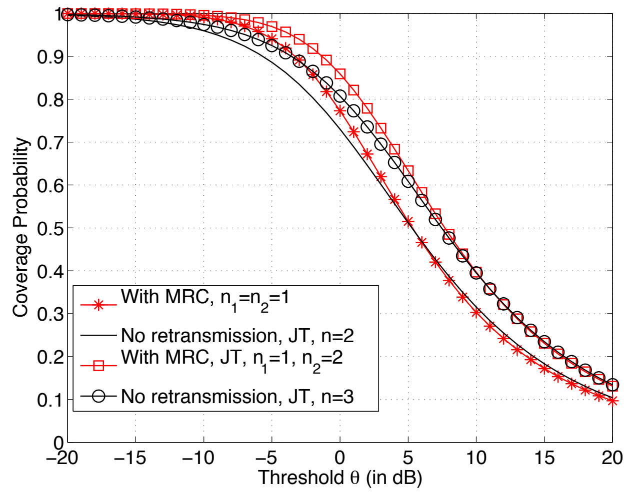

Fig. 4. Coverage probabilities for retransmission with MRC and spatial cooperation using (13), (19) and (22).

the gain due to interference suppression attained by both JT and BSS. Since BSS requires less backhaul overhead than JT, it follows that BSS is a preferable cooperation technique for large values of θ.

C. Retransmission with MRC

Fig. 4 compares the coverage probabilities for spatial cooperation with $n = 2$ cooperating BSs and retransmission with MRC for different values of n_1 and n_2 using (13), (19) and (22). The figure shows that if the receiver has MRC capability, we can reduce number of cooperating BSs by using chase combining compared to spatial cooperation while using same number of resource blocks. For example, when we compare retransmission with MRC using $n_1 = n_2 = 1$ and spatial cooperation using two cooperating BSs, the former provides higher coverage probability up to the threshold of 5 dB and stays comparable to the latter for higher values of thresholds. Similarly, when we compare MRC using $n_1 = 1, n_2 = 2$ and spatial cooperation using three cooperating BSs, MRC provides higher coverage probability up to the threshold of 9 dB and then stays comparable to spatial cooperation. Therefore, retransmission with MRC can provide better or comparable performance to spatial cooperation while using fewer cooperating BSs and hence less backhaul overhead.

VII. Conclusions

In this paper, we considered the problem of spatiotemporal cooperation in interference-limited heterogeneous wireless networks. We focused on two cooperation techniques—JT and BSS—and two decoding techniques—independent attempts and chase combining. For each pair of cooperation/decoding schemes, we derived an integral expression for the coverage probability, that we defined as the probability that the combined SIR across two retransmissions exceeds a threshold value θ. We remark that θ provides an estimate of the attainable spectral efficiency in the network, for instance by a simple inversion of the constrained capacity formula of a Gaussian channel, hence it is related to the achievable data rate by the typical user.

Our analysis reveals the existence of two qualitatively different operating regimes. For small values of θ, the coverage probability is determined, in first approximation, by the probability that the channel fading gain from the cooperating BSs to the receiver is close to zero. In this regime, both retransmissions and joint transmission provide diversity benefit against the channel fading process and hence result in improved coverage compared to the non-cooperation baseline. However, while retransmissions always yield time diversity, channel state information at the cooperating BSs is required in order to achieve spatial diversity. Therefore, we conclude that in this diversity-limited regime link layer retransmission techniques such as HARQ are a viable alternative to spatial cooperation techniques such as joint transmission, unless distributed implementation of space-time codes such as the Alamouti code is possible.

For large values of θ, on the other hand, the coverage probability is determined mainly by the probability that the interference power is large. In this regime, both joint transmission and BSS are effective cooperation techniques in improving the coverage probability as they both suppress part of the interference power. Since our numerical results show that both techniques achieve comparable performance and since BSS requires less overhead traffic in the backhaul network than joint processing, we conclude that in the interference-limited regime BSS is a viable alternative to joint transmission.

It should be remarked that in order to ensure analytical tractability, this paper focused on the single antenna case and on simple space-time cooperation techniques. Nevertheless, we believe that the main insight of the paper, i.e., the existence of two separate regimes, transcends the simplicity of our model. Finally, we wish to remark that while in this paper we assumed that

BSs retransmit erroneous data at most once, most of the proof techniques generalize to the case of arbitrary number of retransmissions, although they will lead to more involved integral expressions for the coverage probability.

APPENDIX A
PROOF OF LEMMA 1

We first map the PPPs to a single one-dimensional PPP whose points represent the inverse of the received power. For every $i = 1, \dots, K$, let $\Xi_i = \{\|x\|^\alpha / P_i,\, x \in \Phi_i\}$ denote the normalized path loss between each BS in Φ_i and the typical user located at the origin. By the mapping theorem [21, Theorem 2.34], Ξ_i is a PPP with intensity $\lambda_i(x) = \lambda_i \frac{2\pi}{\alpha} P_i^{2/\alpha} x^{2/\alpha - 1}$, $x \in \mathbb{R}^+$. From the independence of the PPPs $\Phi_1, \cdots, \Phi_K$, it follows that $\Xi_1, \cdots, \Xi_K$ are also independent and thus the process $\Xi = \bigcup_{i=1}^K \Xi_i$ is a non-homogeneous PPP with intensity $\lambda(x) = \sum_{i=1}^K \lambda_i(x)$. Let the elements of Ξ be indexed in increasing order, such that $\|x_1\|^\alpha / P_{\nu(x_1)} \le \|x_2\|^\alpha / P_{\nu(x_2)} \le \|x_3\|^\alpha / P_{\nu(x_3)} \le \cdots$, and define $\gamma_i = \|x_i\|^\alpha / P_{\nu(x_i)} = l_i^{-2}$ as the normalized path loss between the typical user and the i-th BS in the ordered list.

Assuming $n_1 \le n_2$, the normalized path loss of the cooperating BSs in $\mathcal{C}_2$ is given by $\boldsymbol{\gamma} = \{\gamma_1, \dots, \gamma_{n_2}\}$. Then, by defining $g_{ik} := |h_{ik}|^2$ for $k = 1, 2$ and $\mathbf{g} = (\mathbf{g_1}, \mathbf{g_2})$, the interference in the k-th transmission $I_k = \sum_{i > n_k} g_{ik} \gamma_i^{-1}$. Now, the joint ccdf of SIR_1 and SIR_2, as $\phi(n_1, n_2, \theta_1, \theta_2)$ for JT, is expressed as

$$
\begin{aligned}
&\mathbb{P}\left(\bigcap_{k=1}^{2} \{\mathrm{SIR}_k > \theta_k\} \right) \\
=&\mathbb{P}\left(\bigcap_{k=1}^{2} \Big\{ \Big| \sum_{i \le n_k} \gamma_i^{-1/2} h_{ik} \Big|^2 > \theta_k I_k \Big\} \right) \\
\overset{(a)}{=}& \mathsf{E}_{\boldsymbol{\gamma}, \Xi, \mathbf{g}} \left[\exp\left(-\frac{\theta_1 I_1}{\sum_{i \le n_1} \gamma_i^{-1}} \right) \cdot \exp\left(-\frac{\theta_2 I_2}{\sum_{i \le n_2} \gamma_i^{-1}} \right) \right] \\
=& \mathsf{E}_{\boldsymbol{\gamma}}\, \mathsf{E}_{\Xi, \mathbf{g}} \left[e^{-\frac{\theta_1 \sum_{i > n_1} g_{i1} \gamma_i^{-1}}{\sum_{i \le n_1} \gamma_i^{-1}} - \frac{\theta_2 \sum_{i > n_2} g_{i2} \gamma_i^{-1}}{\sum_{i \le n_2} \gamma_i^{-1}}} \,\Big|\, \gamma_1, \dots, \gamma_{n_2} \right] \\
=& \int\limits_{\substack{0 < y_1 < \dots \\ \dots < y_{n_2} < \infty}} \mathcal{L}\left(\frac{\theta_1}{\sum_{i \le n_1} y_i^{-1}}, \frac{\theta_2}{\sum_{i \le n_2} y_i^{-1}} \right) f_{\boldsymbol{\gamma}}(\mathbf{y})\, \mathrm{d}\mathbf{y},
\end{aligned} \tag{48}
$$

where (a) follows due to the fact that h_{i1} and h_{i2} are mutually independent and $\left|\sum_{i\le n_k}\gamma_i^{-1/2}h_{ik}\right|^2$ is exponentially distributed with mean $\sum_{i=1}^{n_k}\gamma_i^{-1}$ because of the Rayleigh fading assumption; $\mathcal{L}(s_1,s_2)$ is the Laplace transform of the interference vector $[I_1,I_2]$ and $f_{\boldsymbol{\gamma}}(\mathbf{y})$ is the joint distribution of $\boldsymbol{\gamma}$ which can be obtained by following the similar steps as in the derivation of the joint distribution of the nearest points in a homogeneous PPP [22]. It can be easily verified that for any $0<y_1<\ldots<y_{n_2}<\infty$, the joint distribution of $\boldsymbol{\gamma}$ is given by

$$f_{\boldsymbol{\gamma}}(\mathbf{y})=e^{-\pi\sum_{i=1}^{K}\lambda_iP_i^{2/\alpha}y_{n_2}^{2/\alpha}}\prod_{i=1}^{n_2}\left(\sum_{j=1}^{K}\frac{2\pi}{\alpha}\lambda_jP_j^{2/\alpha}y_i^{2/\alpha-1}\right). \tag{49}$$

Given $\boldsymbol{\gamma}=\mathbf{y}$, the Laplace transform of the interference vector, $\mathcal{L}(s_1,s_2)$, can be expressed as

$$\begin{aligned}
&\mathsf{E}_{\Xi,\mathbf{g}}\left[e^{-s_1I_1-s_2I_2}\mid\boldsymbol{\gamma}=\mathbf{y}\right]\\
&=\mathsf{E}_{\Xi,\mathbf{g}}\left[e^{-s_1\sum_{i=n_1+1}^{n_2}g_{i1}y_i^{-1}}\times e^{-\sum_{j>n_2}(s_1g_{j1}+s_2g_{j2})\gamma_j^{-1}}\right]\\
&\overset{(a)}{=}\prod_{i=n_1+1}^{n_2}\left(\frac{1}{1+s_1y_i^{-1}}\right)\mathsf{E}_{\Xi}\left[\prod_{j>n_2}\frac{1}{\left(1+s_1\gamma_j^{-1}\right)\left(1+s_2\gamma_j^{-1}\right)}\right]\\
&\overset{(b)}{=}\prod_{i=n_1+1}^{n_2}\left(\frac{1}{1+s_1y_i^{-1}}\right)e^{-\int_{y_{n_2}}^{\infty}\left(1-\frac{1}{(1+s_1x^{-1})(1+s_2x^{-1})}\right)\lambda(x)\mathrm{d}x}\\
&\overset{(c)}{=}\prod_{i=n_1+1}^{n_2}\left(\frac{1}{1+s_1y_i^{-1}}\right)\times\exp\left(-2\pi\sum_{i=1}^{K}\lambda_iP_i^{2/\alpha}y_{n_2}^{2/\alpha}G\left(s_1y_{n_2}^{-1},s_2y_{n_2}^{-1}\right)\right),
\end{aligned}\tag{50}$$

where (a) uses the fact that g_{i1} and g_{i2} are mutually independent and exponentially distributed with unit mean; (b) is due to the probability generating functional for a PPP [21, Theorem 4.9]; (c) follows from the transformation $x=y_{n_2}t^{\alpha}$ and the definition of $G(x,y)$ in (8). Substituting (49) and (50) in (48) and using the transformation $u_i=\pi\sum_{j=1}^{K}\lambda_jP_j^{2/\alpha}y_i^{2/\alpha}$ gives the result in (8) for JT.

APPENDIX B
PROOF OF THEOREM 2

As in the proof in Appendix A, we define the combined normalized path loss process Ξ with intensity measure $\lambda(x)$. We assume $n_1\le n_2$ and define γ_i, $\mathbf{g}_i$, $\mathbf{g}$ and I_i as in Appendix A. The coverage probability in (7) can be further expressed as

$$\mathsf{P}^{\text{MRC}}=\mathbb{P}(\text{SIR}_1>\theta)+\mathbb{P}(\text{SIR}_1+\text{SIR}_2>\theta,\ \text{SIR}_1<\theta)$$

$$
\begin{aligned}
&\stackrel{(a)}{=} \mathbb{P}(\mathrm{SIR}_1 + \mathrm{SIR}_2 > \theta) \\
&= \mathbb{P}\Big(\theta - \mathrm{SIR}_2 < \mathrm{SIR}_1\Big) \\
&= \mathsf{E}_{\Xi,Z,\mathbf{g}}\left[\mathbb{P}\left((\theta - Z)I_1 < \Big|\sum_{i\le n_1}\gamma_i^{-1/2}h_{i1}\Big|^2 \,\Big|\, \Xi, Z\right)\right],
\end{aligned}
$$

where (a) follows from set theory and the fact that $\mathrm{SIR}_k > 0$ and we define the random variable $Z = \mathrm{SIR}_2$ for simplicity. Using the fact that $|\sum_{i\le n_1}\gamma_i^{-1/2}h_{i1}|^2$ is exponentially distributed with mean $\sum_{i=1}^{n_1}\gamma_i^{-1}$ because of the Rayleigh fading assumption, the coverage probability can be expressed as

$$
\begin{aligned}
&\mathsf{E}_{\Xi,Z,\mathbf{g_1}}\left[\exp\left(-\frac{(\theta - Z)^+ I_1}{\sum_{i\le n_1}\gamma_i^{-1}}\right)\right] \\
&= \mathsf{E}_{\Xi,Z,\mathbf{g_1}}\left[\exp\left(-s_1(\theta - Z)^+\sum_{i>n_1} g_{i1}\gamma_i^{-1}\right)\right] \\
&= \mathsf{E}_{\Xi,Z}\left[\prod_{i>n_1}\frac{1}{1 + s_1(\theta - Z)^+\gamma_i^{-1}}\right] \\
&= \mathsf{E}_{\Xi,Z}\left[\prod_{i>n_1} f(\gamma_i/(\theta - Z)^+)\right]
\end{aligned}
\tag{51}
$$

where we define $s_1 = 1/\sum_{i\le n_1}\gamma_i^{-1}$ and $f(x) := \frac{1}{1+s_1x^{-1}}$. Next, we derive the conditional probability distribution function (pdf) of Z as follows

$$
\begin{aligned}
\mathbb{P}(Z \le z \mid \Xi) &= \mathbb{P}\left(\frac{\Big|\sum_{i=1}^{n_2}\gamma_i^{-1/2}h_{i2}\Big|^2}{\sum_{i>n_2} g_{i2}\gamma_i^{-1}} \le z \,\Big|\, \Xi\right) \\
&= 1 - \mathsf{E}_{\mathbf{g_2}}\left[\exp\left(-\frac{z\sum_{i>n_2} g_{i2}\gamma_i^{-1}}{\sum_{i\le n_2}\gamma_i^{-1}}\right)\right] \\
&= 1 - \prod_{i>n_2}\frac{1}{1 + zs_2\gamma_i^{-1}},
\end{aligned}
$$

where we define $s_2 = 1/\sum_{i\le n_2}\gamma_i^{-1}$. Differentiating the above equation with respect to z, we get the pdf of Z as

$$
f_{Z|\Xi}(z) = \sum_{j>n_2}\left(\frac{s_2\gamma_j^{-1}}{(1 + zs_2\gamma_j^{-1})^2}\prod_{\substack{i>n_2\\ i\ne j}}\frac{1}{1 + zs_2\gamma_i^{-1}}\right). \tag{52}
$$

Now we can compute the expectation over Z in (51) using (52) as

$$
\begin{aligned}
&\int_0^\infty \mathsf{E}_\Xi \left[\prod_{i>n_1} f(\gamma_i/(\theta-z)^+) \times f_{Z|\Xi}(z) \right] \mathrm{d}z \\
&\overset{(a)}{=} \int_0^\infty \mathsf{E}_\gamma \left[\prod_{i=n_1+1}^{n_2} f(a^{-1}\gamma_i) \times \mathsf{E}_\Xi \left[\sum_{j>n_2} \frac{s_2\gamma_j^{-1} f(a^{-1}\gamma_j)}{\left(1+zs_2\gamma_j^{-1}\right)^2} \prod_{\substack{k>n_2\\k\neq j}} \frac{f(a^{-1}\gamma_k)}{1+zs_2\gamma_k^{-1}} \right]\right] \mathrm{d}z \\
&\overset{(b)}{=} \int_0^\infty \mathsf{E}_\gamma \left[\prod_{i=n_1+1}^{n_2} f(a^{-1}\gamma_i) \times \int_{\gamma_{n_2}}^\infty \frac{s_2 x^{-1} f(a^{-1}x)\lambda(x)}{\left(1+zs_2x^{-1}\right)^2} \mathsf{E}_\Xi \left[\prod_{k>n_2} \frac{f(a^{-1}\gamma_k)}{1+zs_2\gamma_k^{-1}} \right] \mathrm{d}x \right] \mathrm{d}z \\
&\overset{(c)}{=} \int_0^\infty \mathsf{E}_\gamma \left[\prod_{i=n_1+1}^{n_2} f(a^{-1}\gamma_i) \times \int_{\gamma_{n_2}}^\infty \frac{s_2 x^{-1} f(a^{-1}x)\lambda(x)}{\left(1+zs_2x^{-1}\right)^2} \exp\left(-\int_{\gamma_{n_2}}^\infty \left(1 - \frac{f(a^{-1}y)}{1+zs_2y^{-1}}\right)\lambda(y)\mathrm{d}y \right) \mathrm{d}x \right] \mathrm{d}z \\
&\overset{(d)}{=} \int_0^\infty \mathsf{E}_\gamma \left[\prod_{i=n_1+1}^{n_2} f(a^{-1}\gamma_i) \times 2\pi \sum_{j=1}^K \lambda_j P_j^{2/\alpha} \gamma_{n_2}^{2/\alpha-1} s_2 \times H\left(zs_2\gamma_{n_2}^{-1}, as_1\gamma_{n_2}^{-1}\right) \times \right. \\
&\qquad \left. \exp\left(-2\pi \sum_{j=1}^K \lambda_j P_j^{2/\alpha} \gamma_{n_2}^{2/\alpha} G\left(zs_2\gamma_{n_2}^{-1}, as_1\gamma_{n_2}^{-1}\right)\right) \right] \mathrm{d}z \\
&\overset{(e)}{=} \int_0^\infty \int_{\substack{0<u_1<\ldots\\ \ldots<u_{n_2}<\infty}} \prod_{i=n_1+1}^{n_2} \frac{1}{1+\frac{a u_i^{-\alpha/2}}{\sum_{k\le n_1} u_k^{-\alpha/2}}} \times \frac{2u_{n_2}^{1-\alpha/2}}{\sum_{k\le n_2} u_k^{-\alpha/2}} \times H\left(\frac{zu_{n_2}^{-\alpha/2}}{\sum_{k\le n_2} u_k^{-\alpha/2}}, \frac{au_{n_2}^{-\alpha/2}}{\sum_{k\le n_1} u_k^{-\alpha/2}} \right) \times \exp(-u_{n_2}) \\
&\qquad \times \exp\left(-2u_{n_2} G\left(\frac{zu_{n_2}^{-\alpha/2}}{\sum_{k\le n_2} u_k^{-\alpha/2}}, \frac{au_{n_2}^{-\alpha/2}}{\sum_{k\le n_1} u_k^{-\alpha/2}} \right)\right) \mathrm{d}\mathbf{u}\mathrm{d}z \\
&= \int_0^\infty \sigma(n_1, n_2, z, (\theta-z)^+)\mathrm{d}z
\end{aligned} \tag{53}
$$

where (a) defines $a = (\theta - z)^+$; (b) is due to the Campbell-Mecke formula [21]; (c) is due to the probability generating functional for a PPP [21, Theorem 4.9]; (d) follows from the transformations $x = \gamma_{n_2} t^\alpha$, $y = \gamma_{n_2} u^\alpha$ and the definitions of $G(x, y)$ in (8) and $H(x, y)$ in (20); (e) uses the values of s_1, s_2 and the pdf of $\boldsymbol{\gamma}$ in (49) and then uses the transformation $u_i = \pi \sum_{j=1}^K \lambda_j P_j^{2/\alpha} y_i^{2/\alpha}$ for $i = 1, \ldots, n_2$. Hence, we get the desired result in Theorem 2 for JT.

Appendix C
Proof of Theorem 6

Given the point processes, the sum in the numerator of the SIR expression in (38) is a sum of Rayleigh distributed random variables. We henceforth denote these random variables as $S_{ik} = l_i\,|h_{ik}|$.

For retransmission without MRC, we can express the outage probability using (38) as

$$\mathbb{P}\left(\bigcap_{k=1}^{2}\left\{\left(\sum_{i=1}^{n_k} S_{ik}\right)^2 < \theta I_k\right\}\right)$$
$$= \mathsf{E}_{l_i,I_k}\left(\prod_{k=1}^{2}\int\limits_{D_{n_k,\sqrt{\theta I_k}}(\mathbf{x_k})}\prod_{i=1}^{n_k} g_{S_{ik}|l_i}(x_{ik})\mathrm{d}\mathbf{x_k}\right),$$

where $g_{S_{ik}|l_i}(x) = \frac{x}{2l_i^2}e^{-x^2/2l_i^2}$ is the probability density function of S_{ik} and we define $D_{n,r}(\mathbf{x}) = \{\mathbf{x} \in (\mathbb{R}^+)^n : \|\mathbf{x}\|_1 < r\}$. The outage probability can be further expressed by substituting $x_{ik} = \sqrt{\theta I_k}t_{ik}$ as:

$$\mathsf{E}_{l_i,I_k}\left(\prod_{k=1}^{2}(\theta I_k)^{n_k}\int\limits_{D_{n_k,1}(\mathbf{t_k})} e^{-\theta I_k\sum\limits_{i=1}^{n_k}\frac{t_{ik}^2}{2l_i^2}}\prod_{i=1}^{n_k}\frac{t_{ik}}{l_i^2}\mathrm{d}\mathbf{t_k}\right)$$
$$\overset{(a)}{\sim} \theta^{n_1+n_2}\,\mathsf{E}_{l_i,I_k}\left(\prod_{k=1}^{2} I_k^{n_k}\int\limits_{D_{n_k,1}(\mathbf{t_k})}\prod_{i=1}^{n_k}\frac{t_{ik}}{l_i^2}\mathrm{d}\mathbf{t_k}\right)$$
$$\overset{(b)}{=} \theta^{n_1+n_2}\,\mathsf{E}_{l_i,I_k}\left(\prod_{k=1}^{2}\frac{I_k^{n_k}\prod_{i=1}^{n_k} l_i^{-2}}{(2n_k)!}\right), \tag{54}$$

where (a) is due to the fact that $\exp(x) \sim 1$ as $x \to 0$ and (b) is due to [23, Equation 4.634]. Applying Lemma 3, we get a diversity gain of $n_1 + n_2$ for retransmission without MRC.

For retransmission with MRC, we define $W_k = \frac{\left(\sum_{i=1}^{n_k} S_{ik}\right)^2}{I_k}$ for $k = 1, 2$ and use (38) to express the outage probability as

$$\mathbb{P}\left(\sum_{k=1}^{2} W_k < \theta\right)$$
$$= \mathsf{E}_{l_i,I_k}\left[\int_0^\infty \mathbb{P}\left(W_1 < \theta - w \mid I_1\right) f_{W_2|I_2}(w)\mathrm{d}w\right] \tag{55}$$

Following similar steps as in (54), we can derive the cdf of W_k given I_k, $F_{W_k|I_k}(x)$ for $x \geq 0$ and $k = 1, 2$ as follows.

$$\mathbb{P}\left(W_k < x \mid I_k\right)$$
$$= (xI_k)^{n_k}\int\limits_{D_{n_k,1}(\mathbf{t_k})} e^{-xI_k\sum\limits_{i=1}^{n_k}\frac{t_{ik}^2}{2l_i^2}}\prod_{i=1}^{n_k}\frac{t_{ik}}{l_i^2}\mathrm{d}\mathbf{t_k} \tag{56}$$

and $f_{W_k|I_k}(x) = \frac{\partial}{\partial x} F_{W_k|I_k}(x)$. Substituting these values in (55), the outage probability is expressed as

$$
\begin{aligned}
&\mathsf{E}_{l_i,I_k}\left[\int_0^\infty F_{W_1|I_1}\left((\theta-w)^+\right) f_{W_2|I_2}(w)\mathrm{d}w\right]\\
&\overset{(a)}{=}\mathsf{E}_{l_i,I_k}\left[\int_0^1 F_{W_1|I_1}\left(\theta(1-t)\right) f_{W_2|I_2}(\theta t)\theta\mathrm{d}t\right]\\
&=\mathsf{E}_{l_i,I_k}\left[\int\limits_0^1 \left(\theta(1-t)I_1\right)^{n_1}\left(\theta I_2\right)^{n_2} t^{n_2-1}\times \int\limits_{D_{n_2,1}(\mathbf{t_2})}\int\limits_{D_{n_1,1}(\mathbf{t_1})} e^{-\theta(1-t)I_1\sum\limits_{i=1}^{n_1}\frac{t_{i1}^2}{2l_i^2}}\prod_{i=1}^{n_1}\frac{t_{i1}}{l_i^2}\mathrm{d}\mathbf{t_1}\times\right.\\
&\qquad\left. e^{-\theta t I_2\sum\limits_{i=1}^{n_2}\frac{t_{i2}^2}{2l_i^2}}\left(n_2-\theta t I_2\sum_{i=1}^{n_2}\frac{t_{i2}^2}{2l_i^2}\right)\prod_{i=1}^{n_2}\frac{t_{i2}}{l_i^2}\mathrm{d}\mathbf{t_2}\mathrm{d}t\right]\\
&\overset{(b)}{\sim}\theta^{n_1+n_2}\,\mathsf{E}_{l_i,I_k}\left[\prod_{k=1}^{2}\frac{I_k^{n_k}}{(2n_k)!\prod_{i=1}^{n_k} l_i^2}\right] n_2 B(n_1+1,n_2),
\end{aligned}
\tag{57}
$$

where (a) is due to the transformation $w=\theta t$ and the fact that $F_{W_1|I_1}(0)=0$; (b) is due to the fact that $\exp(x)\sim 1$ as $x\to 0$ and then we use [23, Equation 4.634] for n_k dimensional integrals and the definition of the beta function for the integral in t. Applying Lemma 3, we get a diversity gain of n_1+n_2 for retransmission with MRC.

Appendix D
Proof of Theorem 7

By defining $\boldsymbol{\gamma}$ as in Appendix A with $n_1=n_2=2$ and $g_i=|h_i|^2$, we can express the coverage probability in (44) as

$$
\begin{aligned}
\mathsf{P}_{\mathrm{c}} &= \mathbb{P}\left(g_1\gamma_1^{-1}+g_2\gamma_2^{-1}>\theta I\right)\\
&\overset{(a)}{=}\mathsf{E}_{I,\boldsymbol{\gamma}}\left[\frac{\gamma_2 e^{-\gamma_1\theta I}-\gamma_1 e^{-\gamma_2\theta I}}{\gamma_2-\gamma_1}\right]\\
&\overset{(b)}{=}\mathsf{E}_{\boldsymbol{\gamma}}\left[\frac{\gamma_2\mathcal{L}(\theta\gamma_1,0)-\gamma_1\mathcal{L}(\theta\gamma_2,0)}{\gamma_2-\gamma_1}\right],
\end{aligned}
\tag{58}
$$

where (a) follows from the fact that g_1 and g_2 are exponentially distributed with unit mean and the cdf of the hypoexponential distribution; (b) follows by the definition of the Laplace transform in (50). Using the value of $\mathcal{L}(.,.)$ in (50) and the joint density function of $\boldsymbol{\gamma}$ in (49), the above coverage probability can be further expressed as

$$\int\limits_{\substack{0<y_1\leq\\ y_2<\infty}} \left(y_2 \exp\left(-2\pi \sum_{i=1}^{K} \lambda_i P_i^{2/\alpha} y_2^{2/\alpha} G\left(\theta y_1/y_2, 0\right) \right) \right.$$

$$\left. - y_1 \exp\left(-2\pi \sum_{i=1}^{K} \lambda_i P_i^{2/\alpha} y_2^{2/\alpha} G\left(\theta, 0\right) \right) \right) \frac{f_\gamma(\mathbf{y})}{y_2 - y_1} \mathrm{d}\mathbf{y}$$

$$\stackrel{(a)}{=} \int\limits_{\substack{0<u_1\leq\\ u_2<\infty}} \left(u_2^{\alpha/2} \exp\left(-2u_2 G\left(\theta(u_1/u_2)^{\alpha/2}, 0\right)\right) - u_1^{\alpha/2} \exp\left(-2u_2 G\left(\theta, 0\right)\right) \right) \frac{e^{-u_2}}{u_2^{\alpha/2} - u_1^{\alpha/2}} \mathrm{d}\mathbf{u}$$

$$\stackrel{(b)}{=} \int_0^\infty \mathrm{d}u_2 \int_0^1 \mathrm{d}t \left(u_2^{\alpha/2} \exp\left(-2u_2 G\left(\theta t^{\alpha/2}, 0\right)\right) - (u_2 t)^{\alpha/2} \exp\left(-2u_2 G\left(\theta, 0\right)\right) \right) \frac{u_2 e^{-u_2}}{u_2^{\alpha/2} - (u_2 t)^{\alpha/2}}$$

$$\stackrel{(c)}{=} \int_0^1 \left(\frac{1}{(1+2G(\theta t^{\alpha/2}, 0))^2} - \frac{t^{\alpha/2}}{(1+2G(\theta, 0))^2} \right) \frac{\mathrm{d}t}{1 - t^{\alpha/2}}, \tag{59}$$

where (a) follows from the change of variable $u_i = \pi \sum_{i=1}^{K} \lambda_i P_i^{2/\alpha} y_i^{2/\alpha}$ for $i = 1, 2$; (b) is due to the substitution $u_1 = u_2 t$ and (c) follows by integrating over u_2.

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