The Local Delay in Poisson Networks

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***Abstract*—Communication between two neighboring nodes is a very basic operation in wireless networks. Yet very little research has focused on the *local delay* in networks with randomly placed nodes, defined as the mean time it takes a node to connect to its nearest neighbor. We study this problem for Poisson networks, first considering interference only, then noise only, and lastly and briefly, interference plus noise. In the noiseless case, we analyze four different types of nearest-neighbor communication and compare the extreme cases of high mobility, where a new Poisson process is drawn in each time slot, and no mobility, where only a single realization exists and nodes stay put forever. It turns out that the local delay behaves rather differently in the two cases. We also provide the low- and high-rate asymptotic behavior of the minimum achievable delay in each case. In the cases with noise, power control is essential to keep the delay finite, and randomized power control can drastically reduce the required (mean) power for finite local delay.**

***Index Terms*—Poisson point process, stochastic geometry, ad hoc networks, interference, delay, power control.**

I. Introduction

Delay and reliability are important performance indicators that measure the quality-of-service provided by a network; they complement the quantity-of-service, usually denoted by throughput or capacity. The triplet of throughput, delay, and reliability (TDR) forms a comprehensive metric for a network's capability to deliver information [1]. The focus of this paper is the *local delay*, defined as the mean time (in numbers of time slots) until a packet is successfully received over a link between nearest neighbors. In terms of the TDR metric, the local delay spans a plane in the TDR space where the reliability is set to 100% and the delay is a function of the throughput or rate. It lower bounds all other types of delays such as single-hop, end-to-end, or round-trip delays, which makes it a fundamental quantity to study. If it is infinite, there is little hope that the network provides any useful service to its users. Furthermore, the local delay is a sensitive indicator of the diversity present in a network model; in particular, it measures the interference correlation in network models with interference.

Focusing on the case where nodes are distributed on the two-dimensional plane as a homogeneous Poisson point process (PPP), we tackle the problem in three steps. First, we analyze the local delay in a Poisson network with interference but without noise. The channel fading and the transmit/receive states are assumed iid over time, while the nodes are either highly mobile, in which case a new realization of the PPP is drawn in each time slot, or completely static, in which case only a single realization exists, and the nodes stay fixed forever.

Secondly, we ignore interference but consider random link distances. so that the "network" is just a collection of independent (orthogonal) links. We take the distances to be static (but random) and derive the mean delay (ensemble average) over the links for different types of fading and power control strategies.

Thirdly, we combine noise and interference and present bounds on the local delay.

A mathematical framework for the analysis of the local delay in Poisson networks is provided in [2, Sect. 17.5] and [3], where it was first observed that the local delay may be infinite for certain network parameters; this phenomenon is called *wireless contention phase transition*. We build on this framework and our preliminary work in [4], [5] to obtain concrete results for the local delay for all four basic types of nearest-neighbor transmission.

Specifically, the main contributions of this paper are:

- Closed-form expressions or bounds on the local delay for the most practical scenarios, including the case where both interference and noise are considered
- Derivation of the optimum ALOHA transmit probabilities and minimum achievable local delays
- Derivation of the delay asymptotics in the low- and high-rate limits
- A systematic comparison of the performance of the four cases of nearest-neighbor communication
- Analysis of the effects of random power control in noise-limited networks

II. Highly Mobile and Static Networks without Noise

A. Network Model

We consider a marked Poisson point process (PPP) $\hat{\Phi} = \{(x_i, t_{x_i})\} \subset \mathbb{R}^2 \times \{0,1\}$, where $\Phi = \{x_i\}$ is a homogeneous PPP of intensity λ, and the marks t_{x_i} are iid Bernoulli with $\mathbb{P}(t=1) = p = 1-q$. A mark of 1 indicates that the node transmits whereas a 0 indicates listening. The large-scale path loss is assumed to be r^α over distance r. A transmission from a node x to a node y is successful if the signal-to-interference ratio (SIR) exceeds a threshold θ. For a transmission from $x \in \Phi$ to $y \in \Phi$, the SIR is

$$\mathrm{SIR}_{xy} \triangleq \frac{S_{xy}}{I_{xy}}\,,$$

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where $S_{xy} \triangleq t_x h_{xy} \|x-y\|^{-\alpha}$ and

$$I_{xy} \triangleq \sum_{(z,t_z)\in\hat{\Phi}\setminus\{(x,t_x)\}} t_z h_{zy} \|z-y\|^{-\alpha} .$$

This definition implies that the transmit powers are normalized to 1, that $I=\infty$ if $t_y=1$ (y is itself transmitting), and $\mathrm{SIR}=0$ if $t_x=0$. The power fading coefficients h_{xy} are exponential with mean 1 and iid for all $x,y\in\Phi$ and over time (block Rayleigh fading). Time is slotted, and transmission attempts are synchronized.

The (normalized) rate of transmission (or spectral efficiency) $\mathcal{R}$ is, slightly optimistically, assumed to be related to the threshold θ by $\mathcal{R}=\log_2(1+\theta)$.

We consider two extremes cases of mobility, the *high-mobility case*, where a new realization of $\hat{\Phi}$ is drawn in each time slot, and the *static case*, where Φ (the node locations) stays fixed forever. The main event of interest is the event that the typical node, situated at the origin $o\triangleq(0,0)\in\mathbb{R}^2$, successfully connects to its nearest neighbor in a single time slot. In the high-mobility case, we denote this event by $\mathcal{C}$. In the static case, we first focus on the success event *conditioned on the point process* Φ, which we call $\mathcal{C}_\Phi$. Success events in different time slots are independent, so there is no need to add a time index to this event. Conditioning on Φ having a point at the origin o implies that the relevant probability measure is the Palm probability $\mathbb{P}^o$, and that expectations that involve the point process are taken with respect to $\mathbb{P}^o$ and denoted by $\mathbb{E}^o$ [6]. The partner node y of the origin will be chosen according to one of the four basic cases of nearest-neighbor communication: nearest-receiver transmission (NRT), nearest-neighbor transmission (NNT), nearest-transmitter reception (NTR), and nearest-neighbor reception (NNR).

In the highly mobile case, we have $\mathbb{P}^o(\mathcal{C})=\mathbb{P}^o(\mathrm{SIR}_{uv}>\theta)$ and in the static case, $\mathbb{P}^o(\mathcal{C}_\Phi)=\mathbb{P}^o(\mathrm{SIR}_{uv}>\theta\mid\Phi)$, where $u=o$, $v=y$ for NRT and NNT, and $u=y$, $v=o$ for NTR and NNR. The link distance $R=\|u-v\|$ is itself a (Rayleigh distributed) random variable. The local delay D is the mean number of slots needed until success. Formally,

$$\text{NRT, NNT:}\quad D\triangleq\mathbb{E}^o\left(\min\left\{k\in\mathbb{N}\colon \mathbf{1}_k\big(o\to\mathrm{NN}(o)\big)\right\}\right)$$
$$\text{NTR, NNR:}\quad D\triangleq\mathbb{E}^o\left(\min\left\{k\in\mathbb{N}\colon \mathbf{1}_k\big(\mathrm{NN}(o)\to o\big)\right\}\right),$$

where $\mathbf{1}_k(x\to y)=1$ if $\mathrm{SIR}_{xy}>\theta$ in time slot k, and 0 otherwise. $\mathrm{NN}(o)$ denotes the origin's nearest node (for NNT and NNR), its nearest receiver (NRT), or its nearest transmitter (NTR).

In the high-mobility case, the local delay is simply $\mathbb{P}^o(\mathcal{C})^{-1}$; in the static case, the success events are only conditionally independent, hence the conditional local delay is geometric with mean $\mathbb{P}^o(\mathcal{C}_\Phi)^{-1}$, and the expectation with respect to the point process yields the local delay:

$$\text{High-mobility:}\quad D=\mathbb{P}^o(\mathcal{C})^{-1} \tag{1}$$

$$\text{Static:}\quad D=\mathbb{E}^o_\Phi\left(\frac{1}{\mathbb{P}^o(\mathcal{C}_\Phi)}\right) \tag{2}$$

In our approach for the static case, we will decondition on Φ in two steps, first with respect to the interferers and then with respect to the link distance. This method can be used whenever conditioning on Φ also fixes the link distance. The static NRT and NTR cases as described above do not meet this requirement, as the link distance would also depend on who is transmitting. So we will make a small amendments to the network model in these cases, namely a fixed partitioning of the point process into point processes of potential transmitters and receivers of the appropriate densities.

Considering D as a function of the transmit probability p, we define the *minimum delay* as

$$D_{\min}\triangleq\min_p\{D(p)\},$$

the *optimum transmit probability* as

$$p_{\mathrm{opt}}\triangleq\arg\min_p\{D(p)\},$$

and the *critical transmit probability* as

$$p_{\mathrm{c}}\triangleq\sup\{p\colon D(p)<\infty\}.$$

An important parameter that will be used throughout the paper is the *spatial contention* γ, introduced in [7] and generalized in [8], [9], which measures a network's capability of spatial reuse by quantifying how quickly the success probability of a transmission (over fixed distance) decreases when the density of interfering nodes is increased. It is defined as the slope of the outage probability of a transmission over unit distance as a function of the interferer density at density zero [8, Def. 2]. It depends on the path loss exponent α, the SIR threshold θ, and the network geometry. For a transmission over distance R in a Poisson field of interferers with Rayleigh fading, the success probability is [10]

$$p_{s|R}=\exp(-C(\alpha)\theta^{2/\alpha}p\lambda R^2), \tag{3}$$

where $C(\alpha)\triangleq 2\pi^2/(\alpha\sin(2\pi/\alpha))$. Asymptotically, as the transmitter density $\lambda p\to 0$, $p_{s|1}\sim 1-C(\alpha)\theta^{2/\alpha}p\lambda$, thus the spatial contention is

$$\gamma=\theta^\delta C(\alpha)=\theta^\delta\pi\frac{\pi\delta}{\sin(\pi\delta)}=\theta^\delta\pi\Gamma(1+\delta)\Gamma(1-\delta), \tag{4}$$

where $\delta\triangleq 2/\alpha$. For $\alpha=4$, $\gamma=\sqrt{\theta}\pi^2/2$, and for $\alpha=3$, $\gamma=\theta^{2/3}4\pi^2/(3\sqrt{3})$. As $\alpha\downarrow 2$, $\gamma\to\infty$, since the interference is infinite a.s. for $\alpha\le 2$.

The asymptotic regimes considered are $\theta\to 0$ and $\theta\to\infty$, or, equivalently, $\mathcal{R}\to 0$ and $\mathcal{R}\to\infty$. Since γ increases monotonically with θ, we may also write $\gamma\to 0$ and $\gamma\to\infty$.

B. High-Mobility Networks

To analyze the four cases of nearest-neighbor transmission, we need the following lemma:

Lemma 1 *Let $\mathcal{H}\subset\mathbb{R}^2$ and*

$$I=\sum_{(x,t_x)\in\hat{\Phi}} t_x h_x\|x\|^{-\alpha}. \tag{5}$$

The conditional Laplace transform of I given that $\mathcal{H}$ does not contain any nodes of Φ is

$$\mathcal{L}_I(s\mid\mathcal{H}\cap\Phi=\emptyset)=\exp\left(-\lambda p\int_{\mathbb{R}^2\setminus\mathcal{H}}\frac{s}{s+\|x\|^\alpha}\mathrm{d}x\right). \tag{6}$$

The success probability for a transmission over distance R is obtained by replacing s by θR^α, i.e., $\mathbb{P}^o(\mathcal{C} \mid R) = \mathcal{L}_I(\theta R^\alpha \mid \mathcal{H} \cap \Phi = \emptyset)$.

Proof: The conditional Laplace transform follows from the probability generating functional for PPPs [6]. The success probability is the Laplace transform evaluated at θR^α, since for Rayleigh fading, given R,

$$\mathbb{P}(\mathrm{SIR} > \theta) = \mathbb{P}(hR^{-\alpha} > \theta I) = \mathbb{E}_I \exp(-\theta R^\alpha I). \quad \blacksquare$$

The distributional properties of the interference I, defined in (5), do not depend on where it is measured. If $\mathcal{H} = \emptyset$, the success probability of a transmission between two nodes at distance R is (cf. (3))

$$\mathbb{P}^o(\mathcal{C} \mid R) = pq\mathbb{E}^o(e^{-\theta R^\alpha I}) = pq\mathcal{L}_I(\theta R^\alpha) = pq\exp(-\gamma p\lambda R^2)\,,$$

1) Nearest-receiver transmission (NRT): In this case, the destination node is always listening, so

$$\mathbb{P}^o(\mathcal{C}^{\mathrm{NRT}} \mid R) = p\exp(-\gamma p\lambda R^2)\,.$$

Since the point process of receivers has intensity λq, the link distance R is is Rayleigh distributed with mean $1/(2\sqrt{q\lambda})$ [11], *i.e.*, $f_R(r) = 2q\lambda\pi r\exp(-q\lambda\pi r^2)$. Hence

$$\mathbb{P}^o(\mathcal{C}^{\mathrm{NRT}}) = \mathbb{E}\big(\mathbb{P}^o(\mathcal{C}^{\mathrm{NRT}} \mid R)\big) = \frac{p\pi}{\pi + \gamma pq^{-1}}$$

and

$$D^{\mathrm{NRT}} = \frac{1}{\mathbb{P}^o(\mathcal{C}^{\mathrm{NRT}})} = \frac{1}{p} + \frac{\gamma}{\pi q}\,. \tag{7}$$

The optimum transmit probability is

$$p_{\mathrm{opt}}^{\mathrm{NRT}} = \frac{\pi - \sqrt{\pi\gamma}}{\pi - \gamma}\,. \tag{8}$$

The local delay is always finite for $p \in (0,1)$, so $p_{\mathrm{c}} = 1$ (as for all high-mobility cases). The minimum delay only depends on the spatial contention:

$$D_{\min}^{\mathrm{NRT}} = 1 + 2\sqrt{\frac{\gamma}{\pi}} + \frac{\gamma}{\pi}\,. \tag{9}$$

2) Nearest-neighbor transmission (NNT): Let y be the typical node's nearest neighbor and $R = \|y\|$. In this case R is distributed as $f_R(r) = 2\lambda\pi r\exp(-\lambda\pi r^2)$, and having the nearest neighbor at distance R implies that *there is no interferer in the ball $B_o(R)$ centered at o with radius R.* So y sees the conditional interference, conditioned on the disk $B_o(R)$ being empty, and the interference observed at the receiver is smaller than at a typical node, or, in other words, the spatial contention is smaller. The following theorem provides bounds on the absolute gain in the spatial contention:

Theorem 1 *The success probability of nearest-neighbor transmission given R is*

$$\mathbb{P}^o(\mathcal{C} \mid R) = pq\exp(-\gamma^{\mathrm{NNT}} p\lambda R^2)\,, \tag{10}$$

with γ^{NNT} denoting the spatial contention for nearest-neighbor transmission. γ^{NNT} is bounded as follows:

(a) $\gamma > \gamma^{\mathrm{NNT}} > \gamma - \pi$, where γ is the unconditioned spatial contention given in (4)*. Also,* $\lim_{\theta\to\infty} \gamma - \gamma^{\mathrm{NNT}} = \pi$.

(b) Letting $\delta = 2/\alpha$ and denoting by $H_\delta(x)$ the Gauss hypergeometric function

$$H_\delta(x) \triangleq {}_2F_1(1, \delta; 1+\delta; x)\,,$$

we have

$$\gamma - \gamma^{\mathrm{NNT}} < \frac{2\pi}{3} H_\delta(-2^\alpha/\theta) + \frac{\pi}{4} H_\delta(-3^{\alpha/2}/\theta) + \frac{\pi}{6} H_\delta(-2^{\alpha/2}/\theta) + \frac{\pi}{6} H_\delta(-1/\theta) \tag{11}$$

$$\gamma - \gamma^{\mathrm{NNT}} > \frac{\pi}{2} H_\delta(-3^{\alpha/2}/\theta) + \frac{\pi}{6} H_\delta(-2^{\alpha/2}/\theta) + \frac{\pi}{12} H_\delta(-1/\theta) \tag{12}$$

(c) A lower bound without special functions is

$$\gamma - \gamma^{\mathrm{NNT}} > \frac{3\pi}{4} - \frac{\pi}{6\theta(2+\alpha)}\left(2\cdot 3^{1+\alpha/2} + 2^{1+\alpha/2} + 1\right)\,. \tag{13}$$

The proof is provided in the appendix.
Remarks:

(i) By construction, the simpler lower bound (13) is looser than (12).

(ii) Since $H_1(x) = -\log(1-x)/x$ and $H_0(x) \equiv 1$, simplified expressions can be obtained for the cases where $\alpha \downarrow 2$ (or $\delta \uparrow 1$) and $\alpha \to \infty$ (or $\delta \downarrow 0$), respectively.

(iii) While the absolute gain in the spatial contention increases with θ, the *relative gain* decreases with θ and approaches 1 as $\theta \to \infty$, see Fig. 1 (a). The plot shows that for small θ, $\gamma^{\mathrm{NNT}} \approx \gamma/2$ (for $\alpha = 4$). In fact, the upper bound on the difference (11) results in a lower bound on the ratio $\gamma^{\mathrm{NNT}}/\gamma$ that approaches $1/2$ as $\theta \to 0$. This follows from

$$H_\delta(-x) \sim x^{-\delta}\frac{\pi\delta}{\sin(\pi\delta)}\,, \quad x \to \infty\,. \tag{14}$$

In particular, for $\alpha = 4$ ($\delta = 1/2$),

$$\lim_{\theta\downarrow 0} \frac{H_{1/2}(-t/\theta)}{\sqrt{\theta}} = \frac{\pi}{2\sqrt{t}}\,.$$

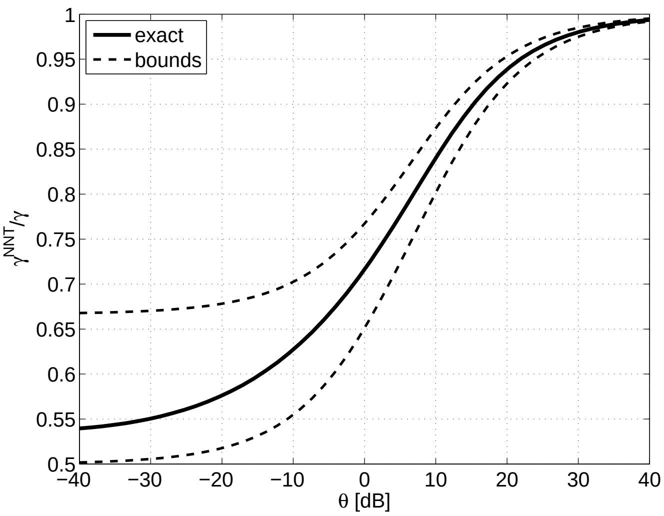
Using this limit in (11) yields $\gamma^{\mathrm{NNT}}/\gamma = \gamma^{\mathrm{NNT}}/(\sqrt{\theta}\pi^2/2) > 1/2$. Applied to (12), we obtain $\gamma^{\mathrm{NNT}}/\gamma < 2/3$ (upper bound in Fig. 1 (a)).

(iv) The qualitative behavior of the spatial contention can also be explained as follows: For small θ, only the closest interferers are relevant. The interference-free region (or hole) $\mathcal{H}$ essentially removes half of them. For large θ, a large area around the receiver must be interferer-free, much larger than $\mathcal{H}$. So the relative gain is small, while the absolute gain is just proportional to the area of the hole.

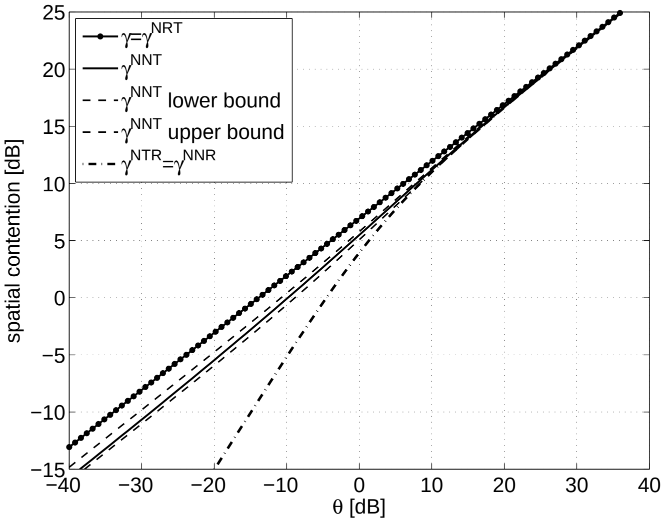
The local delay follows from integration with respect to R, which is Rayleigh with mean $1/(2\sqrt{\lambda})$ in this case:

$$D^{\mathrm{NNT}} = \frac{p\gamma^{\mathrm{NNT}} + \pi}{\pi pq} = \frac{1}{pq} + \frac{\gamma^{\mathrm{NNT}}}{\pi q} \tag{15}$$

The delay is composed of two parts, the *access delay* $1/(pq)$, which is the time it takes for the transmitter to transmit and the

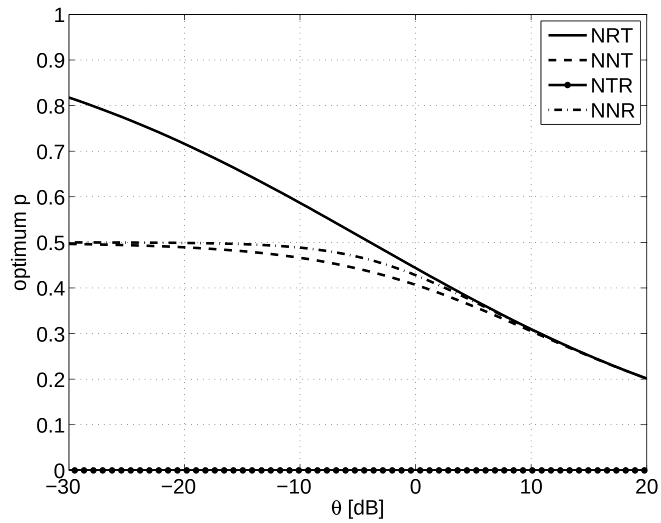


(a) Relative spatial contention.

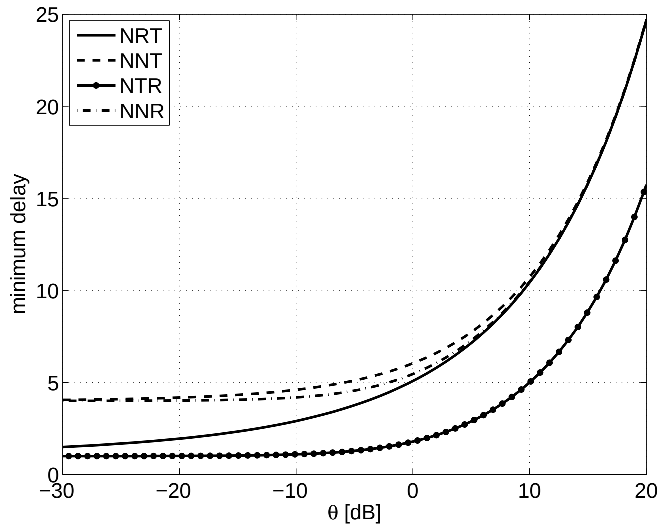


(b) Absolute spatial contention.

Fig. 1. Relative (a) and absolute (b) values of the spatial contention in the high-mobility case for $\alpha = 4$. The bounds are obtained from (11) and (12). In the left figure, the bounds reach $1/2$ and $2/3$, respectively, as $\theta \to 0$.



(a) Optimum transmit probability.



(b) Minimum achievable delay.

Fig. 2. Optimum transmit probability (a) and minimum achievable local delay (b) in the high-mobility case for $\alpha = 4$.

receiver to listen, and the *service time*, which is proportional to the spatial contention $\gamma^{\rm NNT}$. Compared with the nearest-receiver case, we observe the following:

Corollary 1 *For a fixed p and finite θ, $D^{\rm NRT} < D^{\rm NNT}$. Asymptotically, the delays are identical,* i.e., $D^{\rm NRT} \uparrow D^{\rm NNT}$ *as* $\theta \to \infty$.

Proof: The maximum difference $\gamma - \gamma^{\rm NNT}$ is π, achieved as $\theta \to \infty$. Since $p(\gamma - \pi) = p\gamma + \pi q$, the two delays are then identical. For finite θ, the difference is smaller and thus $D^{\rm NRT} < D^{\rm NNT}$. ■

So at high rates, the gain in the spatial contention in the NNT case is exactly offset by the fact that the nearest neighbor is only listening with probability q. The optimum transmit probability is

$$p_{\rm opt}^{\rm NNT} = \frac{\sqrt{1+\gamma^{\rm NNT}/\pi} - 1}{\gamma^{\rm NNT}/\pi}\,, \tag{16}$$

and the minimum delay is

$$D_{\rm min}^{\rm NNT} = \frac{g^2}{g + 2(1-\sqrt{1+g})}\,, \tag{17}$$

where $g = \gamma^{\rm NNT}/\pi$. As $\theta \to 0$,

$$D_{\rm min}^{\rm NNT} \sim 4 + \frac{2\gamma^{\rm NNT}}{\pi} \sim 4 + c\gamma/\pi\,, \tag{18}$$

where $c \in (1, 4/3)$. Fig. 2 shows the optimum transmit probability p and the minimum delay. As $\theta \to 0$, the optimum transmit probability for NRT approaches 1, whereas for NNT, it approaches $1/2$. The difference is due to the fact that the *receiver* density is less critical in NRT. In the delay plot, it is observed that $D^{\rm NNT} > 4$, since $1/(pq)$ is at least 4.

When comparing the NRT and NNT schemes, it also needs to be factored in that the distance to the nearest receiver is a factor $q^{-1/2}$ larger than the distance to the nearest neighbor. So a significant advantage of NRT is that more distance is covered. It is, however, an opportunistic scheme since the receiver may change from slot to slot, and it can therefore not be used in standard routing algorithms, where routing tables are maintained at relay nodes.

3) Nearest-transmitter reception (NTR): Next we consider the case where the typical node at o receives from its nearest transmitter, say y. This implies that there are no interferers in the disk of radius $R = \|y\|$ around the receiver. So in this case, we apply Lemma 1 with $\mathcal{H} = B_o(R)$. Using the hypergeometric function defined in Thm. 1,

$$2\pi \int_0^R \frac{rs}{r^\alpha + s} \mathrm{d}r = \pi R^2 H_\delta(-R^\alpha/s)\,,$$

so $\gamma^{\mathrm{NTR}} = \gamma - \pi H_\delta(-1/\theta)$ and

$$\mathbb{P}^o(\mathcal{C}^{\mathrm{NTR}} \mid R) = q \exp\left(-\lambda p \pi R^2 (\gamma - \pi H_\delta(-1/\theta))\right)\,.$$

As $\theta \to \infty$, the gain in the spatial contention approaches π, as in the NNT case, hence $\gamma^{\mathrm{NTR}} \sim \gamma - \pi \sim \gamma$. This is to be expected, since for large θ, an area much larger than the disk of radius R needs to be free of interferers, so it does not matter whether the disk is centered at the receiver or translated by R. As $\theta \to 0$, it follows from (14) that $\pi H_\delta(-1/\delta) \to \gamma$, which indicates that the spatial contention vanishes faster than θ^δ. In fact,

$$\gamma^{\mathrm{NTR}} \sim \frac{2\pi}{\alpha - 2}\theta = \frac{\delta\pi}{1-\delta}\,, \quad \theta \to 0\,. \tag{19}$$

The two asymptotic regimes are clearly visible in Fig. 1 (b) in the case $\alpha = 4$. For $\theta < 1$, the slope is about one (or 10dB/decade), whereas for $\theta > 1$ it is about 5dB/decade. For $\alpha = 4$, the success probability simplifies to

$$\mathbb{P}^o(\mathcal{C}^{\mathrm{NTR}} \mid R) = q \exp\left(-\lambda p \pi R^2 \sqrt{\theta}\left[\frac{\pi}{2} - \arctan\left(\frac{1}{\sqrt{\theta}}\right)\right]\right)\,. \tag{20}$$

In the NTR case, R is distributed as $f_R(r) = 2\pi p \lambda r \exp(-p\lambda\pi r^2)$. Integration and inversion yields

$$D^{\mathrm{NTR}} = \frac{1}{q} + \frac{\gamma^{\mathrm{NTR}}}{\pi q}\,, \tag{21}$$

which is monotonically decreasing as $p \downarrow 0$. This indicates that, without noise, the benefit of reducing the interferer density compensates for the increased transmission distance. (For $p = 0$, the delay is undefined since there is no nearest transmitter in this case.) For small θ, we have the particularly simple result

$$D^{\mathrm{NTR}}_{\min} \sim 1 + \frac{2}{\alpha-2}\theta = 1 + \frac{\delta}{1-\delta}\theta\,, \quad \theta \to 0\,.$$

4) Nearest-neighbor reception (NNR): This case is quite similar to NTR, with the difference that the nearest neighbor is at distance $1/(2\sqrt{\lambda})$ on average and that the delay increases by a factor $1/p$ since the nearest neighbor only transmits with probability p. So $\gamma^{\mathrm{NNR}} = \gamma^{\mathrm{NTR}}$, and

$$D^{\mathrm{NNR}} = \frac{1}{pq} + \frac{\gamma^{\mathrm{NNR}}}{\pi q}\,.$$

The expression has the same form as the one for NNT, the only difference being the spatial contention. So the optimum p and the minimum delay follow from (16) and (17), respectively, with γ^{NNR} instead of γ^{NNT}. As $\theta \to 0$,

$$D^{\mathrm{NNR}}_{\min} \sim 4 + \frac{4}{\alpha-2}\theta = 4 + \frac{2\delta}{1-\delta}\theta\,. \tag{22}$$

The results for all four cases are shown in Fig. 2.

C. Static Networks

In the static case, only a single realization of the point process is drawn. Comparing (1) and (2), we obtain a bound on the local delay in the static case by Jensen's inequality: $D > \mathbb{P}(\mathcal{C})^{-1}$. As we shall see, this bound is often very loose. In particular, the actual delay may be infinite while the lower bound is always finite. The reason is the correlation of the interference in the static case [12]. To analyze static networks, we need a lemma similar to Lemma 1:

Lemma 2 *Let I denote the interference as defined in* (5), $\mathcal{H} \subset \mathbb{R}^2$*, and let*

$$\mathcal{L}_I(s \mid \Phi, \mathcal{H}) = \mathbb{E}^o(\exp(-sI \mid \Phi, \Phi \cap \mathcal{H} = \emptyset))$$

be the conditional Laplace transform given Φ and given that there is no transmitter in $\mathcal{H}$. Then

$$\mathbb{E}^o\left(\frac{1}{\mathcal{L}_I(s \mid \Phi, \mathcal{H})}\right) = \exp\left(\lambda \int_{\mathbb{R}^2 \setminus \mathcal{H}} \frac{ps}{sq + \|x\|^\alpha} \mathrm{d}x\right)\,, \tag{23}$$

which for $\mathcal{H} = \emptyset$ evaluates to

$$= \exp\left(\frac{p\lambda C(\alpha) s^\delta}{q^{1-\delta}}\right)\,,$$

with $C(\alpha)$ as defined in (4)*. The local delay conditioned on a link distance R is obtained by replacing s by θR^α.*

Proof: Follows from [2, Lemma 17.30 and Prop. 17.31]. ■

1) Nearest-receiver transmission (NRT): Here we consider the case where the partitioning into potential transmitters and receivers is fixed, *i.e.*, the transmitters are chosen from Φ with probability p, as before, but there exists another, independent PPP of receivers Φ_{r} of intensity $\lambda_{\mathrm{r}} = q\lambda$. So, in this model, the nodes in Φ that do not transmit are not available as receivers. This assumption maintains the same density of (actual) transmitters and receivers as in the other models. As pointed out at the beginning of this section, the reason for this slightly modified model is that the NRT scheme where transmitters and receivers are chosen dynamically from the same point process cannot be analyzed in the same way since the link distance would not be fixed if Φ was fixed.

Theorem 2 *For independent point processes $\hat{\Phi}$ of intensity λ of* potential *transmitters and Φ_{r} of intensity $\lambda_{\mathrm{r}} = q\lambda$ of receivers, let D^{NRT} be the local delay from the typical node in Φ to its nearest neighbor in Φ_{r}. We have*

$$D^{\mathrm{NRT}} = \frac{1}{p}\,\frac{\pi}{\pi - \gamma p q^{\delta-2}}\,, \quad pq^{\delta-2} < \pi/\gamma \tag{24}$$

and the bounds $\underline{D}^{\mathrm{NRT}} < D^{\mathrm{NRT}} < \overline{D}^{\mathrm{NRT}}$ *where*

$$\overline{D}^{\mathrm{NRT}} = \frac{1}{p}\frac{\pi}{\pi - \gamma p q^{-2}}\,, \quad \gamma p < q^2\pi \tag{25}$$

$$\underline{D}^{\mathrm{NRT}} = \frac{1}{p}\frac{\pi}{\pi - \gamma p q^{-1}}\,, \quad \gamma p < q\pi\,. \tag{26}$$

Proof: Replacing s in (23) by θR^α yields the success probability given a link distance R. Since $C(\alpha)(\theta R^\alpha)^{2/\alpha} = \gamma R^2$, deconditioning on R yields

$$D^{\mathrm{NRT}} = \frac{1}{p}2\pi q\lambda \int_0^\infty \exp\left(\frac{\lambda p \gamma r^2}{q^{1-\delta}}\right) r \exp(-\pi q \lambda r^2)\mathrm{d}r\,, \tag{27}$$

which evaluates to (24). Since $\delta - 2 \in (-2,-1)$, we obtain upper and lower bounds for the delay by replacing the exponent $\delta - 2$ by -1 and -2, respectively. ∎

Remarks.

(i) The upper delay bound becomes tight as α increases ($\delta \downarrow 0$), whereas the lower delay bound becomes tight as $\alpha \downarrow 2$ ($\delta \uparrow 1$).

(ii) Interestingly, the lower bound $\underline{D}^{\mathrm{NRT}}$ is the delay that would result from using the unconditional Laplace transform of I instead of the conditional one (23). This would be the delay resulting from a network model with highly mobile interferers but static (yet random) link distance R.

The local delay together with the bounds are illustrated in Fig. 3.

The condition for a finite upper delay bound $\overline{D}$ yields a *lower* bound for p_{c}, and vice versa. Therefore, $\overline{p}_{\mathrm{c}} \geqslant p_{\mathrm{c}} \geqslant \underline{p}_{\mathrm{c}}$ for

$$\overline{p}_{\mathrm{c}}^{\mathrm{NRT}} = \frac{\pi}{\gamma + \pi}\,; \quad \underline{p}_{\mathrm{c}}^{\mathrm{NRT}} = 1 + \frac{\gamma}{2\pi}\left(1 - \sqrt{1 + \frac{4\pi}{\gamma}}\right)\,. \tag{28}$$

Since $\overline{p}_{\mathrm{c}}^{\mathrm{NRT}} \sim \pi/\gamma$ and $\underline{p}_{\mathrm{c}}^{\mathrm{NRT}} \sim \pi/\gamma$ as $\gamma \to \infty$, we have $p_{\mathrm{c}}^{\mathrm{NRT}} \sim \pi/\gamma$. For the case $\alpha = 4$, for which $\delta - 2 = -3/2$ is exactly in between the two extremes -2 and -1, p_{c} can be calculated in closed form as the solution to a cubic equation. It lies quite exactly in between the two bounds.

Asymptotically, as $\gamma \to 0$ (or $\theta \to 0$), the exact p_{c} can be obtained from $(1-p_{\mathrm{c}})^{2-2\alpha} = (\gamma/\pi)^\alpha$, since $p_{\mathrm{c}} \to 1$ as $\gamma \to 0$. This yields

$$p_{\mathrm{c}}^{\mathrm{NRT}} \sim 1 - \left(\frac{\gamma}{\pi}\right)^{\frac{1}{2-\delta}}\,, \quad \gamma \to 0\,.$$

Minimizing the lower bound $\underline{D}^{\mathrm{NRT}}(p)$ yields an upper bound on the optimum p:

$$\overline{p}_{\mathrm{opt}}^{\mathrm{NRT}} = 1 - \sqrt{\frac{\gamma}{\gamma+\pi}}\,.$$

Hence $\overline{p}_{\mathrm{opt}}^{\mathrm{NRT}} \sim \pi/(2\gamma)$ as $\gamma \to \infty$. Minimizing the upper bound yields a solution of a cubic equation with the same asymptotic behavior, from which we conclude that $p_{\mathrm{opt}}^{\mathrm{NRT}} \sim \pi/(2\gamma) = p_{\mathrm{c}}^{\mathrm{NRT}}/2$ asymptotically. For finite γ, this is an upper bound on $p_{\mathrm{opt}}^{\mathrm{NRT}}$, which is quite tight as soon as $\gamma > 13$dB. So a good rule of thumb is to choose the transmit probability

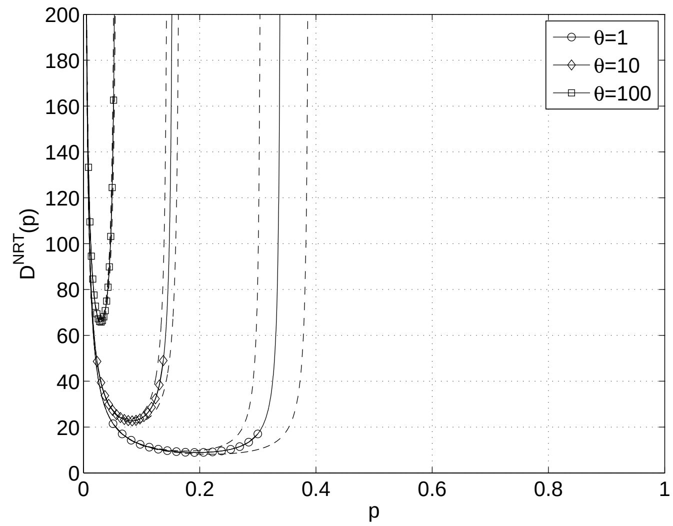


Fig. 3. The local delay for the static NRT case as a function of the channel access probability p for $\alpha = 4$ and $\theta = 1, 10, 100$. The dashed lines are the upper and lower bounds (25) and (26), respectively.

slightly smaller than $\pi/(2\gamma)$. For $\alpha = 4$, this is $p \lessapprox 1/(\pi\sqrt{\theta})$ if $\theta > 10$. As $\gamma \to 0$, we have

$$\overline{p}_{\mathrm{opt}}^{\mathrm{NRT}} \sim 1 - \sqrt{\frac{\gamma}{\pi}}\,; \qquad \underline{p}_{\mathrm{opt}}^{\mathrm{NRT}} \sim 1 - \left(\frac{2\gamma}{\pi}\right)^{1/3}\,, \tag{29}$$

so there is a gap between the bounds.

The expression for the minimum delay $D_{\min}^{\mathrm{NRT}} = D(p_{\mathrm{opt}})$ is not very compact and therefore omitted. Asymptotically, since $p_{\mathrm{opt}} \sim \pi/(2\gamma)$,

$$D_{\min}^{\mathrm{NRT}} \sim 4\left(1 - \frac{1}{\alpha} + \frac{\gamma}{\pi}\right)\,, \quad \gamma \to \infty\,. \tag{30}$$

On the other hand, as $\gamma \to 0$, the minimum delay is smaller than the delay achieved if p is chosen from either of the two bounds (29). From the series expansions of the two resulting expressions,

$$D_{\min}^{\mathrm{NRT}} = 1 + O(\gamma^{\max\{1/3,1/\alpha\}})\,, \quad \gamma \to 0\,. \tag{31}$$

2) Nearest-neighbor transmission (NNT): Applying Lemma 2 to the case where $\mathcal{H} = B_{(R,0)}(R)$ gives for the local delay given R

$$D_R^{\mathrm{NNT}} = \frac{1}{pq}\exp\left(\lambda p R^2(\gamma/q^{1-\delta} - \kappa^{\mathrm{NNT}})\right)\,,$$

where

$$\kappa^{\mathrm{NNT}} = \int_{\mathcal{H}} \frac{s}{sq + \|x\|^\alpha}\mathrm{d}x\,.$$

Apart from the additional factor q in the denominator, this is the same integral as in the highly mobile NNT case, cf. (66). Using the hypergeometric function, we can express the integral in polar form as

$$\int_0^b \frac{rs}{sq + r^\alpha}\mathrm{d}r = \frac{b^2}{2q}H_\delta(-b^\alpha/(sq))\,. \tag{32}$$

Theorem 3 *The local delay of nearest-neighbor transmission in static networks given R can be expressed as*

$$D_R^{\rm NNT} = \frac{1}{pq}\exp(\lambda p R^2(\gamma/q^{1-\delta} - \kappa^{\rm NNT}))\,, \qquad (33)$$

where $\kappa^{\rm NNT}$ depends on q, δ, and θ and accounts for the hole. $\kappa^{\rm NNT}$ is bounded as follows:

(a) $0 < \kappa^{\rm NNT} < \pi/q$, and $\lim_{\theta\to\infty} \kappa^{\rm NNT} = \pi/q$.

(b) Letting $\theta' = \theta q$, we have

$$\begin{aligned} q\kappa^{\rm NNT} <& \frac{2\pi}{3}H_\delta(-2^\alpha/\theta') + \frac{\pi}{4}H_\delta(-3^{\alpha/2}/\theta') + \\ & \frac{\pi}{6}H_\delta(-2^{\alpha/2}/\theta') + \frac{\pi}{6}H_\delta(-1/\theta') \qquad (34)\\ q\kappa^{\rm NNT} >& \frac{\pi}{2}H_\delta(-3^{\alpha/2}/\theta') + \frac{\pi}{6}H_\delta(-2^{\alpha/2}/\theta') + \\ & \frac{\pi}{12}H_\delta(-1/\theta') \qquad (35)\end{aligned}$$

(c) A lower bound without special functions is

$$q\kappa^{\rm NNT} > \frac{3\pi}{4} - \frac{\pi}{6\theta'(2+\alpha)}\left(2\cdot 3^{1+\alpha/2} + 2^{1+\alpha/2} + 1\right)\,. \qquad (36)$$

Proof: The proof is similar to the proof of Thm. 1. The form of the expression (33) follows from Lemma 2 and (32). The bounds (a) follow from $\lim_{s\to\infty} A(R,s,q) = \pi R^2/q$. The bounds (b) and (c) follow from (32) and the inner and outer bounds on the hole shown in Fig. 11. ∎

Remarks.

(i) As $\theta q \to 0$, it follows from (14) that

$$\frac{\pi}{3}\theta^\delta q^{\delta-1}\frac{\pi\delta}{\sin(\pi\delta)} < \kappa^{\rm NNT} < \frac{\pi}{2}\theta^\delta q^{\delta-1}\frac{\pi\delta}{\sin(\pi\delta)}\,,$$

or

$$\frac{1}{3}\frac{\gamma}{q^{1-\delta}} < \kappa^{\rm NNT} < \frac{1}{2}\frac{\gamma}{q^{1-\delta}}\,. \qquad (37)$$

So the gain due to the hole is asymptotically between a factor $1/3$ to $1/2$ in the exponent, as in the highly-mobile NNT case.

(ii) Better bounds that do not contain special functions can be obtained by including more terms from the expansion (67) of the hypergeometric function.

Deconditioning on R yields the local delay

$$D^{\rm NNT} = \frac{1}{pq}\cdot\frac{\pi}{\pi - p(\gamma/q^{1-\delta} - \kappa^{\rm NNT})}\,. \qquad (38)$$

The bounds on the local delay obtained from (34) and (35) are shown in Fig. 4.

Due to the different exponents of p and q and the dependence of κ on p, (38) does not lend itself easily for the derivation of the optimum transmit probability and minimum achievable delay. Asymptotically, as in the NRT case, it can be shown that $p_{\rm c}^{\rm NNT} \sim \pi/\gamma$. This is not surprising since the relative effect of the hole vanishes as γ increases.

For the optimum p, setting $\delta = 0$ and $\kappa^{\rm NNT} = \pi/q$ yields

$$\underline{p}_{{\rm opt},\theta\to\infty}^{\rm NNT} = \frac{1}{2}\frac{\pi}{\gamma}\,. \qquad (39)$$

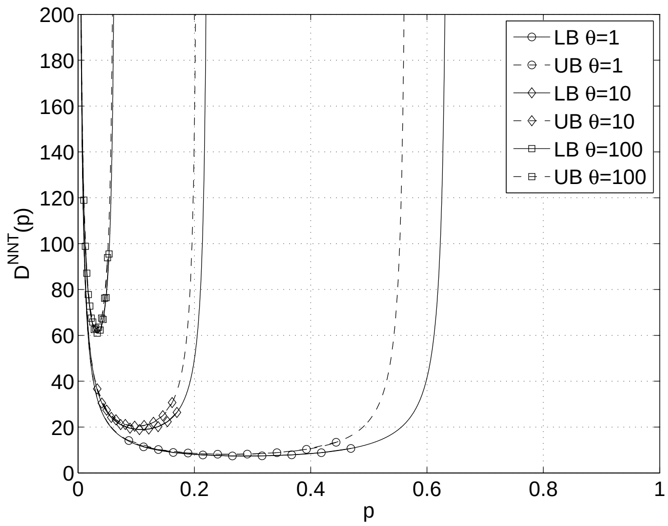


Fig. 4. Bounds on the local delay for the static NNT case as a function of the channel access probability p for $\alpha = 4$ and $\theta = 1, 10, 100$. The bounds are obtained by inserting the bounds on $\kappa^{\rm NNT}$ in Thm. 3 in the delay expression (38).

And for $\delta = 1$,

$$\overline{p}_{{\rm opt},\theta\to\infty}^{\rm NNT} = \frac{1}{3}\left(1 - \sqrt{1 - \frac{3\pi}{\gamma}}\right)\,. \qquad (40)$$

Again the scaling is the same, so $p_{\rm opt}^{\rm NNT} \sim \pi/(2\gamma)$. Numerically obtained bounds on $p_{\rm opt}^{\rm NNT}$ and the asymptotic expressions are illustrated in Fig. 5. Inserting the asymptotic value in the delay expression, we find

$$D_{\rm min}^{\rm NNT} \sim \frac{4\gamma}{\pi}\,, \quad \gamma\to\infty\,. \qquad (41)$$

On the other hand, for small γ (small θ), the limiting value of $\gamma/q^{1-\delta} - \kappa^{\rm NNT}$ is less than $2\gamma/(3q^{1-\delta})$ by (37). Setting $\delta = 0$ yields the simple asymptotic lower bound

$$\underline{p}_{{\rm opt},\theta\to 0}^{\rm NNT} = \frac{3\pi}{6\pi + 2\gamma}\,. \qquad (42)$$

As $\gamma \to 0$, $D_{\rm min}^{\rm NNT} = 4 + \Theta(\gamma)$. The numerically obtained $D_{\rm min}^{\rm NNT}$ and its asymptotic behavior are shown in Fig. 6.

Generally, as $\gamma \to \infty$, there is no difference between the NRT and NNT in terms of interference, but only in the availability of the destination node as a receiver and in the link distance distribution.

3) Nearest-transmitter reception (NTR): Similarly to the static NRT case, we pre-partition transmitters and receivers. In this case, receivers do not matter (except for the typical receiver considered). We take a fixed point process of transmitters of intensity λp, which implies there is no actual ALOHA involved, or, in terms of the marked point process $\hat{\Phi}$, we take the marks to be fixed also.

Recall from the high-mobility case that here the interferer-free hole is centered at the receiver, so we apply Lemma 2 with $\mathcal{H} = B_o(R)$. From (32) we have

$$D_R^{\rm NTR} = \frac{1}{q}\exp\left(\lambda p R^2(\gamma/q^{1-\delta} - \kappa^{\rm NTR})\right)\,,$$

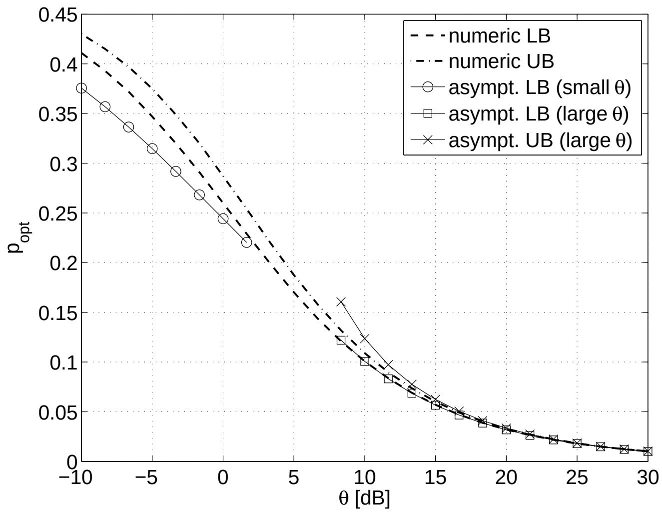


Fig. 5. The optimum transmit probability for the static NNT case as a function of the SIR threshold θ for $\alpha = 4$. The numeric bounds are obtained by numerically optimizing over p when the upper and lower bounds from Thm. 3 for $\kappa^{\rm NNT}$ are used. The lower and upper bounds for large θ are the ones in (39) and (40), respectively. The asymptotic lower bound as $\theta \to 0$ is from (42).

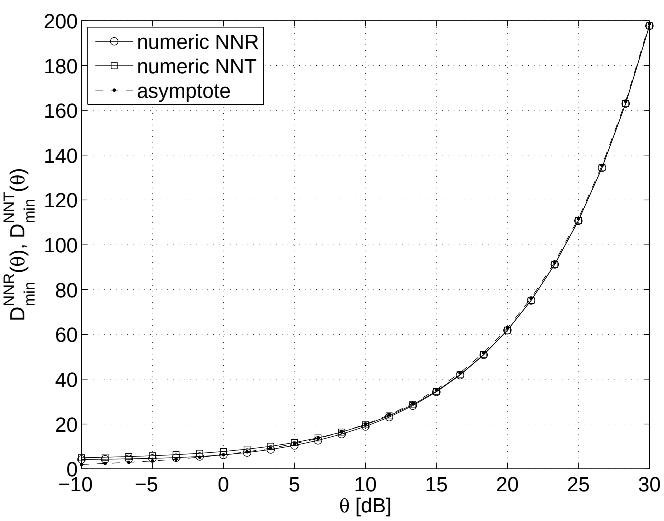


Fig. 6. The minimum local delay for the static NNT and NNR cases as a function of the SIR threshold θ for $\alpha = 4$. The curves are very close; at small θ, the delay for NNT is slightly larger. The asymptote is $4\gamma/\pi$, per (41), in both cases. It is quite tight as soon as $\theta > 1$.

with $\kappa^{\rm NTR} = \frac{\pi}{q} H_\delta(-1/(\theta q))$. For $\alpha = 4$, this simplifies to

$$D_R^{\rm NTR} = \frac{1}{q} \exp\left(\lambda p \pi R^2 \sqrt{\frac{\theta}{q}} \left[\frac{\pi}{2} - \arctan\left(\frac{1}{\sqrt{\theta q}}\right)\right]\right). \tag{43}$$

From (23) follows that, for general α,

$$\gamma/q^{1-\delta} - \kappa^{\rm NTR} \sim \frac{2\pi}{\alpha - 2}\theta, \quad \theta \to 0. \tag{44}$$

Deconditioning on R yields

$$D^{\rm NTR} = \frac{1}{q} \frac{\pi}{\pi - \gamma/q^{1-\delta} + \kappa^{\rm NTR}}, \quad \gamma/q^{1-\delta} - \kappa^{\rm NTR} < \pi. \tag{45}$$

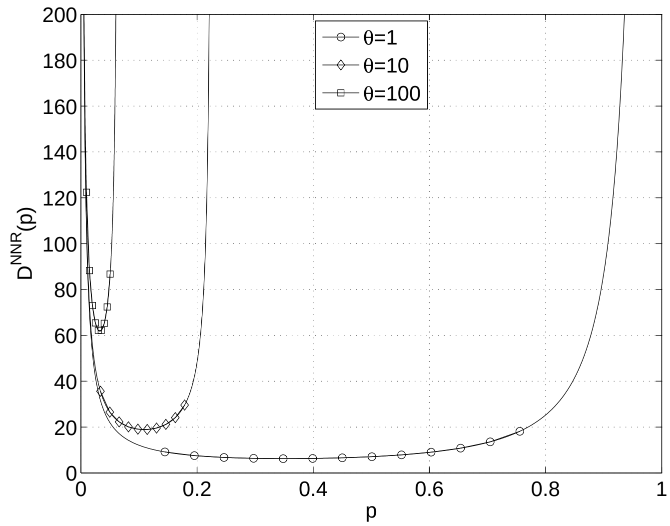


Fig. 7. The local delay for the static NNR case as a function of the channel access probability p for $\alpha = 4$ and $\theta = 1, 10, 100$. The delay is given in (48).

Since the delay is monotonically decreasing as $p \downarrow 0$ (and thus $q \uparrow 1$),

$$D_{\min}^{\rm NTR} = \frac{\pi}{\pi - \gamma + \kappa^{\rm NTR}} = \frac{1}{1 + H_\delta(-1/\theta) - \gamma/\pi}. \tag{46}$$

What is interesting about this case is that there is a *hard* phase transition in the sense that a finite local delay cannot be achieved for any p as soon as θ exceeds some critical value $\theta_{\rm c}$, determined by $1 + H_\delta(-1/\theta_{\rm c}) - \gamma/\pi = 0$. While reducing p reduces the interference, it also increases the link distance in proportion to $p^{-1/2}$, and the net gain is negative if θ is larger than $\theta_{\rm c}$. For $\alpha = 4$, $\theta_{\rm c} \approx 1.351$. So, the maximum rate that can be supported for finite local delay is $\mathcal{R}_{\max} \approx 1.2333$. As α decreases, $\theta_{\rm c}$ decreases also. Since $\alpha < 4.95$ in most environments, the rate supported by NTR cannot exceed 4/3 bits/s/Hz. Therefore the high-θ asymptotics do not exist. The NTR case is relevant for the downlink of cellular networks when mobile users connect to the nearest base station [13]. For small θ, it follows from (46) and (44) that

$$D_{\min}^{\rm NTR} \sim 1 + \frac{2}{\alpha - 2}\theta = 1 + \frac{\delta}{1 - \delta}\theta, \quad \theta \to 0. \tag{47}$$

4) Nearest-neighbor reception (NNR): In this case, pre-partitioning is unnecessary. As in the highly mobile case, the difference to NTR is the factor $1/p$ in the delay and the link distance distribution. We have $D_R^{\rm NNR} = \frac{1}{pq} \exp\left(\lambda p R^2 (\gamma/q^{1-\delta} - \kappa^{\rm NNR})\right)$ with $\kappa^{\rm NNR} = \kappa^{\rm NTR}$, and deconditioning yields

$$D^{\rm NNR} = \frac{1}{pq} \cdot \frac{\pi}{\pi - p(\gamma/q^{1-\delta} - \kappa^{\rm NNR})} \quad \text{for } p(\gamma/q^{1-\delta} - \kappa^{\rm NNR}) < \pi. \tag{48}$$

This expression has the same form as (38), so for large θ, all bounds derived in the NNT section apply. For small θ, the difference between $\kappa^{\rm NNT}$ and $\kappa^{\rm NNR}$ becomes significant, as in the highly mobile case. Fig. 7 shows the local delay as a

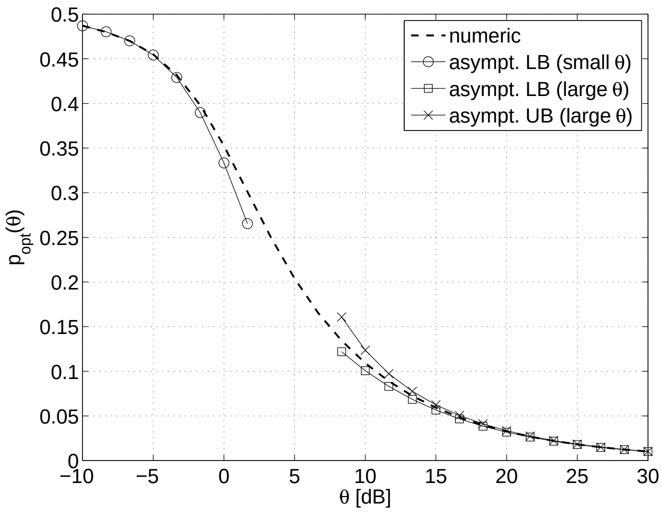


Fig. 8. The optimum transmit probability for the static NNR case as a function of the SIR threshold θ for $\alpha = 4$. The numeric curve is obtained by numerically optimizing over p. The lower and upper bounds for large θ are the ones in (39) and (40), respectively (same as for the NNT case). The asymptotic lower bound as $\theta \to 0$ is from (50).

function of p. Certainly $p_{\rm opt}^{\rm NNR} \to 1/2$ as $\theta \to 0$, hence we have

$$D_{\rm min}^{\rm NNR} \sim 4\left(1+\frac{\delta/2}{1-\delta}\theta\right) . \tag{49}$$

For $\alpha = 4$, the sharper expression

$$p_{\rm opt}^{\rm NNR} \sim \frac{1}{3}\left(1+\frac{1}{\theta}-\sqrt{1-\frac{1}{\theta}+\frac{1}{\theta^2}}\right) , \quad \theta \to 0, \tag{50}$$

can be derived. In this case, $D_{\rm min}^{\rm NRT} \sim 4 + 2\theta$.

The numerically optimized transmit probability is shown in Fig. 8, together with the asymptotic curves, and the resulting minimum delay is plotted in Fig. 6.

D. Asymptotic Delays

We first summarize the results on the asymptotic delays in a theorem.

Theorem 4 *As $\theta \to \infty$, the minimum local delay in all four highly mobile cases scales as γ/π or*

$$D_{\rm min} \sim \theta^{\delta} \frac{\pi\delta}{\sin(\pi\delta)} .$$

In the static NRT, NNT, and NNR cases, the scaling behavior is $4\gamma/\pi$ or

$$D_{\rm min} \sim 4\theta^{\delta} \frac{\pi\delta}{\sin(\pi\delta)} .$$

The exception is the static NTR case, where the delay becomes infinite for all values of p as soon as θ exceeds some critical value $\theta_{\rm c}$.

As $\theta \to 0$, the scaling laws of the minimum local delay are listed in Table I.

Remarks.

(i) The benefits of the interferer-free disk around the transmitter or receiver are apparent as $\theta \to 0$. Compared to NRT, where interferers can be located anywhere, the delay scaling improves from $\theta^{1/\alpha}$ to $\theta^{2/\alpha}$ for the NNT case, where the hole is centered at the transmitter, and further to θ for the NTR and NNR cases, where the hole is centered at the receiver.

(ii) It does not matter for small θ whether the nodes are static or highly mobile when the disk around the receiver is known to be interferer-free.

	High mobility	Static
NRT	$1+2\sqrt{\gamma/\pi}$; $1+\Theta(\theta^{1/\alpha})$	$1+O(\gamma^{\max\{1/3,1/\alpha\}})$
NNT	$4+c\gamma/\pi$; $4+\Theta(\theta^{\delta})$	$4+\Theta(\gamma)$; $4+\Theta(\theta^{\delta})$
NTR	$1+\frac{\delta}{1-\delta}\theta$	$1+\frac{\delta}{1-\delta}\theta$
NNR	$4+\frac{2\delta}{1-\delta}\theta$	$4+\frac{2\delta}{1-\delta}\theta$

TABLE I
SCALING BEHAVIOR OF THE MINIMUM LOCAL DELAY AS $\theta \to 0$. WITHOUT A O OR Θ SYMBOL, THE ASYMPTOTIC RESULTS ARE SHARP, *i.e.*, "$\sim$". THE CONSTANT c DEPENDS ON α AND ASSUMES VALUES $c \in (1, 4/3)$.

Expressed in terms of the transmission rate, the scaling behavior can be summarized as follows.

Corollary 2 *Irrespective of the level of mobility in the network and the choice of the nearest-neighbor transmission scheme, the minimum local delay scales at high rates as*

$$D_{\rm min} = \Theta(2^{\delta\mathcal{R}}) , \quad \mathcal{R} \to \infty .$$

Again the exception is the static NTR *case. As $\mathcal{R} \to 0$,*

$$D_{\rm min} = K + O(\mathcal{R}^{1/\alpha}) \quad \textit{and} \quad D_{\rm min} = K + \Omega(\mathcal{R}) ,$$

where $K = 1$ for NRT and NTR and $K = 4$ for NNT and NNR.

The constant K is the minimum achievable access delay. If one of the nodes is known to be transmitting or listening, $K = 1$.

E. Delay Distributions

1) Highly mobile networks: Let Δ be the delay random variable, s.t. $D = \mathbb{E}(\Delta)$. The delay Δ is geometric, since each transmission attempt is independent. For nearest-receiver transmission (NRT),

$$\mathbb{P}(\Delta^{\rm NRT} = k) = (1-\xi)^{k-1}\xi , \quad \xi = \frac{\pi p}{\pi + \gamma p/q} .$$

2) Static networks: Let $D(R)$ be the local delay as a function of the link distance random variable R.

For NRT, $\mathbb{P}(R^2 \le x) = 1 - \exp(-\lambda\pi q x)$, hence in the static case, the delay $D(R)$ has a continuous distribution with a heavy tail:

$$\mathbb{P}(D^{\rm NRT}(R) \le x) = 1 - \left(\frac{1}{xpq}\right)^{\frac{\pi q^{2-2/\alpha}}{p\gamma}} , \quad x \ge 1/(pq) . \tag{51}$$

In any case $D^{\rm NRT}(R)$ is finite, but the local delay $\mathbb{E}_R(D^{\rm NRT}(R))$ does not exist if $\pi q^{2-2/\alpha} \le p\gamma$, which recovers the condition in Thm. 3. While the distribution (51) is not the delay distribution, since it includes averaging over the point process (given the link distance), it indicates that the delay distribution is fundamentally different in the static than in the highly mobile case in two aspects: It is continuous, and it has a heavy tail. Both are consequences of the temporal dependence of the transmission success events, as analyzed in [12].

III. Static Networks without Interference

In this section, we consider noise but not interference. This scenario is appropriate if the links use orthogonal channels or, more generally, if the distance between concurrent transmitters is much larger than the distance of the typical link. The resulting network is a collection of independent wireless links.

A. System Model

The links have random distances that are spatially iid but temporally fixed or static, and they are subject to fading that is iid across both space and time. In the presence of noise, the transmit power, denoted by P, becomes relevant. Focusing on a single link, the received power is $P_r = PhR^{-\alpha}$, where h is the (power) fading coefficient and R is the link distance. We will allow h to be more general than Rayleigh fading in this section, but always iid temporally and across links. The transmission is assumed successful if $P_r > \theta$ for some threshold θ that is proportional to the noise power. Given R, the success probability is

$$p_{s|R} = \mathbb{P}(h > \theta R^\alpha/P) = 1 - F_h(\theta R^\alpha/P)\,,$$

where $F_h(x) = \mathbb{P}(h \leq x)$ is the cumulative fading distribution function. Since each node has a pre-defined partner, we refer to the mean delay until success simply as the mean delay instead of the local delay. We also ignore channel access delays, which are trivial in the interference-free case. The mean delay of successful transmission, conditioned on R, is $p_{s|R}^{-1}$. If R was also temporally iid, the (unconditioned) mean delay would simply be

$$D = 1/\mathbb{E}_R(p_{s|R})\,.$$

In this case, we could define $\tilde{h} \triangleq hR^{-\alpha}$ and consider the fading coefficient to be $\tilde{h}$, combining the distance and fading uncertainties [14]. Here we focus on the case of fixed R, in which case the mean delay is the ensemble average $D = \mathbb{E}_R(1/p_{s|R})$. This static case is more interesting and practical.

B. The gamma/Rayleigh case

We first consider the case where the link distance is gamma distributed, parametrized by an integer n:

$$f_{R_n}(r) = \frac{2}{\Gamma(n)}(\lambda\pi)^n r^{2n-1}\exp(-\lambda\pi r^2) \quad r \geq 0,\ n \in \mathbb{N}$$

We will refer to this link distance model as the gamma(n) model. The fading is Rayleigh.

The gamma distribution models the case where a node transmits to its n-th nearest neighbor in a Poisson network [11]. The mean is $\mathbb{E}(R_n) = \sqrt{n/\lambda}/2$. The local delay as a function of n is denoted by D_n. We start with the case $n = 1$, where the distance is Rayleigh distributed, and then relate the local delay for general n to the case $n = 1$. The success probability of a transmission over distance R_1 is $p_{s|R_1} = \exp(-\theta R_1^\alpha/P)$. For constant P, the mean delay

$$D_1 = 2\pi\lambda\int_0^\infty \exp(\theta r^\alpha/P) r\exp(-\lambda\pi r^2)\mathrm{d}r$$

diverges to infinity as soon as $\alpha > 2$. For $\alpha = 2$ and all constant P, we have

$$D_1 = \frac{\lambda\pi}{\lambda\pi - \theta/P}\,. \tag{52}$$

For $\alpha > 2$, the transmit power needs to be chosen as a function of the link distance R_1. So we will henceforth assume that the transmitter knows R_1, which is a reasonable assumption given that the distance remains constant forever, and that it chooses its transmit power as

$$P \triangleq aR_1^{\alpha-2+b}\,, \tag{53}$$

where a the *power control factor*, and b the *power control exponent*. In this case, the success probability becomes $p_{s|R_1} = \exp(-\frac{\theta}{a}R_1^{2-b})$, and the mean delay, now a function of a and b, is

$$D_1(a,b) = 2\lambda\pi\int_0^\infty \exp\left(\frac{\theta}{a}r^{2-b}\right) r\exp(-\lambda\pi r^2)\mathrm{d}r\,. \tag{54}$$

For $b < 0$, the integral diverges for all values of the remaining parameters (not enough power if the nearest neighbor is far). For $b > 2$, the integral diverges since there is not enough power for receivers that are very close ($R \ll 1$). This second type of divergence is due to the singularity of the path loss law at the origin. If a bounded path loss law is used, say $(1+R)^{-\alpha}$ (and the corresponding transmit power), the first exponential in (54) is to be replaced by $\exp(\frac{\theta}{a}(1+r)^{2-b})$, which results in a finite delay for all $b \geq 0$. This integral does not admit a closed-form expression, though. We will therefore continue with the unbounded path-loss law and restrict ourselves to the regime $b \in [0,2]$, knowing that for bounded path loss the delay could be further reduced by choosing $b > 2$, *i.e.*, by *over-compensating* for the large-scale path loss.

1) $b = 0$: We obtain

$$D_1(a,0) = \frac{\lambda\pi}{\lambda\pi - \theta/a}\,, \qquad \theta < a\lambda\pi\,, \tag{55}$$

which shows that the mean delay exhibits a phase transition even in the interference-free case. There is tension between the delay given the distance r, call it $D_1(r)$, for which $\log D_1(r) = c_2 r^2$, and the density $f_{R_1}(r)$, for which $\log f_{R_1}(r) \sim -c_1 r^2$ as $r \to \infty$. Hence the local delay is finite if $c_1 > c_2$, which is exactly the condition in (55).

So, if the power control factor a is large enough, the local delay will be finite even if the power is adjusted in proportion to $R_1^{\alpha-2}$ only — thus the compensation for the large-scale path loss does not have to be complete. In particular, for $\alpha = 2$, the transmit power can be chosen to be the same for all nodes, irrespective of their nearest-neighbor distance (see (52)). As a consequence, the distances do not need to be known at the transmitter for $\alpha = 2$.

2) $b=2$: With complete compensation for the large-scale path loss, the integration in (54) becomes obsolete since the success probability does not depend on the distance R_1, and we obtain immediately

$$D_1(a,2)=\exp\left(\frac{\theta}{a}\right)\,. \tag{56}$$

In this case, the delay increases exponentially in θ, or $\log D_1=\Theta(\theta)$ as $\theta\to\infty$.

3) $b=1$: Let $t\triangleq\frac{\theta}{2a\sqrt{\lambda\pi}}$. Then

$$\begin{aligned}D_1(a,1)&=1+\frac{\theta\exp(t^2)\,(1+\operatorname{erf} t)}{2a\sqrt{\lambda}}\\&=1+\sqrt{\pi}t\exp(t^2)(1+\operatorname{erf} t)\,.\end{aligned} \tag{57}$$

So in this case, $\log D_1=\Theta(\theta^2)$. We observe that there is no phase transition for $b=1$ or $b=2$.

4) General $b\in(0,2]$:

Proposition 1 (Rayleigh distance distribution and Rayleigh fading) *If $P\propto R_1^{\alpha-2+b}$, for any $0<b\leq 2$, the links can support arbitrary rates at finite mean delays.*

Proof: Letting $x\triangleq r^2$, the delay (54) is of the form

$$c_1\int_0^\infty \exp\big(-x(c_1-c_2x^{-b/2})\big)\mathrm{d}x\,,\quad c_1,c_2>0\,.$$

For $b\leq 2$, the integral can only diverge due to the upper integration bound. To show that it converges even for $b\ll 1$, we compare the integrand with $\exp(-bx)$. We observe that

$$\exp\left(-x(c_1-c_2x^{-b/2})\right)<\exp\left(-\frac{c_1x}{2}\right)\quad\text{for } x>\left(\frac{2c_2}{c_1}\right)^{2/b},$$

which proves finiteness of the delay for all $0<b\leq 2$, $c_1>0$ and $0<c_2<\infty$. $c_2=\theta/a$ is finite for all rates $\mathcal{R}$. ∎

The delays will become extremely large as $b\to 0$, $a\to 0$, and/or $\theta\to\infty$, but there is no phase transition.

For general n, calculating $\mathbb{E}\exp(\frac{\theta}{a}R_n^{2-b})$, we find that the delay increases geometrically in n:

Proposition 2 (Gamma distance distribution and Rayleigh fading) *For a transmit power $P=aR_n^{\alpha-2}$, the mean delay D_n is*

$$D_n(a,0)=(D_1(a,0))^n\,,\qquad n\in\mathbb{N}\,. \tag{58}$$

If the path loss is fully compensated for, i.e., $P=aR_n^\alpha$, *$D_n(a,2)=\exp(\theta/a)$, irrespective of n.*

In this result, the transmit powers are adjusted according to n, so the nearest-neighbor and the second-nearest-neighbor delays, related by (58), are achieved using different powers. If the transmit power is chosen according to the distance to the second-nearest neighbor, the time to connect to the nearest neighbor is bounded as $D_1(a,0)<\sqrt{D_2(a,0)}$ and $D_1(a,2)<D_2(a,2)$ since $R_2>R_1$ a.s.

The mean transmit power for general a, b is

$$a\mathbb{E}(R_n^{\alpha-2+b})=a(\lambda\pi)^{1-\alpha/2-b/2}\frac{\Gamma(n+\alpha/2+b/2-1)}{\Gamma(n)}\,. \tag{59}$$

C. The Rayleigh/Nakagami case

Here we restrict ourselves to Rayleigh (or gamma(1)) link distances but allow the fading to be Nakagami-m.

Proposition 3 (Rayleigh distance distribution and Nakagami fading) *With Nakagami-m fading, $m\geq 1/2$, and $b=0$, the mean delay is finite if*

$$\theta<\frac{a\lambda\pi}{m} \tag{60}$$

and infinite if

$$\theta>\frac{a\lambda\pi}{m}\,. \tag{61}$$

For $b=2$, the mean delay is

$$D(a,2)=\frac{\Gamma(m)}{\Gamma(m,m\theta/a)}\,, \tag{62}$$

where $\Gamma(\cdot,\cdot)$ is the upper *incomplete gamma function.*

Proof: Let h be a Nakagami-m (power) fading random variable. From $\mathbb{P}(h<x)=\frac{\Gamma(m,mx)}{\Gamma(m)}$ follows

$$p_{s|R}=\frac{\Gamma(m,m\theta R^{2-b}/a)}{\Gamma(m)}\,,$$

and examining the range where $\mathbb{E}(p_{s|R}^{-1})$ is finite yields the result for $b=0$. For $b=2$, the delay is simply p_s^{-1}, which is independent of R. ∎

Remarks.

(i) For $b=0$ it is interesting to note that the phase transition occurs at a value of θ that is directly proportional to the amount of fading or the variance of the fading random variable h. The stronger the fading (the smaller m), the higher the threshold can be chosen while still achieving finite delay. If $m>a\lambda\pi/\theta$, the delay becomes infinite due to a lack of diversity.

(ii) The condition (60) generalizes the condition in (55) for finite delay.

(iii) For $b=2$, the delay is decreasing (to 1) with increasing m if $\theta/a<1$ and increasing (diverging to ∞) if $\theta/a>1$. This is intuitive since without fading, the delay is 1 if $\theta/a<1$, in which case transmissions always succeed, and infinity otherwise.

For $b<0$ the mean delay is infinite. Hence we have the following fact.

Fact 1 *With Nakagami-m fading, power control is needed as soon as the path loss exponent is larger than 2. In other words, any constant power is not sufficient to keep the mean delay finite if $\alpha>2$.*

D. Induced fading: Random power control

We focus on the case of Rayleigh distances. Comparing the expression for temporally iid link distances, $1/\mathbb{E}_R(p_{s|R})$, and the expression for the static case, $\mathbb{E}_R(1/p_{s|R})$, it is apparent from Jensen's inequality that much can be gained by temporal fluctuations in the received power. With static link distances, such an effect can be realized by *random power control*, even if no fading is present. It seems plausible that inducing fading by randomly varying the transmit power would help keep the mean delay finite. Since heavier-tailed distributions can be expected to yield better results due to their larger variance, we use the Pareto distribution given by

$$\mathbb{P}(H > x) = \left(\frac{k-1}{kx}\right)^k , \quad k > 1,\ x \geq 1 - 1/k ,$$

parametrized with a single parameter k such that $\mathbb{E}(H) = 1$ for all $k > 1$. The transmit power is then chosen to be $P = HR^{\alpha-2+b}$, with H temporally independently Pareto. Assuming no channel fading, we thus have the following result.

Proposition 4 (Pareto power control) *Without fading but Pareto power control,*

$$p_s(R) = \begin{cases} \left(\frac{k-1}{k\theta R^{2-b}/a}\right)^k & \text{for } R^{2-b} > \frac{a(k-1)}{\theta k} \\ 1 & \text{otherwise.} \end{cases}$$

For $b = 0$ and integer $k \geq 2$, the mean delay is of the form

$$D(a, 0) = 1 + Q(\xi) \exp\left(-\frac{k-1}{k\xi}\right) ,$$

where $\xi \triangleq \theta/(\lambda\pi a)$ and $Q(\xi) = c_1\xi + c_2\xi^2 + \ldots + c_k\xi^k$ is a polynomial of order k with coefficients

$$c_j = \frac{k^{j+1}}{(k-1)^{j-1}} \frac{\Gamma(k-1)}{\Gamma(k-j+1)} , \quad j \in \{1, 2, \ldots, k\} .$$

Proof: Straightforward (yet somewhat tedious) calculation. ■

Unless $a \gg \theta$, which is impractical, the minimum mean delay is attained at $k = 2$, as expected, since this (integer) choice of k produces the heaviest tail. In this case,

$$D(a, 0) = 1 + (4\xi + 8\xi^2) \exp(-1/(2\xi)) ,$$

which is finite for all choices of θ and a, and $D(a, 0) = \Theta(\theta^2)$, $\theta \to \infty$! So, inducing fading with a polynomial-tail distribution ensures the finiteness of the mean delay for all choices of parameters, and it achieves much better asymptotic scaling of the delay with respect to θ than Rayleigh fading, where the delay scales at least exponentially in θ. So we observe that *fading with exponential tail appears to result in a delay that increases at least exponentially in θ, whereas fading with a polynomial tail results in a delay that increases only polynomially in θ.*

Fig. 9 shows a comparison of the mean delay in the case of Rayleigh fading and Pareto induced fading. For small power levels, only the Pareto delay is finite, where for larger power levels, the Rayleigh delay is slightly smaller. In the limit, as the power increases, the delay approaches 1 in both cases, as expected.

For $b = 2$,

$$D(a, 2) = \begin{cases} \left(\frac{k\theta/a}{k-1}\right)^k & \text{if } \theta k > a(k-1) \\ 1 & \text{otherwise.} \end{cases}$$

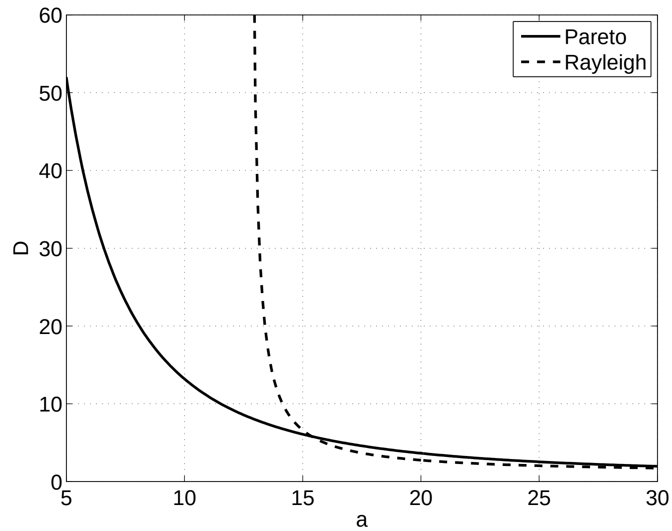


Fig. 9. The mean delay for Rayleigh fading case and Pareto random power control ($k = 2$) for $b = 0$, $\theta = 10$, $\lambda = 1/4$ (such that the mean link distance $\mathbb{E}(R) = 1$). The phase transition in the Rayleigh case occurs at $a = \theta/(\lambda\pi) = 40/\pi \approx 12.7$.

which is again minimized for $k = 2$. The asymptotic scaling with respect to θ is not improved by the larger b. The fact that $D(a, 2) = \Theta(\theta^k)$ is interesting; it confirms that the delay scaling is closely tied to how fast the tail of the (complementary) fading distribution decays. In conclusion:

Fact 2 *Drawing the transmit power from a Pareto distribution in an iid fashion in each time slot drastically reduces the mean power required to keep the delay finite.*

Intuitively, the reason why random power control yields a substantially lower delay at low mean power is that transmissions at low fixed power are bound to fail. Hence the only way to achieve a finite mean delay is to occasionally transmit at higher power. Essentially the delay-minimizing strategy is to maximize the variance of the received power, as explained in detail in [15], [16]. The same conclusion was reached in [17] for a different network model and metric.

The disadvantage of Pareto power control is the high peak-to-average power ratio.

IV. Static Networks with Noise and Interference

In this section, we add interference back to the network model and re-focus on Rayleigh fading. First, we address the case of fixed transmit power, then the case where only a single node selfishly uses power control, and lastly the case of network-wide power control. We focus on the case of nearest-receiver transmission (NRT).

A. Fixed Transmit Power

With noise, we have seen in Section III that constant transmit power only result in finite local delay for $\alpha = 2$ if interference is ignored. But for $\alpha = 2$, the spatial contention $\gamma = \infty$, so the local delay with interference is trivially infinite. So we can state the following result.

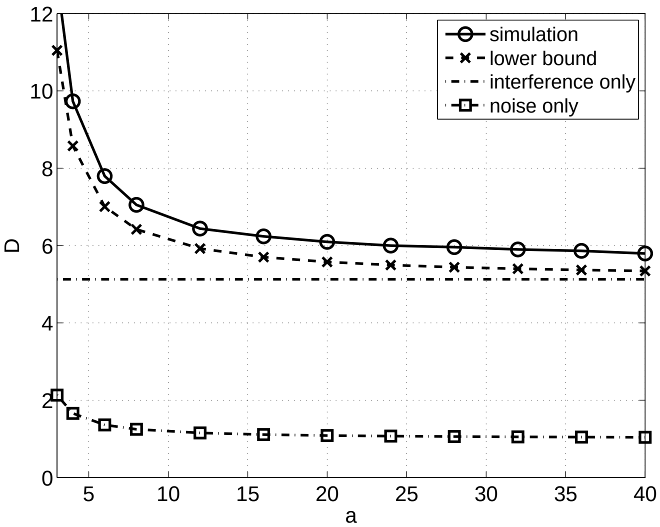


Fig. 10. The local delay in the NRT case with noise and interference as a function of the power control factor a. The solid curve (top) is a simulation, the dashed curve below is obtained by numerical integration of (64). The dash-dotted curve is the local delay if noise is ignored, which is independent of a. The bottom curve is the delay if interference and channel access delays are ignored. The potential transmitter density is $\lambda = 1/4$, and the receiver density is $\lambda_{\rm r} = 2$. Other parameters are $b = 0$, $p = 1/5$, $\alpha = 4$, and $\theta = 10$. The local delay is lower bounded by the ALOHA channel access delay $1/p = 5$.

Fact 3 *In a static network with noise and interference with the same transmit power at all nodes, the local delay is infinite for all path loss exponents, rates, and transmit probabilities.*

Clearly, power control is needed.

B. Power Control at a Single Transmitter

If only the node under consideration uses the power control scheme $P = aR^{\alpha-2+b}$ while the other nodes transmit at unit constant power, the interference is unchanged, and the local delay (with noise and interference) follows from the combination of (54) and (27):

$$D^{\rm NRT}(a,b) = 2\pi q\lambda \int_0^\infty \exp\left(\frac{\theta}{a} r^{2-b}\right) \cdot \exp\left(\frac{\lambda p a^{-\delta}\gamma r^{4/\alpha - 2b/\alpha}}{q^{1-2/\alpha}}\right) r \exp(-\pi q\lambda r^2) \mathrm{d}r\,, \quad (63)$$

which is finite whenever $\alpha > 2$ and $b > 0$ or, if $\alpha > 2$ and $b = 0$, for small enough p and large enough a. For $b = 2$, the first two exponentials do not depend on r, and the local delay is given by their product. Of course this is a selfish approach that only works for a single transmitter in the network.

C. Power Control at All Nodes

Since it appears impossible to get an exact closed-form solution for the case with full power control, we replace the interferer's transmit powers by their averages (59) (for $n = 1$), which, due to the convexity of the exponential and by Jensen's inequality, yields a lower bound on the local delay:

$$\underline{D}^{\rm NRT}(a,b) = \frac{1}{p}\mathbb{E}_R\left(\exp\left(\frac{\theta}{a} r^{2-b}\right)\exp(c_3 r^{(4-2b)/\alpha})\right) \quad (64)$$

where

$$c_3 = \frac{\lambda p \Big((q\lambda\pi)^{1-\alpha/2-b/2}\Gamma(\alpha/2+b/2)\Big)^\delta \gamma}{q^{1-\delta}}\,.$$

Fig. 10 shows a simulation result and the result of the numerical integration of (64). As expected, the analytical result is a lower bound on the delay. For comparison, also shown are the curves for the cases where noise and interference only are considered.

V. Conclusions

A. General remarks

We have provided a comprehensive analysis of the local delay in Poisson networks. The stochastic geometry-based mathematical framework permits the derivation of concrete results for different types of nearest-neighbor communication and mobility levels. While we focused on the two-dimensional case, the results can be extended to an arbitrary number of dimensions d in a fairly straightforward manner. In most cases, the only necessary changes are to define the parameter δ as d/α and to replace the factor π in the spatial contention by the volume of the d-dimensional unit ball. The condition for finite interference is still $\delta < 1$.

B. Interference only

In the noise-free case, we observe the following:

1) None of the delays depend on the node density. The increased interference in a network of higher density is exactly offset by the decreased transmission distance.
2) In the highly mobile cases, the local delay is finite for all values of the SIR threshold θ and the access probability p. It decomposes into a sum of access delay and service time. The optimum transmit probability is proportional to $\gamma^{-1/2}$ or $\theta^{-\delta/2}$ in the NRT, NNT, and NNR cases.
3) In the static cases, there is a phase transition, *i.e.*, the local delay becomes infinite if p or θ exceeds a certain critical transmit probability. This is a consequence of the correlation of the interference, which leads to a heavy tail in the delay distribution. In the NRT, NNT, and NNR cases, the optimum transmit probability $p_{\rm opt}$ is roughly half the critical transmit probability. As $\theta \to \infty$, it approaches $\pi/(2\gamma)$ in all three cases, hence it is proportional to γ^{-1} or $\theta^{-\delta}$. So $p_{\rm opt}$ decreases quadratically faster in the static case than in the highly mobile case, which is due to the smaller diversity in the static case that needs to be compensated for by a smaller transmit probability[1].
4) We focused on the two extreme cases of mobility. Any practical level of mobility will fall in between, so we can expect that the results obtained are upper and lower bounds for all levels of mobility. At low rates, both extremes behave very similarly, so any Poisson network with finite mobility exhibits the same scaling behavior.
5) The NRT and NTR cases benefit from the fact that the destination node is known to be listening (NRT) or the

[1]As shown in [12], the correlation coefficient of the interference in static networks is proportional to p.

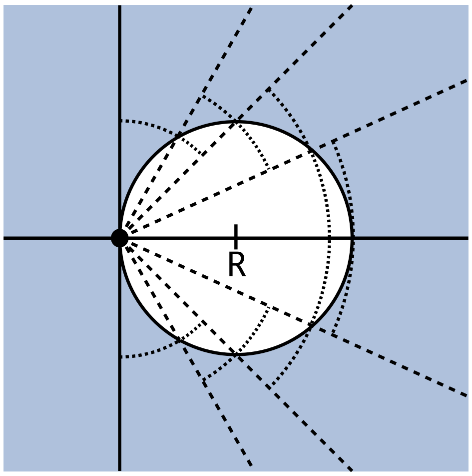


Fig. 11. Illustration for the proof of Thm. 1. The white disk is the interference-free region of nearest-neighbor transmission (NNT). The receiver is located at the origin, and the four sectors (per quadrant) indicate the inner and outer bounds on the disk that are used to obtain analytical bounds on the spatial contention. The angles of the radial beams are $\pi/6$, $\pi/4$. and $\pi/3$.

source node is known to be transmitting (NTR). Hence the minimum delay as $\theta \to 0$ is $K = 1$ for NRT and NTR, while it is $K = 4$ for NNT and NNR.

6) The NTR, NNT, and NNR cases benefit from an interferer-free disk centered at the transmitter (NNT) or centered at the receiver (NTR and NNR). Asymptotically as $\theta \to 0$, this manifests itself in a larger exponent ν in the delay expression $K + \theta^\nu$ (Table I).

C. Noise only

If power control of the form $P = aR^{\alpha-2+b}$ is used, the local delay for Rayleigh link distances and Rayleigh fading is finite for $b = 0$ and some conditions on θ, a, and λ, and it is always finite for $b > 0$. For $b = 0$, a similar condition holds for Nakagami fading. If power control is randomized with a distribution with polynomial tail, the local delay is finite even for $b = 0$. So, induced fading can greatly increase the stability region. On the other hand, with a peak power constraint, there is no power control scheme that can keep the local delay finite as soon as $\alpha > 2$.

Extensions from nearest-neighbor communication to n-th nearest neighbor communication are possible in a fairly straightforward manner.

D. Interference and noise

Power control is needed to overcome the noise, which complicates the analysis since it affects the interference distribution. In static networks, a further difficulty is that only the fading states vary in an iid fashion, whereas power control is static over time, as the distance to the nearest neighbor stays constant. (The situation would change if information on the channel state was also available at the transmitter.) We resorted to deriving a reasonably tight lower bound by replacing the interferers' actual powers by their averages and invoking Jensen's inequality.

Appendix
Proof of Theorem 1

Proof: (a) By stationarity of Φ, the situation is statistically the same if the transmitter is located at $(R, 0)$ and its nearest neighbor at o, as shown in Fig. 11, with the receiver at the origin. Hence we can apply Lemma 1 with $\mathcal{H} = B_{(R,0)}(R)$. The integral in (6) is

$$\int_{\mathbb{R}^2\setminus\mathcal{H}} \frac{s}{s + \|x\|^\alpha} \mathrm{d}x = C(\alpha)s^{2/\alpha} - A(R, s)\,,$$

where $A(R, s)$ is the integral over the interferer-free region $\mathcal{H}$. In polar coordinates,

$$A(R, s) = \int_{-\pi/2}^{\pi/2} \int_0^{2R\cos\phi} \frac{rs}{r^\alpha + s} \mathrm{d}r\mathrm{d}\phi\,. \qquad (65)$$

First, we note that $A(R, R^\alpha\theta) \propto R^2$, and if the proportionality constant is $\gamma - \gamma^{\mathrm{NNT}}$, the success probability is indeed of the form (10). Next, $A(R, 0) = 0$, and for fixed R, $A(R, s)$ is monotonically increasing to πR^2 as $s \to \infty$. Using this upper bound in the conditional success probability yields

$$\begin{aligned}\mathbb{P}^o(\mathcal{C} \mid R) &= pq\mathcal{L}_I(\theta R^\alpha \mid B_{(R,0)}(R) \cap \Phi = \emptyset)\\ &< pq\exp(-p\lambda R^2(\gamma - \pi))\,,\end{aligned}$$

which proves the first bound. (b) For the tighter bounds, we use

$$\int_0^b \frac{rs}{r^\alpha + s}\mathrm{d}r = \frac{s^\delta}{\alpha}\int_{sb^{-\alpha}}^\infty \frac{\mathrm{d}y}{y^\delta(1+y)} = \frac{b^2}{2}H_\delta(-b^\alpha/s) \qquad (66)$$

to bound the integral (65) by integrating over four sectors with fixed radius. The four sectors are $|\phi| \leq \pi/6, \pi/6 < |\phi| \leq \pi/4$, $\pi/4 < |\phi| \leq \pi/3$, and $\pi/3 < |\phi| \leq \pi/2$, see Fig. 11. At the angles bordering these sectors, $\cos\phi$ assumes the simple values $\sqrt{n}/2$, $n = 0, 1, 2, 3$. For the lower (inner) bound, only three sectors are used. For the sector $|\phi| \leq \pi/6$, letting

$$A_1 = \int_{-\pi/6}^{\pi/6} \int_0^{2R\cos\phi} \frac{rs}{r^\alpha + s}\mathrm{d}r\mathrm{d}\phi\,,$$

we obtain

$$\frac{\pi}{3}\int_0^{2R\cos(\pi/6)} \frac{rs}{r^\alpha + s}\mathrm{d}r < A_1 < \frac{\pi}{3}\int_0^{2R} \frac{rs}{r^\alpha + s}\mathrm{d}r\,.$$

These bounds can be expressed using the hypergeometric function, and the bounds for the other sectors follow analogously. The result is obtained by the substitution $s = \theta R^\alpha$. (The first terms in (11) and (12) are the ones pertaining to the A_1 term calculated here.) (c) For the third bound, we use a series expansion of the hypergeometric function at $x = 0$:

$$H_\delta(x) = \sum_{k=0}^\infty \frac{2}{2 + k\alpha}x^k = \sum_{k=0}^\infty \frac{\delta}{\delta + k}x^k\,. \qquad (67)$$

Truncated at $k = 1$, we obtain the bound

$$H_\delta(-t/\theta) \geq 1 - \frac{2t}{2+\alpha} \cdot \frac{1}{\theta}\,, \quad t \geq 0\,, \qquad (68)$$

which, when used in (12), yields the result. ∎

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