

Outage, Local Throughput, and Capacity of Random Wireless Networks

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Abstract—Outage probabilities and single-hop throughput are two important performance metrics that have been evaluated for certain specific types of wireless networks. However, there is a lack of comprehensive results for larger classes of networks, and there is no systematic approach that permits the convenient comparison of the performance of networks with different geometries and levels of randomness.

The *uncertainty cube* is introduced to categorize the uncertainty present in a network. The three axes of the cube represent the three main potential sources of uncertainty in interference-limited networks: the node distribution, the channel gains (fading), and the channel access scheme (set of transmitting nodes). For the performance analysis, a new parameter, the so-called *spatial contention*, is defined. It measures the slope of the outage probability in an ALOHA network as a function of the transmit probability p at $p = 0$. Outage is defined as the event that the signal-to-interference ratio (SIR) is below a certain threshold in a given time slot. It is shown that the spatial contention is sufficient to characterize outage and throughput in large classes of wireless networks, corresponding to different positions on the uncertainty cube. Existing results are placed in this framework, and new ones are derived.

Further, interpreting the outage probability as the SIR distribution, the ergodic capacity of unit-distance links is determined and compared to the throughput achievable for fixed (yet optimized) transmission rates.

Index Terms—Wireless networks, interference, channel access protocols, outage, capacity.

I. INTRODUCTION

A. Background

IN many large wireless networks, the achievable performance is limited by the interference. Since the seminal paper [1] the *scaling behavior* of the network throughput or transport capacity has been the subject of intense investigations, see, e.g., [2] and references therein. Such “order-of” results are certainly important, but they do not provide design insight when different protocols lead to the same scaling behavior. On the other hand, relatively few *quantitative* results on outage and local (per-link) throughput are available. While such results provide only a microscopic view of the network, we can expect concrete performance measures that permit, for example, the fine-tuning of channel access probabilities or transmission rates.

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TABLE I
SPECIFICATION OF THE UNCERTAINTY CUBE.

Node location	$u_l = 0$ $u_l = 1$	Deterministic node placement Poisson point process
Channel (fading)	$u_f = 0$ $u_f = 1$	No fading Rayleigh (block) fading
Channel access	$u_a = 0$ $u_a = 1$	TDMA slotted ALOHA

B. Contributions

We introduce the concept of the *uncertainty cube* to illustrate the difference sources of uncertainty in a network and a new parameter, the *spatial contention*, as a general metric that captures the network’s capability of spatial reuse and provides a systematic approach to evaluate and compare network performance. Using this metric, we classify and extend the outage results in [3]–[6] to general stochastic wireless networks with up to three dimensions of uncertainty: node placement, channel characteristics, and channel access. Finally, we define and derive the probabilistic throughput and the ergodic capacity for different types of networks.

C. The uncertainty cube

The level of uncertainty of a network is determined by its position in the *uncertainty cube*. The three coordinates (u_l, u_f, u_a) , $0 \leq u_l, u_f, u_a \leq 1$, denote the degree of uncertainty in the node placement, the channels, and the channel access scheme, respectively. Values of 1 indicate complete uncertainty (and independence), as specified in Table I.

The value of the u_f -coordinate corresponds to the fading figure (amount of fading). For the Nakagami- m fading model, for example, we may define $u_f \triangleq 1/m$. A network with $(u_l, u_f, u_a) = (1, 1, 1)$ has its nodes distributed according to a Poisson point process (PPP), all channels are Rayleigh (block) fading, and the channel access scheme is slotted ALOHA. The other extreme is the $(0, 0, 0)$ network where the node positions are deterministic, there is no fading, and there is a deterministic scheduling mechanism. Any point in the unit cube corresponds to a meaningful practical network—the three axes are independent, and they introduce an ordering in terms of the amount of uncertainty present in the network. Note that this ordering does not translate into an ordering in terms of network performance, i.e., a network with higher uncertainty may perform better. Our objective is to characterize the outage probabilities and the local throughput for the relevant corners of this uncertainty cube.

We focus on the interference-limited case, so we do not

consider noise¹. It is assumed that all nodes transmit at the same power level that can be set to 1 without loss of generality since only relative received powers matter. The performance results are also independent of the absolute scale of the network since only relative distances matter.

D. Models, notation, and definitions

Channel model. For the large-scale path loss (deterministic channel component), we assume the standard power law where the received power decays with $r^{-\alpha}$ for a path loss exponent α . For the fading model (random channel component), we consider Rayleigh and Nakagami- m fading. The case where all channels are Rayleigh is sometimes referred to as a ‘‘Rayleigh/Rayleigh’’ model; we denote this case as ‘‘1/1’’ fading. If either only the desired transmitter or the interferers are subject to fading, we speak of *partial fading*, denoted as ‘‘1/0’’ or ‘‘0/1’’ fading, respectively.

Network model. Let S denote the signal power of the desired transmitter measured at the receiver. We consider a single link of distance 1, so that $\mathbb{E}S = 1$, in a large network with $n \in \{1, 2, \dots, \infty\}$ other nodes as potential interferers. The distances to the interferers are denoted by r_i . In the case of a PPP as the node distribution, the intensity is 1. For regular line networks, the inter-node distance between nearest neighbors is 1.

Transmit probability p . In slotted ALOHA, every node transmits independently with probability p in each timeslot. Hence if the nodes form a PPP of unit intensity, the set of transmitting nodes in each timeslot forms a PPP of intensity p . The interference from node i is $I_i = B_i G_i r_i^{-\alpha}$, where B_i is iid Bernoulli with parameter p and $\mathbb{E}G_i = 1$ (fading).

Success probability p_s . Denoting the total interference power by $I = \sum_{i=1}^n I_i$, a transmission is assumed successful if the (instantaneous) SIR S/I exceeds a certain threshold θ : $p_s \triangleq \mathbb{P}(SIR > \theta)$. This is the reception probability given that the desired transmitter-receiver pair transmits and listens, respectively.

Effective distances ξ_i . The *effective distance* ξ_i of a node to the receiver is defined as $\xi_i \triangleq r_i^\alpha / \theta$.

Spatial contention γ and spatial efficiency σ . For a network using ALOHA with transmit probability p , define

$$\gamma \triangleq -\left. \frac{dp_s(p)}{dp} \right|_{p=0}, \quad (1)$$

i.e., the slope of the outage probability $1 - p_s$ at $p = 0$, as the *spatial contention* measuring how concurrent transmissions (interference) affect the success probability. γ depends on the SIR threshold θ , the geometry of the network, and the path loss exponent α . Its inverse $\sigma \triangleq 1/\gamma$ is the *spatial efficiency* which quantifies how efficiently a network uses space as a resource; a large σ indicates that the success probabilities are not significantly affected by higher transmit probabilities (higher spatial reuse).

(Local) probabilistic throughput p_T . The probabilistic throughput is the unconditioned success probability, defined

¹In the Rayleigh fading case, the outage expressions factorize into a noise part and an interference part, see (5). So, the noise term is simply a multiplicative factor to the interference-limited success probability.

to be the success probability multiplied by the probability that the transmitter actually transmits and, in addition in half-duplex operation, the receiver actually listens. This is the throughput achievable with a simple ARQ scheme (with error-free feedback) [7]. For the ALOHA scheme, the half-duplex probabilistic throughput is $p_T^h \triangleq p(1-p)p_s$, and for full-duplex it is $p_T^f \triangleq p p_s$. For a TDMA line network where nodes transmit in every m -th timeslot, $p_T \triangleq p_s/m$.

Throughput T . The throughput is defined as the product of the probabilistic throughput and the rate of transmission, assuming that capacity-achieving codes are used, *i.e.*, $T \triangleq p_T \log(1 + \theta)$.

Ergodic capacity C . Finally, we define the *ergodic capacity* of the link to be $C \triangleq \mathbb{E} \log(1 + SIR)$. Multiplying C by the spatial density of transmissions yields the *spatial capacity*.

II. RELATED WORK

The study of outage and throughput performance is related to the problem of interference characterization. Important results on the interference in large wireless systems have been derived by [5], [8]–[11]. In [4], outage probabilities for cellular networks are calculated for channels with Rayleigh fading and shadowing while [3] determines outage probabilities to determine the optimum transmission range in a Poisson network. [12] combined the two approaches to determine the optimum transmission range under Rayleigh fading and shadowing. [6] provides a detailed analysis on outage probabilities and routing progress in Poisson networks with ALOHA.

For our study of $(1, 0, 1)$, $(0, 1, 1)$, and $(1, 1, 1)$ networks, we will draw on results from [3], [5], [6], [12], as discussed in the rest of this section.

A. $(1, 0, 1)$: Infinite non-fading random networks with $\alpha = 4$ and slotted ALOHA

This case is studied in [3]. The characteristic function of the interference is determined to be²

$$\mathbb{E}e^{j\omega I} = \exp(-\pi p \Gamma(1 - 2/\alpha) e^{-j\pi/\alpha} \omega^{2/\alpha}) \quad (2)$$

and, for $\alpha = 4$,

$$= \exp(-\pi \sqrt{\pi/2} (1-j)p \sqrt{\omega}). \quad (3)$$

B. $(0, 1, 1)$: Regular fading networks with $\alpha = 2$ and slotted ALOHA

In [5], the authors derive the distribution of the interference power for one- and two-dimensional Rayleigh fading networks with slotted ALOHA and $\alpha = 2$. Closed-form expressions are derived for infinite regular line networks with $r_i = i$, $i \in \mathbb{N}$. The Laplace transform of the interference is [5, Eqn. (8)]

$$\mathcal{L}_I(s) = \frac{\sinh(\pi \sqrt{s(1-p)})}{\sqrt{1-p} \sinh(\pi \sqrt{s})}. \quad (4)$$

The Laplace transforms of the interference are particularly convenient for the determination of outage probabilities in Rayleigh fading. As was noted in [4], [6], [12], the success

²Note that their notation is adapted to ours. Also, a small mistake in [3, Eqn. (18)] is corrected here.

probability p_s can be expressed as the product of the Laplace transforms of the interference and noise:

$$p_s = \int_0^\infty e^{-s\theta} d\mathbb{P}(N + I \leq s) = \mathcal{L}_I(\theta) \cdot \mathcal{L}_N(\theta). \quad (5)$$

This holds in great generality; the only condition is that the desired link is subject to Rayleigh fading.

In the interference-limited regime, the Laplace transform of the interference itself is sufficient. Otherwise an exponential factor for the noise term (assuming noise with fixed variance) needs to be added.

C. (1, 1, 1): Random fading networks with slotted ALOHA

In [6], [12], (5) was calculated for a two-dimensional random network with Rayleigh fading and ALOHA. Ignoring the noise, they obtained (see [6, Eqn. (3.4)], [12, (Eqn. (A.11))])

$$p_s = e^{-p\theta^{2/\alpha} C_2(\alpha)} \quad (6)$$

with

$$C_2(\alpha) = \pi\Gamma(1 + 2/\alpha)\Gamma(1 - 2/\alpha) = \frac{2\pi^2}{\alpha} \csc\left(\frac{2\pi}{\alpha}\right). \quad (7)$$

The subscript 2 in C_2 indicates that this is a constant for the two-dimensional case. Useful values include $C_2(3) = 4\pi^2/3\sqrt{3} \approx 7.6$ and $C_2(4) = \pi^2/2 \approx 4.9$. $C_2(2) = \infty$, so $p_s \rightarrow 0$ as $\alpha \rightarrow 2$ for any θ . The spatial contention is $\gamma = \theta^{2/\alpha} C_2(\alpha)$.

III. THE CASE OF A SINGLE INTERFERER

To start, we consider the case of a single interferer at effective distance $\xi = r^\alpha/\theta$ transmitting with probability p , which is the simplest case of a $(0, u_f, 1)$ -network. For the fading, we allow the desired channel and the interferer's channel to be fading or static. If both are Rayleigh fading (the 1/1 case), the success probability is

$$p_s^{1/1} = \mathbb{P}(\text{SIR} > \theta) = 1 - \frac{p}{1 + \xi}. \quad (8)$$

For a fading desired link and non-fading interferers (1/0 fading), $I = Br^{-\alpha}$ with B Bernoulli with parameter p and thus

$$p_s^{1/0} = \mathbb{P}(S > B/\xi) = 1 - p(1 - e^{-1/\xi}). \quad (9)$$

In the case of 0/1 fading (non-fading desired link, fading interferer),

$$p_s^{0/1} = \mathbb{P}(I < \theta^{-1}) = 1 - pe^{-\xi}. \quad (10)$$

For comparison, transmission success in the non-fading (0/0) case is guaranteed if $\xi > 1$ or the interferer does not transmit, i.e., $p_s^{0/0} = 1 - p\mathbf{1}_{\xi \leq 1}$.

Hence in all cases the outage probability $1 - p_s(p)$ is increasing linearly in p with slope γ . The values of γ are summarized in Table II.

The ordering is $\gamma^{1/0} \geq \gamma^{1/1} \geq \gamma^{0/1}$, with equality only if $\xi = 0$, corresponding to an interferer at distance 0 that causes an outage whenever it transmits, in which case all γ 's are one. The ordering follows from Jensen's inequality: $\gamma^{1/0} \geq \gamma^{1/1}$ since $\mathbb{E}(\exp(-I\theta)) \geq \exp(-\theta\mathbb{E}I)$, and $\gamma^{1/1} \geq \gamma^{0/1}$ since $\mathbb{E}(1 - \exp(-S\xi)) \leq 1 - \exp(-\xi\mathbb{E}S)$. To summarize:

TABLE II
SPATIAL CONTENTION γ IN THE SINGLE-INTERFERER CASE.

Case	Spatial contention γ
1/1	$\frac{1}{1+\xi}$
1/0	$1 - \exp(-1/\xi)$
0/1	$\exp(-\xi)$
0/0	$\mathbf{1}_{\xi \leq 1}$

Fact 1 *In the single-interferer case, fading in the desired link is harmful whereas fading in the channel from the interferer is helpful.*

We also observe that for small ξ , $\gamma^{1/1} \approx \gamma^{0/1}$, whereas for large ξ , $\gamma^{1/1} \approx \gamma^{1/0}$. So if the interferer is relatively close, it does not matter whether the desired link is fading or not, and if it is relatively far, it hardly matters whether the interferer's channel is fading.

The results can be generalized to Nakagami- m fading in a straightforward manner. If the interferer's channel is Nakagami- m fading, while the desired link is Rayleigh fading, we obtain

$$p_s^{1/m-1} = 1 - p \left(1 - \frac{m^m}{(\xi^{-1} + m)^m} \right). \quad (11)$$

Here the notation $p_s^{1/m-1}$ is chosen for consistency with the previous binary notation: For $m = 1$, we retrieve the Rayleigh fading case, whereas $m \rightarrow \infty$ corresponds to the case of no fading. As a function of m , this is decreasing for all $\theta > 0$, and in the limit converges to $p_s^{1/0}$ as $m \rightarrow \infty$ (see (9)). On the other hand, if the desired link is Nakagami- m , the success probability is

$$p_s^{m-1/1} = 1 - p \left(\frac{m\xi^{-1}}{1 + m\xi^{-1}} \right)^m \quad (12)$$

which *increases* as m increases for fixed $\theta > 0$ and approaches (10) as $m \rightarrow \infty$.

IV. NETWORKS WITH RANDOM NODE DISTRIBUTION

A. (1, 1, 1): d -dimensional fading random networks with slotted ALOHA

Evaluating (5) in the one-dimensional (and noise-free) case yields

$$p_s = \exp\left(-\int_0^\infty \frac{2p}{1+r^\alpha/\theta} dr\right) = \exp(-p\theta^{1/\alpha} C_1(\alpha)), \quad (13)$$

where $C_1(\alpha) = 2\pi \csc(\pi/\alpha)/\alpha$ for $\alpha > 1$. So the spatial contention is $\gamma = \theta^{1/\alpha} C_1(\alpha)$. Mapping a d -dimensional PPP to a one-dimensional process first (see, e.g., [13]) and then evaluating (13) yields the general result for d dimensional networks:

$$\gamma = \theta^{d/\alpha} C_d(\alpha), \quad (14)$$

where $C_d = c_d(d\pi/\alpha) \csc(d\pi/\alpha)$ and $c_d = \pi^{d/2}/\Gamma(1+d/2)$ the volume of the d -dim. unit ball. $\alpha > d$ is necessary for finite γ .

B. (1, 1, 1): Partially fading two-dimensional random networks with slotted ALOHA

If only the desired link is subject to fading (1/0 fading) and $\alpha = 4$, we can exploit (2), replacing $j\omega$ by $-\theta$, to get

$$p_s^{1/0} = \mathcal{L}_I(\theta) = e^{-p\pi\Gamma(1-2/\alpha)\theta^{2/\alpha}}. \quad (15)$$

For $\alpha = 4$, $p_s^{1/0} = \mathcal{L}_I(\theta) = e^{-p\sqrt{\theta}\pi^{3/2}}$. So $\gamma = \pi\Gamma(1-2/\alpha)\theta^{2/\alpha}$ which is larger than for the case with no fading at all. So, as in the single-interferer case, it hurts the desired link if interferers do not fade.

C. (1, 0, 1): Non-fading two-dimensional random networks with $\alpha = 4$ and slotted ALOHA

From [3, Eqn. (21)], I^{-1} has the cdf

$$F_{I^{-1}}(\theta) = \mathbb{P}(1/I < \theta) = 1 - p_s = \operatorname{erf}\left(\frac{\pi^{3/2}p\sqrt{\theta}}{2}\right), \quad (16)$$

which is the outage probability for non-fading channels for a transmitter-receiver distance 1. For the spatial contention we obtain $\gamma = \pi\sqrt{\theta}$, and it can be verified (e.g., by comparing Taylor expansions) that $1 - \gamma p < p_s(p) < \exp(-\gamma p)$ holds.

D. (1, 1, 1): Fully random two-dimensional networks with exponential path loss

To determine the effect of an exponential path loss law, as suggested in [14], on the spatial contention, we replace $r^{-\alpha}$ by $\exp(-\delta r)$, which yields

$$\begin{aligned} p_s &= \exp\left(-2\pi p \int_0^\infty \frac{r}{1 + \exp(\delta r)/\theta} dr\right) \\ &= \exp\left(-2\pi p \frac{-\operatorname{dilog}(\theta + 1)}{\delta^2}\right), \end{aligned} \quad (17)$$

where dilog is the dilogarithm function defined as $\operatorname{dilog}(x) = \int_1^x \log t/(1-t) dt$. So $\gamma = -2\pi \operatorname{dilog}(\theta + 1)/\delta^2$. The (negative) dilog function is bounded by $-\operatorname{dilog}(x) < \log(x)^2/2 + \pi^2/6$ [15], so

$$\gamma < \frac{\pi}{\delta^2} \left(\log^2(1 + \theta) + \frac{\pi^2}{3} \right), \quad (18)$$

indicating that the spatial contention grows more slowly (with $\log \theta$ instead of $\theta^{2/\alpha}$) for large θ than for the power path loss law, and finiteness of the integral is guaranteed for any $\delta > 0$, in contrast to the power law where α needs to exceed the number of network dimensions.

V. NETWORKS WITH DETERMINISTIC NODE PLACEMENT

In this section, we assume that n interferers are located at fixed distances r_i .

A. (0, 1, 1): Fading networks with slotted ALOHA

In this case, $p_s = \mathbb{P}(S \geq \theta I)$ for $I = \sum_{i=1}^n G_i r_i^{-\alpha}$ and G_i iid exponential with mean 1. For general r_i and α , we obtain from (5)

$$p_s = \mathbb{E}[e^{-\theta I}] = \prod_{i=1}^n \left(1 - \frac{p}{1 + \xi_i}\right), \quad (19)$$

where $\xi_i = r_i^\alpha/\theta$ is the effective distance. We find for the spatial contention

$$\gamma \triangleq -\left.\frac{dp_s(p)}{dp}\right|_{p=0} = \sum_{i=1}^n \frac{1}{1 + \xi_i}. \quad (20)$$

Since dp_s/dp is decreasing, $p_s(p)$ is convex, so $1 - p\gamma$ is a lower bound on the success probability. On the other hand, $e^{-p\gamma}$ is an upper bound, since

$$\log p_s = \sum_{i=1}^n \log\left(1 - \frac{p}{1 + \xi_i}\right) \approx \sum_{i=1}^n -\frac{p}{1 + \xi_i}. \quad (21)$$

The upper bound is tight for small p or ξ_i large for most i , i.e., if most interferers are far.

B. (0, 1, 1): Infinite regular line networks with fading and ALOHA

Here we specialize to the case of regular one-dimensional (line) networks, where $r_i = i$, $i \in \mathbb{N}$. For $\alpha = 2$, we obtain from (4) (or by direct calculation of (20))

$$\gamma = \frac{1}{2} \left(\pi\sqrt{\theta} \coth(\pi\sqrt{\theta}) - 1 \right). \quad (22)$$

Since $x \coth x - 1 < x < x \coth x$, this is bounded by $(\pi\sqrt{\theta} - 1)/2 < \gamma < \pi\sqrt{\theta}/2$, with the lower bound being very tight as soon as $\theta > 1$. Again the success probability is bounded by $1 - \gamma p < p_s(p) < \exp(-p\gamma)$, and both these bounds become tight as $\theta \rightarrow 0$, and the upper bound becomes tight also as $\theta \rightarrow \infty$.

For $\alpha = 4$, we first establish the analogous result to (4).

Proposition 1 (Success probability in line networks with ALOHA and $\alpha = 4$.)

For one-sided infinite regular line networks ($r_i = i$, $i \in \mathbb{N}$) with slotted ALOHA and $\alpha = 4$,

$$p_s = \frac{\cosh^2(y(1-p)^{1/4}) - \cos^2(y(1-p)^{1/4})}{\sqrt{1-p}(\cosh^2 y - \cos^2 y)} \quad (23)$$

where $y \triangleq \pi\theta^{1/4}/\sqrt{2}$.

Proof: Rewrite (19) as

$$p_s = \frac{\prod_{i=1}^n (1 + (1-p)\theta/i^4)}{\prod_{i=1}^n (1 + \theta/i^4)}. \quad (24)$$

The factorization of both numerator and denominator according to $(1 - z^4/i^4) = (1 - z^2/i^2)(1 + z^2/i^2)$ permits the use of Euler's product formula $\sin(\pi z) \equiv \pi z \prod_{i=1}^{\infty} (1 - z^2/i^2)$ with $z = \sqrt{\pm j}((1-p)\theta)^{1/4}$ (numerator) and $z = \sqrt{\pm j}\theta^{1/4}$ (denominator). The two resulting expressions are complex conjugates, and $|\sin(\sqrt{j}x)|^2 = \cosh^2(x/\sqrt{2}) - \cos^2(x/\sqrt{2})$. ■

The spatial contention is

$$\gamma = \frac{1}{8} \frac{(y-1)e^{2y} + 4\cos^2 y + 4y \cos y \sin y - 2 - (y+1)e^{-2y}}{\cosh^2 y - \cos^2 y}. \quad (25)$$

For $y \gtrsim 2$, the e^{2y} (numerator) and $\cosh^2 y$ (denominator) terms dominate, so $\gamma \approx (y-1)/2$ for $y > 2$. In terms of θ , this implies that

$$\gamma \approx \pi\theta^{1/4}/(2\sqrt{2}) - 1/2, \quad (26)$$

which is quite accurate as soon as $\theta > 1$. The corresponding approximation

$$p_s \approx e^{-p(\pi\theta^{1/4}/(2\sqrt{2})-1/2)}. \quad (27)$$

can be derived from (23) noting that for y not too small and p not too close to 1, the cosh terms dominate the cos terms and $\cosh^2(x) \approx e^{2x}/4$, $1 - (1-p)^{1/4} \approx p/4$, and $(1-p)^{-1/2} \approx e^{p/2}$. For two-dimensional networks, γ simply doubles, as is apparent from (20), and the success probability is squared.

For general α , the bound $\gamma < \zeta(\alpha)\theta$, where ζ is the Riemann zeta function, is obtained by noting that the sum (20) would be $\zeta(\alpha)\theta$ if the 1 in the denominator was not present. This bound is only tight if $\theta < 1$.

C. (0, 1, 1): Partially fading regular networks

If only the desired link is subject to fading, the success probability is given by

$$p_s = \exp\left(-p\theta \sum_{i=1}^n r_i^{-\alpha}\right), \quad (28)$$

thus $\gamma = \sum_{i=1}^n 1/\xi_i$. Compared with (20), $1 + \xi$ is replaced by ξ . For regular line networks $\xi_i = i^\alpha/\theta$, so $\gamma = \theta\zeta(\alpha)$ and $p_s = e^{-p\theta\zeta(\alpha)}$. So the spatial contention is larger than in the case of full fading, *i.e.*, fading in the interferers' channels helps, as in the single-interferer case. For $\theta > 1$, the difference becomes significant.

D. (0, 1, 0): Regular line networks with fading and TDMA

If, in a TDMA scheme, only every m -th node transmits, the relative distances of the interferers are increased by a factor of m . Fig. 1 shows a two-sided regular line network with $m = 2$. Since $(mr)^\alpha/\theta = r^\alpha/(\theta m^{-\alpha})$, having every m -th node transmit is equivalent to reducing the threshold θ by a factor m^α and setting $p = 1$.

Proposition 2 (Success probability for TDMA line networks for $\alpha = 2, 4$)

The success probability for one-sided infinite regular line networks with Rayleigh fading and m -phase TDMA is: For $\alpha = 2$:

$$p_s = \frac{y}{\sinh y}, \quad \text{where } y \triangleq \frac{\pi\sqrt{\theta}}{m}, \quad (29)$$

and for $\alpha = 4$:

$$p_s = \frac{2y^2}{\cosh^2 y - \cos^2 y}, \quad \text{where } y \triangleq \frac{\pi\theta^{1/4}}{\sqrt{2}m}. \quad (30)$$

Proof: Apply L'Hôpital's rule for $p = 1$ in (4) and (23) (for $\alpha = 2, 4$, respectively) and replace θ by $\theta m^{-\alpha}$. ■

The following proposition establishes sharp bounds for arbitrary α .

Proposition 3 (Bounds on the success probability for TDMA line networks)

The success probability for one-sided infinite regular line networks, Rayleigh fading, and m -phase TDMA is bounded by

$$e^{-\zeta(\alpha)\theta/m^\alpha} \lesssim p_s \lesssim \frac{1}{1 + \zeta(\alpha)\frac{\theta}{m^\alpha}}. \quad (31)$$

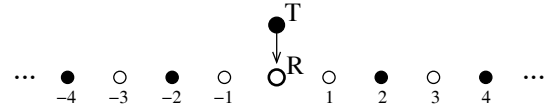


Fig. 1. Two-sided regular line network with TDMA with $m = 2$, *i.e.*, every second node transmits. The filled circles indicate the transmitters. The transmitter denoted by T is the intended transmitter, the others are interferers. The receiver at the origin, denoted by R, is the intended receiver. In the one-sided case, the nodes at positions $x < 0$ do not exist.

A tighter upper bound is

$$p_s \lesssim \frac{1}{1 + \zeta(\alpha)\frac{\theta}{m^\alpha} + (\zeta(\alpha) - 1)\frac{\theta^2}{m^{2\alpha}}}. \quad (32)$$

Proof: Upper bound: We only need to prove the tighter bound. Let $\theta' \triangleq \theta/m^\alpha$. The expansion of the product (19), $p_s^{-1} = \prod_{i=1}^{\infty} 1 + \theta'/i^\alpha$, ordered according to powers of θ' , has only positive terms and starts with $1 + \theta'\zeta(\alpha) + \theta'^2(\zeta(\alpha) - 1)$. There are more terms with θ'^2 , but their coefficients are relatively small, so the bound is tight. The lower bound is a special case of (21). ■

Note that all bounds approach the exact p_s as θ/m^α decreases. Interestingly, for $\alpha = 2, 4$, the upper bound (31) corresponds exactly to the expressions obtained when the denominators in (29) and (30) are replaced by their Taylor expansions of order 2α . Higher-order Taylor expansions, however, deviate from the tighter bound (32).

In order to use the spatial contention framework for TDMA networks, Let $\tilde{p} \triangleq 1/m$ be the fraction of time a node transmits. Now $dp_s/d\tilde{p}|_{\tilde{p}=0} = 0$ since p_s depends on m^α rather than m itself. So for TDMA, we define

$$\gamma_{\text{TDMA}} \triangleq -\left.\frac{dp_s}{d(\tilde{p}^\alpha)}\right|_{\tilde{p}=0}. \quad (33)$$

Since both bounds in (31) yield the same slope, the spatial contention for the (0, 1, 0) line network is $\gamma_{\text{TDMA}} = \zeta(\alpha)\theta$, which is identical to the spatial contention of the ALOHA line network with non-fading interferers.

Table III summarizes the results on the spatial contention established in this section.

VI. LOCAL THROUGHPUT AND CAPACITY

A. $(u_t, u_f, 1)$: Networks with slotted ALOHA

For networks with slotted ALOHA, define the *probabilistic throughput* as

$$\begin{aligned} \text{full-duplex: } p_T^f &\triangleq p p_s(p) \\ \text{half-duplex: } p_T^h &\triangleq p(1-p) p_s(p). \end{aligned}$$

Proposition 4 (Maximum probabilistic throughput in ALOHA networks with fading)

Consider a network with ALOHA and Rayleigh fading with spatial contention γ such that $p_s = e^{-p\gamma}$. Then in the full-duplex case

$$p_{\text{opt}} = 1/\gamma; \quad p_{T \text{ max}}^f = \frac{1}{e\gamma} \quad (34)$$

and in the half-duplex case

$$p_{\text{opt}} = \frac{1}{\gamma} + \frac{1}{2} \left(1 - \sqrt{1 + \frac{4}{\gamma^2}}\right) \quad (35)$$

TABLE III
SPATIAL CONTENTION PARAMETERS FOR DIFFERENT TYPES OF SLOTTED ALOHA NETWORKS. FOR COMPARISON, THE TDMA CASE IS ADDED. "REGULAR NETWORK" REFERS TO AN INFINITE LINE NETWORK WITH UNIT NODE SPACING.

Uncertainty	Spatial contention γ	Eqn.	#dim.	Remark
(1, 1, 1)	$(d/\alpha)\pi^{1+d/2}\theta^{d/\alpha} \text{csc}(d\pi/\alpha)/\Gamma(1+d/2)$	(14)	d	General result
	$\pi^2\sqrt{\theta}/2$	(6)	2	Special case for $d=2, \alpha=4$
	$\pi\Gamma(1-2/\alpha)\theta^{2/\alpha}$	(15)	2	Non-fading interferers
(1, 0, 1)	$\pi\sqrt{\theta}$	(16)	2	No fading, $\alpha=4$
(0, 1, 1)	$\sum_{i=1}^n 1/(1+\xi_i)$	(20)	d	n nodes
	$\pi\sqrt{\theta} \coth(\pi\sqrt{\theta})/2 - 1/2$	(22)	1	Regular one-sided network, $\alpha=2$
	$\approx \pi\theta^{1/4}/(2\sqrt{2}) - 1/2$	(26)	1	Regular one-sided network, $\alpha=4$
	$\sum_{i=1}^n 1/\xi_i$	(28)	d	Non-fading interferers
	$\theta\zeta(\alpha)$	(28)	1	Regular network, non-fading interf.
(0, 1, 0)	$\gamma_{\text{TDMA}} = \theta\zeta(\alpha)$	(31)	1	Regular one-sided network

and

$$p_{T \max}^h \gtrsim \frac{1+\gamma}{(2+\gamma)^2} \exp\left(-\frac{\gamma}{2+\gamma}\right), \quad (36)$$

Proof: Full-duplex: $p_{\text{opt}} = 1/\gamma$ maximizes $p \exp(-p\gamma)$. Half-duplex: Maximizing $\log p_T^h(p)$ yields the quadratic equation $p_{\text{opt}}^2 - p_{\text{opt}}(1+2\sigma) + \sigma = 0$ whose solution is (35). Any approximation of p_{opt} yields a lower bound on p_T^h . Since $p_{\text{opt}}(0) = 1/2$, and $p_{\text{opt}} = \Theta(\gamma^{-1})$ for $\gamma \rightarrow \infty$, a simple yet accurate choice is $p_{\text{opt}} \gtrsim 1/(2+\gamma)$ which results in the bound in the proposition. ■

Numerical calculations show that the lower bound (36) is within 1.4% of the true maximum over the whole range $\gamma \in \mathbb{R}^+$.

B. (0, 1, 0): Two-sided regular line networks with TDMA

Here we consider a two-sided infinite regular line network with m -phase TDMA (see Fig. 1). To maximize the throughput $p_T \triangleq p_s/m$, we use the bounds (31) for p_s . Since the network is now two-sided, the expressions need to be squared. Let $\hat{m}_{\text{opt}} \in \mathbb{R}$ and $\hat{m}_{\text{opt}} \in \mathbb{N}$ be estimates for the true $m_{\text{opt}} \in \mathbb{N}$. We find

$$(\theta\zeta(\alpha)(2\alpha-1))^{1/\alpha} < \hat{m}_{\text{opt}} < (\theta\zeta(\alpha)2\alpha)^{1/\alpha}, \quad (37)$$

where the lower and upper bounds stem from maximizing the upper and lower bounds in (31), respectively. The factor 2 in 2α indicates that the network is two-sided. Rounding the average of the two bounds to the nearest integer yields a good estimate for m_{opt} :

$$\hat{m}_{\text{opt}} = \lceil (\theta\zeta(\alpha)(2\alpha-1/2))^{1/\alpha} \rceil \quad (38)$$

Fig. 2(a) shows the bounds (37), \hat{m}_{opt} , and the true m_{opt} (found numerically) for $\alpha=2$ as a function of θ . For most values of θ , $\hat{m}_{\text{opt}} = m_{\text{opt}}$. The resulting difference in the maximum achievable throughput $p_{T \max}$ is negligibly small. We can obtain estimates on the success probability p_s by inserting (37) into (31):

$$\left(1 - \frac{1}{2\alpha}\right)^2 \approx p_s \approx e^{-1/\alpha}. \quad (39)$$

In Fig. 2(b), the actual $p_s(\theta)$ is shown with the two approximations for $\alpha=2$. Since m_{opt} is increasing with θ , the relative

error $|\hat{m}_{\text{opt}} - m_{\text{opt}}|/m_{\text{opt}} \rightarrow 0$, so we expect $\lim_{\theta \rightarrow \infty} p_s(\theta)$ to lie between the approximations (39).

C. Rate optimization

So far we have assumed that the SIR threshold θ is fixed and given. Here we address the problem of finding the optimum θ , i.e., the optimum rate of transmission for networks for which $\gamma \propto \theta^{d/\alpha}$, where $d=1,2$ indicates the number of network dimensions. We define the throughput as the product of the probabilistic throughput p_T and the (normalized) rate of transmission $\log(1+\theta)$ (in nats/s/Hz). As before, we distinguish the cases of half-duplex and full-duplex operation, i.e., we maximize $p_T^f(\theta) \log(1+\theta)$ (full-duplex) or $p_T^h(\theta) \log(1+\theta)$ (half-duplex), respectively.

Proposition 5 (Optimum SIR threshold for full-duplex operation)

The throughput $T = p \exp(-p\gamma) \log(1+\theta)$ is maximized at the SIR threshold

$$\theta_{\text{opt}} = \exp\left(\mathcal{W}\left(-\frac{\alpha}{d}e^{-\alpha/d}\right) + \frac{\alpha}{d}\right) - 1, \quad (40)$$

where \mathcal{W} is the principal branch of the Lambert W function and $d=1,2$ is the number of network dimensions.

Proof: Given γ , the optimum p is $1/\gamma$. With $\gamma = c\theta^{d/\alpha}$, we need to maximize

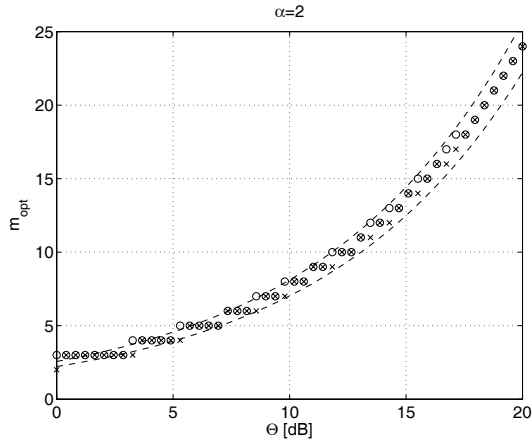
$$T(\alpha, \theta) = \frac{1}{c\theta^{d/\alpha}} \log(1+\theta), \quad (41)$$

where $d=1,2$ is the number of dimensions. Solving $\partial T/\partial \theta = 0$ yields (40). ■

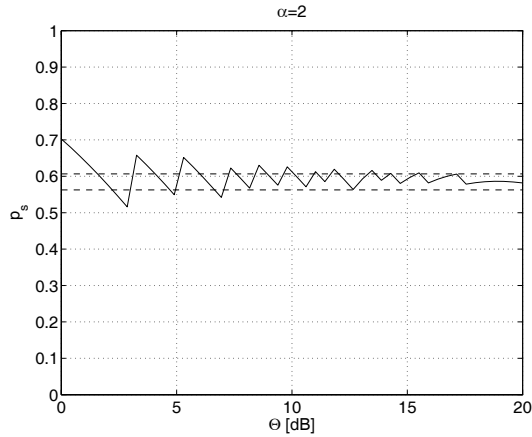
Remark. θ_{opt} in the two-dimensional case for a path loss exponent α equals θ_{opt} in the one-dimensional case for a path loss exponent $\alpha/2$. In the two-dimensional case, the optimum threshold is smaller than one for $\alpha < 4 \log 2 \approx 2.77$.

The optimum (normalized) transmission rate (in nats/s/Hz) is

$$R_{\text{opt}}(\alpha) = \log(1+\theta_{\text{opt}}) = \mathcal{W}\left(-\frac{\alpha}{d}e^{-\alpha/d}\right) + \frac{\alpha}{d}, \quad d=1,2. \quad (42)$$



(a) Optimum TDMA parameter m as a function of θ [dB] for $\alpha = 2$. The dashed lines show the bounds (37), the circles indicate the true optimum m_{opt} , the crosses the estimate \hat{m}_{opt} in (38)



(b) p_s for the optimum m as a function of θ [dB] for $\alpha = 2$. The dashed lines show the approximations (39), the solid line the actual value obtained numerically.

Fig. 2. Optimum TDMA parameter and resulting success probability.

$R_{\text{opt}}(\alpha)$ is concave for $\alpha > d$, and the derivative at $\alpha = d$ is 2 for $d = 1$ and 1 for $d = 2$. So we have $R_{\text{opt}}(\alpha) < \alpha - 2$ for $d = 2$ and $R_{\text{opt}}(\alpha) < 2(\alpha - 1)$ for $d = 1$.

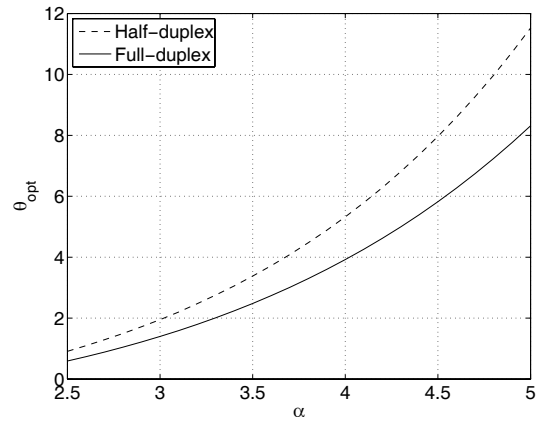
In the half-duplex case, closed-form solutions are not available. The results of the numerical throughput maximization are shown in Fig. 3, together with the results for the full-duplex case. As can be seen, the maximum throughput scales almost linearly with $\alpha - d$. The optimum transmit probabilities do not depend strongly on α and are around 0.105 for full-duplex operation and 0.08 for half-duplex operation. The achievable throughput for full-duplex operation is quite exactly 10% higher than for half-duplex operation, over the entire practical range of α .

D. (1, 1, 1): Ergodic capacity

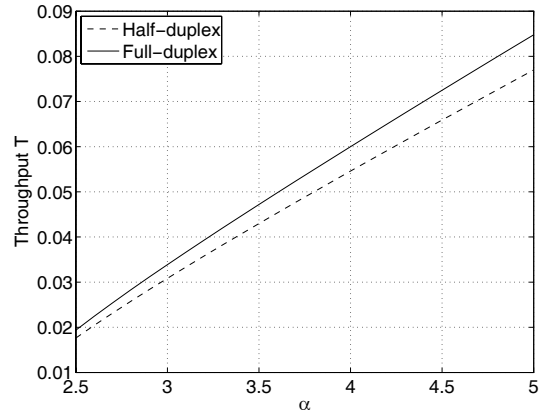
Based on our definitions, the ergodic capacity can be generally expressed as

$$C = \mathbb{E} \log(1 + \text{SIR}) = \int_0^\infty -\log(1 + \theta) dp_s, \quad (43)$$

where $p_s(\theta)$ is the ccdf of the SIR.



(a) Optimum threshold θ_{opt} for full- and half-duplex operation as a function of α .



(b) Maximum throughput as a function of α .

Fig. 3. Optimum threshold θ_{opt} and maximum throughput in two-dimensional network.

Proposition 6 (Ergodic capacity for (1, 1, 1) networks)

Let C be the ergodic capacity of a link in a two-dimensional (1, 1, 1) network with transmit probability p . For $\alpha = 4$,

$$C = 2\Re\{q\} \cos(c_p) - 2\Im\{q\} \sin(c_p), \quad q \triangleq \text{Ei}(1, jc_p), \quad (44)$$

where $c_p = pC_2(\alpha)$ and $\text{Ei}(1, z) = \int_1^\infty \exp(-xz)x^{-1}dx$ is the exponential integral. For general $\alpha > 2$, C is lower bounded as

$$C > \log 2 \cdot \left(c_p^{-\alpha/2} \Gamma(1 + \alpha/2, c_p) + \left(\frac{\alpha}{4} - 1 \right) \exp(-\sqrt{2}c_p) + \exp(-c_p) \right) + \frac{\alpha}{2} \text{Ei}(\sqrt{2}c_p), \quad (45)$$

where $\Gamma(a, x) = \int_0^x t^{a-1} \exp(-t)dt$ is the lower incomplete gamma function.

The one-dimensional network with path loss exponent α (and $c_p = pC_1(\alpha)$) has the same capacity as the two-dimensional network with path loss exponent 2α .

Proof: Let $c_p \triangleq p\gamma\theta^{-2/\alpha} = pC_2(\alpha)$. We have

$$C = \frac{2c_p}{\alpha} \int_0^\infty \log(1 + \theta) \theta^{2/\alpha - 1} \exp(-c_p \theta^{2/\alpha}) d\theta \quad (46)$$

$$= c_p \int_0^\infty \log\left(1 + t^{\alpha/2}\right) \exp(-c_p t) dt. \quad (47)$$

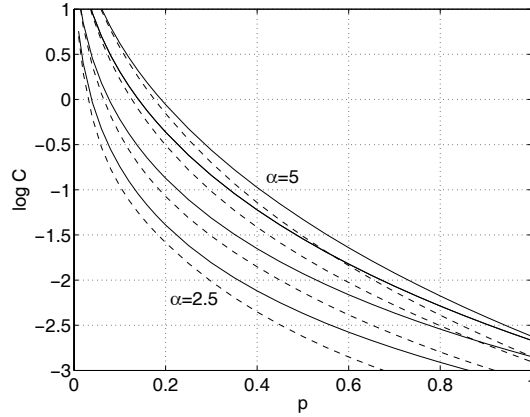


Fig. 4. Ergodic capacity for a two-dimensional fading network with ALOHA for $\alpha = 2.5, 3, 4, 5$ as a function of p . The solid lines are the actual capacities (47), the dashed lines the lower bounds (45).

So, the $2/\alpha$ -th moment of the SIR is exponentially distributed with mean $1/c_p$. As a consequence, the capacity of the ALOHA channel is the capacity of a Rayleigh fading channel with mean SIR c_p^{-1} with an ‘‘SIR boost’’ exponent of $\alpha/2 > 1$. Note that since a significant part of the probability mass may be located in the interval $0 \leq \theta < 1$, this does not mean that the capacity is larger than for the standard Rayleigh case. This is only true if the SIR is high on average.

For general p and α , the integral does not have a closed-form expression. For $\alpha = 4$, direct calculation of (47) yields

$$C = \exp(-jc_p) \text{Ei}(1, jc_p) + \exp(-jc_p) \text{Ei}(1, -jc_p), \quad (48)$$

which equals (44). To find an analytical lower bound, rewrite (47) as (by substituting $t \leftarrow t^{-1}$)

$$C = c_p \int_0^\infty \frac{\log(1 + t^{-\alpha/2}) \exp(-c_p/t)}{t^2} dt \quad (49)$$

and lower bound $\log(1 + t^{-\alpha/2})$ by $L(t)$ given by

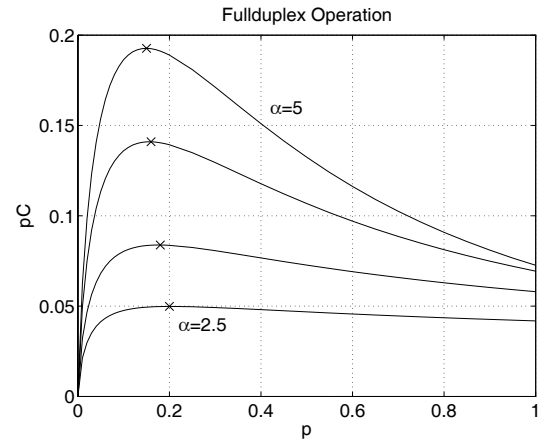
$$L(t) = \begin{cases} -\frac{\alpha}{2} \log t & \text{for } 0 \leq t < \sqrt{2}/2 \\ \log 2 & \text{for } \sqrt{2}/2 \leq t < 1 \\ \log(2)t^{-\alpha/2} & \text{for } 1 \leq t. \end{cases} \quad (50)$$

This yields the lower bound (45). ■

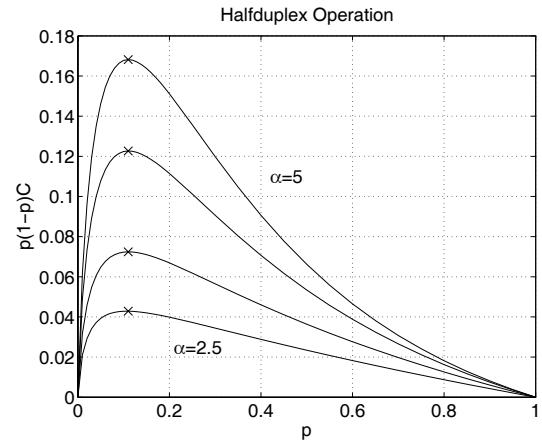
For rational values of α , pseudo-closed-form expressions are available using the Meijer G function. Fig. 4 displays the capacities and lower bounds for $\alpha = 2.5, 3, 4, 5$. For small c_p (high SIR on average), a simpler bound is

$$C > \int_1^\infty -\log(\theta) dp_s = \frac{\alpha}{2} \text{Ei}(1, pC(\alpha)), \quad (51)$$

To obtain the *spatial capacity*, the ergodic capacity needs to be multiplied by the probability (density) of transmission. It is expected that there exists an optimum p maximizing the product pC in the case of full-duplex operation or $p(1-p)C$ in the case of half-duplex operation. The corresponding curves are shown in Fig. 5. Interestingly, in the full-duplex case, the optimum p is *decreasing* with increasing α . In the half-duplex case, $p_{\text{opt}} \approx 1/9$ quite exactly — independent of α .



(a) Full-duplex



(b) Half-duplex

Fig. 5. Spatial capacities for $\alpha = 2.5, 3, 4, 5$ as a function of p . The star marks the optimum p .

E. TDMA line networks

Proposition 7 (Ergodic capacity bounds for TDMA line networks)

For $\alpha = 2$,

$$2 \log\left(\frac{2m}{\pi}\right) < C < \log\left(1 + \frac{7\zeta(3)}{\pi^2} m^2\right) \quad (52)$$

and

$$\mathbb{E}\sqrt{\text{SIR}} = \frac{\pi}{4} m; \quad \mathbb{E}\text{SIR} = \frac{7\zeta(3)}{\pi^2} m^2. \quad (53)$$

For general $\alpha > 1$,

$$C > e^{\zeta(\alpha)/m^\alpha} \text{Ei}(1, \zeta(\alpha)/m^\alpha) \quad (54)$$

and

$$\mathbb{E}\text{SIR} > \frac{1}{\zeta(\alpha)} m^\alpha. \quad (55)$$

Proof: $\alpha = 2$: Using (43) and (29) and substituting $t \leftarrow \pi\sqrt{\theta}/m$ yields

$$C = \int_0^\infty \log\left(1 + \left(\frac{mt}{\pi}\right)^2\right) \frac{t \cosh t - \sinh t}{\sinh^2 t} dt \quad (56)$$

Replacing $\log(1+x)$ by $\log x$ results in the lower bound which gets tighter as m increases. It also follows that $\pi\sqrt{\text{SIR}}/m$ is

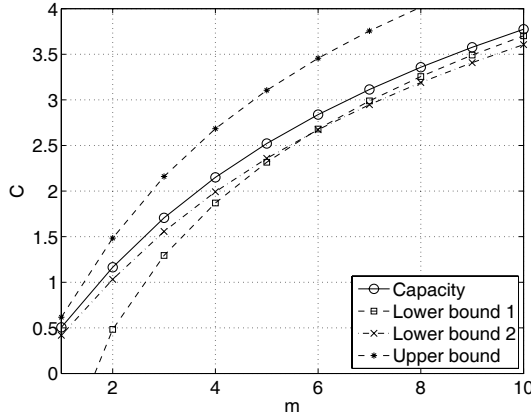


Fig. 6. Ergodic capacity for TDMA line network for $\alpha = 2$ as a function of the reuse parameter m . The solid line is the actual capacity (47), lower bound 1 and the upper bound are from (52), and lower bound 2 is (54).

distributed as

$$\mathbb{P}(\pi\sqrt{\text{SIR}}/m < t) = \frac{e^{2t} - 2te^t - 1}{e^{2t} - 1} \quad (57)$$

from which the moments of the SIR follow. The upper bound in (52) stems from Jensen's inequality. The result for general α follows from the lower bound (31) on p_s . ■

Fig. 6 shows the ergodic capacity for the TDMA line network for $\alpha = 2$, together with the lower bounds (52) and (54) and the upper bound from (52). As can be seen, the lower bound specific to $\alpha = 2$ gets tighter for larger m . Using the lower bound (55) on the SIR together with Jensen's inequality results in a very good approximation $C \approx \log(1 + m^\alpha/\zeta(\alpha))$.

From the slope of $C(m)$ it can be seen that the optimum spatial reuse factor $m = 2$ maximizes the spatial capacity C/m for $\alpha = 2$. For $\alpha = 4$, $m = 3$ yields a slightly higher C/m . This is in agreement with the observation made in Fig. 5 (left) that in ALOHA p_{opt} slightly decreases as α increases.

VII. CONCLUSIONS

We have introduced the uncertainty cube to classify wireless networks according to their underlying stochastic processes. For large classes of networks, the outage probability $\mathbb{P}(\text{SIR} < \theta)$ of a unit-distance link is determined by the spatial contention γ . Summarizing the outage results:

- For $(1, u_f, 1)$ networks (PPP networks with ALOHA), $\gamma \propto \theta^{d/\alpha}$. With Rayleigh fading, $p_s = \exp(-p\gamma)$, otherwise $p_s \leq \exp(-p\gamma)$.
- For two-dimensional regular line networks with ALOHA (a class of $(0, 1, 1)$ networks), $\gamma \approx c\theta^{d/\alpha} - 1$. So, the regularity is reflected in the shift in γ by 1, i.e., γ becomes affine in $\theta^{d/\alpha}$ rather than linear. The difference or ratio between the spatial contention values can be used to define a *regularity gain*, i.e., the gain in spatial reuse that is obtained by placing nodes more regularly.
- Quite generally, with the exception of deterministic networks without fading interferers, γ is a function of θ only through $\theta^{d/\alpha}$ (see Table III).
- For regular line networks with m -phase TDMA, $p_s \approx \exp(-\tilde{p}^\alpha \zeta(\alpha)\theta)$, where $\tilde{p} = 1/m$. So the increased effi-

ciency of TDMA scheduling in line networks is reflected in the exponent α of \tilde{p} .

The following interpretations of $\gamma = \sigma^{-1}$ demonstrate the fundamental nature of this parameter:

- γ determines how fast $p_s(p)$ decays as p increases from 0: $\partial p_s / \partial p|_{p=0} = -\gamma$.
- For any ALOHA network with Rayleigh fading, there exists a unique parameter γ such that $1 - p\gamma \leq p_s \leq \exp(-p\gamma)$. This could serve as an alternative definition of the spatial contention.
- In a PPP network, the success probability equals the probability that a disk of area γ around the receiver is free from concurrent transmitters. So an *equivalent disk model* could be devised where the interference radius is $\sqrt{\gamma/\pi}$ (in the two-dimensional case). For a transmission over distance R , the disk radius would scale to $R\sqrt{\gamma/\pi}$.
- In full-duplex operation, the probabilistic throughput is $p_T^f = pe^{-p\gamma}$, and $p_{\text{opt}} = \min\{\sigma, 1\}$. So the spatial efficiency equals the optimum transmit probability in ALOHA, and $p_T^f = \sigma/e$. The throughput is proportional to σ .
- The transmission capacity, introduced in [16], is defined as the maximum spatial density of concurrent transmission allowed given an outage constraint ϵ . In our framework, for small ϵ , $p_s = 1 - p\gamma = 1 - \epsilon$, so $p = \epsilon\sigma$. So the transmission capacity is proportional to the spatial efficiency.
- Even if the channel access protocol that is eventually used is different from ALOHA, the spatial contention for ALOHA offers a single-parameter characterization of the network's capabilities to use space. In this case, the parameter p needs to be replaced by the corresponding parameter that captures the interferer density (such as $1/m$ in m -phase TDMA).

Further, $p_s(\theta)$ enables determining both the optimum θ (rate of transmission) and the ergodic capacity. For the cases where $\gamma \propto \theta^{d/\alpha}$, $\text{SIR}^{d/\alpha}$ is exponentially distributed. The optimum rates and the throughput are roughly linear in $\alpha - d$, the spatial capacity is about $2.5\times$ larger than the throughput, and the penalty for half-duplex operation is 10-20%. The optimum transmit probability p_{opt} is around 1/9 for both optimum throughput (Fig. 3, right) and maximum spatial capacity (Fig. 4, right). The mean distance to the nearest interferer is $1/(2\sqrt{p_{\text{opt}}}) = 3/2$, so for optimum performance the nearest interferer is, on average, 50% further away from the receiver than the desired transmitter. In line networks with m -phase TDMA, ESIR grows with m^α .

The results obtained can be generalized for (desired) link distances other than one in a straightforward manner. Many other extensions are possible, such as the analysis of other MAC protocols such as CSMA or CDMA (although the analytical tractability may suffer), the inclusion of power control, multiple-antenna nodes, directional transmissions, as well as node distributions whose uncertainty lies *inside* the uncertainty cube rather than on its surface. A first attempt in this direction has been taken in [17], where the interference and outage of clustered networks have been analyzed.

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