Interference and Outage in Poisson Cognitive Networks

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Abstract

Consider a cognitive radio network with two types of users: primary users (PUs) and cognitive users (CUs), whose locations follow two independent Poisson point processes. The cognitive users follow the policy that a cognitive transmitter is active only when it is outside the primary user exclusion regions. We found that under this setup the active cognitive users form a point process called the *Poisson hole process*. Due to the interaction between the primary users and the cognitive users through exclusion regions, an exact calculation of the interference and the outage probability seems unfeasible. Instead, two different approaches are taken to tackle this problem. First, bounds for the interference (in the form of Laplace transforms) and the outage probability are derived, and second, it is shown how to use a Poisson cluster process to model the interference in this kind of network. The bipolar network model with different exclusion region settings is analyzed.

Keywords- Cognitive Radio, Cognitive Network, Interference Modeling, Poisson Point Process, Poisson Cluster Process, Stochastic Geometry.

I. Introduction

The inefficiency in the spectrum usage of current wireless systems has led to significant research activities in cognitive radio. One of the ideas in cognitive radio is that a cognitive (secondary) user is allowed to share the spectrum with primary users as long as the interference is below a threshold (the underlay type of cognitive network) [1]. In wireless networks, a cognitive user can take advantage of either the time (when a primary user is not transmitting), the frequency (when a primary user is transmitting at a different frequency band), or the space (when a primary user is far away). The latter is a form of spatial reuse, thus the geometry plays a key role in this type of cognitive network. A cognitive user may transmit when the neighboring primary users are idle, but the signals of several secondary users could still cause harmful interference at primary users further away. As a result, there is a need to characterize the *aggregate* interference in order to satisfy the interference temperature metric [2].

This paper considers a cognitive radio network with two types of users: primary users (PUs) and cognitive users (CUs). Primary users are licensed users while cognitive users are allowed to transmit only if the performance of the primary network is not harmfully affected. This is the so-called underlay type of cognitive network. The cognitive users employ the following "cognition" in order to control their interference: a cognitive user may transmit only when it is outside the *primary exclusion regions*. For this setup, the primary metrics of interest are the aggregate interference and the outage probabilities at the primary and secondary users.

In this paper, we assume that the locations of PUs and CUs follow two independent Poisson point processes. The advantages and validity of using spatial Poisson process for modeling the locations of the wireless devices have been stated in many articles[1]. Quite often the user locations are time-varying, and we would like to determine the average performance over a large population of users for a class of random networks [3]. Stochastic geometry, a field focusing on the study of random spatial patterns, provides an elegant way of analyzing large networks. The spatial points, representing the locations of users, are constructed according to a spatial point process model. Without any prior knowledge, the user locations are often assumed independent and completely random. The spatial Poisson process is thus a natural (and a popular) choice in such situations because, given that a user is inside a region B, the PDF of its location is conditionally uniform over B [4]. In addition, the Poisson process is a fundamental point process that is easy to handle analytically, and it provides bounds for the performance of more general network models. The performance in clustered networks is lower than for the PPP [5], whereas the performance in more regular networks is higher [6], [7]. The Poisson bipolar network model was considered in [8], in which further justification of using the PPP model is given.

This stochastic geometry model also applies to multi-channel networks. If multiple channels are available, our model captures the situation in a single channel. Moreover, stochastic geometry permits spatial averaging and thus inherently considers all possible network realizations, weighed by their likelihood of occurring. As a result, time, space, and frequency sharing in the cognitive

[1]See the *IEEE Journal of Selected Areas on Communications Special Issue: Stochastic Geometry and Random Graphs for Wireless Networks* (September 2009) and the references therein.

network are all included in the stochastic geometry model presented in this paper.

A. Contributions

Due to the interaction between the primary and the cognitive users, an exact calculation of the interference and outage probability seems unfeasible. Instead, two different approaches are taken in this paper: the first approach is to derive bounds for the outage probability, and the second approach is to approximate the point process formed by the active cognitive users using Poisson cluster processes. From the first approach, the interference and outage for the bipolar Poisson cognitive network model are analyzed and bounded. Variations of the model are also discussed. In the second approach, it will be shown that under the exclusion region setup, the cognitive users form a *Poisson hole process* [6], which exhibits properties similar to a Poisson cluster process.

Our main contributions are the following: (1) This paper analyzes *all four types of aggregate interference* between primary and cognitive users, including the auto-interference between primary users among themselves and secondary users among themselves as well as the cross-interference from secondary to primary users and vice versa, in spectrum sensing cognitive networks, considering simultaneously the Rayleigh fading, the Poisson point process (PPP) model, and the exclusion regions. (2) A novel approach is proposed to estimate the interference between cognitive users, namely, *approximations based on the Poisson cluster process*.

B. Related Work

Point process theory has been successfully applied to wireless network analysis in the last two decades [9]. Recently, with the prosperity of research on cognitive radio, point process models find applications to cognitive networks. Pinto *et al.* considered a stochastic geometry-based mathematical model for coexistence in networks composed of both narrowband and ultra-wideband (UWB) wireless nodes [4]. In the paper by Huang *et al.* [10], the capacity trade-off between the coexisting cellular uplink and mobile ad hoc networks under spectrum underlay and spectrum overlay was analyzed based on the transmission capacity of a network with Poisson interferers. Ren *et al.* studied power control in cognitive networks and qualitatively characterized the impacts of the transmission power of secondary users on the occurrence of spectrum opportunities and the reliability of opportunity detection [11]. Riihijarvi and Mahonen utilized spatial statistics to improve the performance of cognitive radio networks [12].

Although there is already a vast body of research on cognitive networks, very few papers have focused on the *aggregate interference* caused by multiple secondary users, together with the interference that the primary users cause among themselves in the Poisson point process setup. Three papers are closest to our work. Hong *et al.* [13] and Ghasemi and Sousa [14] modeled the aggregate interference from the cognitive users outside the primary exclusion regions in fading channel, but both papers only considered a *single* primary receiver (instead of multiple primary transmitters and receivers). Yin *et al.* [15] derived the maximum primary and secondary transmitter densities given outage constraints for the overlaid network with multiple primary and cognitive users, but they considered *non-fading channel* and *no exclusion regions*.

C. Mathematical Preliminaries

Here we give a brief overview of some terminology and mathematical tools for stochastic geometry. Readers are referred to [9], [16]–[18] for further details.

Definition 1. The *Poisson point process* with uniform intensity $\lambda > 0$ is a point process in $\mathbb{R}^2$ such that [16]

1) For every bounded closed set B, the counting measure (number of points) $\mathsf{N}(\mathsf{B})$ has a Poisson distribution with mean $\lambda \cdot |\mathsf{B}|$, where $|\mathsf{B}|$ denotes the area of B.
2) If $\mathsf{B}_1, ..., \mathsf{B}_\mathsf{m}$ are disjoint regions, then $\mathsf{N}(\mathsf{B}_1), ..., \mathsf{N}(\mathsf{B}_\mathsf{m})$ are independent.

This definition leads to the following property: given $\mathsf{N}(\mathsf{B}) = n$, then the n points are independently, uniformly distributed in B. This point process is thus a good model when the user locations are independent and completely random.

Definition 2. A *hard-core point process* is a point process in which the points are forbidden to lie closer than a certain minimum distance [18].

Definition 3. A *Poisson cluster process* is formed by taking a Poisson process Φ of parent points and replacing each point $x \in \Phi$ by a random cluster Z_x which is a finite point process. The superposition of all clusters yields the Poisson cluster process $Y = \bigcup_{x\in\Phi} Z_x$ [16].

Definition 4. The Laplace transform $\mathcal{L}$ of X is defined as $\mathcal{L}_X(s) = \mathbb{E}[\exp(-sX)]$ [17].

In the case of Rayleigh fading, the received signal power S is exponentially distributed. Let the transmit power be μ, the transmission distance r, and the path loss $r^{-\alpha}$ with a path loss exponent α. Then $\mathbb{E}[S] = \mu r^{-\alpha}$. Denoting the interference by I and ignoring the noise, the success probability $p_s(\theta)$ is a function of the threshold θ as $p_s(\theta) = \mathbb{P}\left[\frac{S}{I} > \theta\right] = \mathbb{E}\left[\exp\left(-\frac{\theta r^\alpha}{\mu} I\right)\right]$. Since $\mathbb{E}[\exp(-sI)]$ is the Laplace transform of the interference, the success probability can be obtained by setting $s = \theta\mu^{-1}r^\alpha$. As a result, the Laplace transform characterizes the interference *and* the success probability in Rayleigh fading. We will frequently use the property that the Laplace transform of the sum of independent random variables is the product of the individual Laplace transforms. See [17] for further details on using the Laplace transform.

Definition 5. Let $v(x) : \mathbb{R}^2 \to [0, \infty)$ be measurable. The probability generating functional (PGFL) of the point process Φ is defined as $\mathcal{G}[v] = \mathbb{E}\left[\prod_{x \in \Phi} v(x)\right]$ [17].

For example, it can be found that for PPP, $\mathcal{G}[v] = \exp\left[-\int_{\mathbb{R}^2} (1 - v(x))\, \lambda(\mathrm{d}x)\right]$ [17].

D. Organization

Section II describes the network model. Section III derives bounds of interference and outage probability for the bipolar network setup. Section IV then generalizes the results to variations of the bipolar model. Section V introduces the Poisson cluster process as an approximation model for the Poisson hole process. Finally, the paper is concluded in Section VI.

II. NETWORK MODEL

Let us consider an underlay type of cognitive network with all the primary and cognitive users operating at the same frequency band. Assume that the cognitive users can perfectly detect the primary receivers[2], so that the cognitive users have full knowledge of the locations of the primary users. The cognitive users also know the transmission parameters of the primary users in order to set up the exclusion regions (described later). Since the cognitive users will avoid

[2]How to detect the primary users is outside the scope of this paper, and many schemes have been proposed. If the primary receivers are passive, detecting the power leakage of local oscillator is a possible way. See [19] for a survey.

the exclusion regions to limit their interference, the primary users do not need any information about the cognitive users.

The bipolar network model is considered in this paper. In this model, transmitters are assumed to have receivers at a fixed distance. This model provides an insight into how the network performance depends on the link distance. The results obtained thus can also be interpreted as the performance of networks with random link distances *conditioned on the link distance having a certain value*.

A. Bipolar Model

The bipolar (BP) model is shown in Fig. 1(a). The locations of the primary transmitters follow a homogeneous Poisson point process (PPP) $\Phi_p = \{x_1, x_2, \ldots\} \subset \mathbb{R}^2$ of density λ_p, and the locations of the potential cognitive transmitters follow another, independent, homogeneous Poisson point process $\Phi_c = \{y_1, y_2, \ldots\} \subset \mathbb{R}^2$ of density λ_c. Assume that all the primary transmitters use the same transmission power μ_p, and all the primary receivers are at a distance r_p from the corresponding primary transmitters in a random direction. Similarly, all the cognitive transmitters use the same transmission power μ_c, and all the cognitive receivers are at a distance r_c from the corresponding cognitive transmitters. The locations of the primary and the cognitive receivers are also PPPs with density λ_p and λ_c, respectively. r_c is assumed to be small relative to the mean nearest-neighbor distance of Φ_c ($r_c \ll \lambda_c^{-\frac{1}{2}}$) since the transmission power and the range of the cognitive users are usually small. The activation of the cognitive users depends on the exclusion region setup of the primary users. The exclusion regions are circular regions with radius D designed to guarantee that cognitive transmitters will, on average, not generate an aggregate interference resulting in the outage of primary users, which occurs when the instantaneous signal-to-interference ratio (SIR)[3] is lower than θ_p. Similarly, the SIR threshold for the cognitive users is denoted as θ_c.

The radius D of the exclusion region in the bipolar model is chosen as

$$D = r_p \left[\theta_p \left(\frac{\beta \mu_c}{\mu_p} \right) \right]^{\frac{1}{\alpha}}, \tag{1}$$

[3]Throughout the paper, the noise is neglected since interference is what causes the interaction between primary and cognitive users. Hence the focus is on the SIR instead of the signal-to-interference-and-noise ratio (SINR).

where α is the path loss exponent and β is a design factor such that β cognitive transmitters will, on average, not generate an aggregate interference resulting in an SIR below the threshold. The expression for D reflects the fact that in order to protect the primary users, the exclusion region must grow along with the increase in the transmission power of the cognitive users, the SIR threshold of the primary network, and the primary user transmission distance. On the other hand, the exclusion region shrinks when the transmission power of the primary user increases such that more cognitive users can be active. Besides, the path loss exponent must also be taken into account. Let us assume that D is larger than $r_p + r_c$, ensuring that the primary transmitters are inside the exclusion regions such that a cognitive receiver and a primary transmitter cannot be arbitrarily close.

B. Interference Model

Define $I(y) = \sum_{x \in \Phi} \mu_x h_x \ell(y - x)$ as the total interference at y resulting from the interferers positioned at the points of the process Φ, where $\ell(x) = \|x\|^{-\alpha}$ is the large-scale path loss model, and assume the power fading coefficients h_x are i.i.d. exponential (Rayleigh fading) with $\mathbb{E}[h] = 1$. μ_x is either μ_p or μ_c (thus a fixed value), depending on which interference is considered.

The interference to the primary users and the interference to the cognitive users are considered separately. For each case, the interference is comprised of contributions by both primary transmitters and cognitive transmitters, so there are four types of interference: the interference from the primary transmitters to the primary receivers I_{pp}, the interference from the primary transmitters to the cognitive receivers I_{pc}, the interference from the cognitive transmitters to the primary receivers I_{cp}, and the interference from the cognitive transmitters to the cognitive receivers I_{cc}. To calculate the interference to the primary users, we condition on having a primary receiver at the origin, the *typical receiver*, which yields the Palm distribution for the primary transmitters. By Slivnyak's theorem [18], this conditional distribution is the same as the original one for the rest of the primary network. For the secondary network, however, conditioning on a typical cognitive receiver generally changes the distance distribution since the activation of the cognitive transmitters is determined by the locations of the primary users. This is the reason why only bounds can be obtained for any interference involving the cognitive users.

III. Analysis of the Bipolar Model

In this section, the bipolar model with the exclusion regions around the primary receivers is discussed[4]. Let us define $\delta \triangleq 2/\alpha$. The following two lemmas are used as building blocks for the analysis of the bipolar model.

Lemma 1. ((3.21) in [17]) *Let* $I(y) = \sum_{x\in\Phi} \eta h_x \left\|x-y\right\|^{-\alpha}$ *where* Φ *is a PPP with density* ν *and* h_x*'s are i.i.d. exponential with* $\mathbb{E}\left[h\right]=1$*,* η *is the transmission power, and*

$$\mathcal{L}_0(\nu,\eta,s) \quad \triangleq \quad \exp\left\{-\nu\frac{\pi^2\delta}{\sin(\pi\delta)}\eta^\delta s^\delta\right\}. \tag{2}$$

Then the Laplace transform of the interference I *is* $\mathcal{L}_0(\nu,\eta,s)$*.*

Lemma 2. ((3.46) in [17]) *Let*

$$\mathcal{L}_1(\nu,\eta,\rho,s) \triangleq \exp\left\{-\nu\pi\left(\eta^\delta s^\delta \mathbb{E}_h\left[h^\delta\gamma\left(1-\delta, s\eta h\rho^{-\alpha}\right)\right] - \frac{s\eta\rho^{2-\alpha}}{1+s\eta\rho^{-\alpha}}\right)\right\}, \tag{3}$$

where $\gamma(a,z)=\int_0^z \exp(-t)t^{a-1}\mathrm{d}t$ *is the* lower *incomplete gamma function. Following the setup in Lemma 1, except that now the interference from the users within the distance* ρ *is not included, the Laplace transform of the interference* I *is* $\mathcal{L}_1(\nu,\eta,\rho,s)$*.*

A. Interference to Primary Users

The interference to a primary user is composed of two parts: the interference to a primary receiver from other primary transmitters, denoted as I_{pp}, and the interference to a primary receiver from the cognitive transmitters, denoted as I_{cp}.

Since the fading is Rayleigh and the primary transmitters are distributed as a PPP, the Laplace transform of I_{pp}, denoted as $\mathcal{L}_{I_{pp}}(s)$, is obtained from Lemma 1 with density λ_p and transmission power μ_p, i.e.,

$$\mathcal{L}_{I_{pp}}(s) = \mathcal{L}_0(\lambda_p,\mu_p,s). \tag{4}$$

The interference to a primary receiver from the cognitive transmitters, denoted as I_{cp}, is hard to calculate exactly. Instead, a bound can be derived as follows. Let Φ_a and $\Phi_{a'}$ be the partition

[4]In Section IV, the case with exclusion regions around the primary transmitters will be considered.

of Φ_c into active and inactive nodes depending on whether the cognitive transmitters are outside or inside the exclusion regions. Let Φ_D include all the points in Φ_c except the points that are within the exclusion region of the typical primary receiver. Since $\Phi_a \subset \Phi_D$, the interference I_{cp} caused by the active cognitive transmitters is stochastically dominated[5] by the interference $\hat{I}_{cp}$ caused by Φ_D (denoted as $I_{cp} \overset{s}{<} \hat{I}_{cp}$). Since the cognitive transmitters are at least at distance D, the Laplace transform of $\hat{I}_{cp}$, denoted as $\mathcal{L}_{\hat{I}_{cp}}(s)$, is given by Lemma 2 with density λ_c and transmission power μ_c, i.e.,

$$\mathcal{L}_{\hat{I}_{cp}}(s) = \mathcal{L}_1(\lambda_c, \mu_c, D, s). \tag{5}$$

Now we are ready to bound the outage probability of the primary users.

Theorem 1. *The outage probability of the primary users ϵ_p is upper-bounded as*

$$\epsilon_p < 1 - \exp\left\{ -\theta_p^\delta r_p^2 \left[\lambda_p \frac{\pi^2 \delta}{\sin(\pi\delta)} + \lambda_c \pi \left(\frac{\mu_c}{\mu_p}\right)^\delta \left(\mathbb{E}_h \left[h^\delta \gamma \left(1 - \delta, \frac{h}{\beta}\right)\right] - \frac{\beta^\delta}{1+\beta} \right)\right]\right\}. \tag{6}$$

Proof: With Rayleigh fading, the transmission success probability of the primary users is the Laplace transform evaluated at $s = \theta_p \mu_p^{-1} r_p^\alpha$. Since the interference from the primary transmitters and the interference from the cognitive transmitters are independent, the outage probability ϵ_p is upper-bounded by $\hat{\epsilon}_p = 1 - \mathcal{L}_{I_{pp}}\left(\theta_p \mu_p^{-1} r_p^\alpha\right) \cdot \mathcal{L}_{\hat{I}_{cp}}\left(\theta_p \mu_p^{-1} r_p^\alpha\right)$. ■

When $\alpha = 4$ ($\delta = \frac{1}{2}$) and $\beta = 1$, the upper bound for the outage probability of the primary users ϵ_p can be simplified to

$$\epsilon_p < 1 - \exp\left\{ -\sqrt{\theta_p} r_p^2 \left(\lambda_p \frac{\pi^2}{2} + \lambda_c \frac{\pi^2}{4} \sqrt{\frac{\mu_c}{\mu_p}} \right)\right\}, \tag{7}$$

which follows from $\mathbb{E}_h[h^\delta \gamma\left(1-\delta, vh\right)] = \frac{\pi}{2} - \arctan\left(\frac{1}{\sqrt{v}}\right) + \frac{\sqrt{v}}{1+v}$.

Note that the point process of active cognitive users Φ_a is not a PPP but a *Poisson hole process* (see Def. 6 and Prop. 1 in Section V). Nonetheless, independent thinning of the cognitive users outside the exclusion regions with probability $\exp(-\lambda_p \pi D^2)$ yields a good approximation on I_{cp}, since the higher-order statistics of the point process, which govern the interaction between

[5] A random variable $\mathtt{A}$ stochastically dominates a random variable $\mathtt{B}$ if $\mathbb{P}[\mathtt{A} > x] \geq \mathbb{P}[\mathtt{B} > x]$ for all x, or equivalently, $F_{\mathtt{A}}(x) \leq F_{\mathtt{B}}(x)$ for cumulative distribution functions $F_{\mathtt{A}}(x)$ and $F_{\mathtt{B}}(x)$.

nodes, become less relevant if D is not too small. Thus we obtain the approximation $\tilde{I}_{cp}$ with Laplace transform

$$\mathcal{L}_{\tilde{I}_{cp}}(s) = \mathcal{L}_1\left(\lambda_c \exp(-\lambda_p \pi D^2), \mu_c, D, s\right). \tag{8}$$

An approximation to the outage probability of the primary users ϵ_p is, therefore, given by

$$\epsilon_p \approx 1 - \exp\Bigg\{ -\theta_p^\delta r_p^2 \Bigg[\lambda_p \frac{\pi^2 \delta}{\sin(\pi\delta)} + \lambda_c \pi \exp\left(-\lambda_p \pi r_p^2 \theta_p^\delta \left(\frac{\beta \mu_c}{\mu_p} \right)^\delta \right) \times \left(\frac{\mu_c}{\mu_p} \right)^\delta \left(\mathbb{E}_h \left[h^\delta \gamma \left(1-\delta, \frac{h}{\beta} \right) \right] - \frac{\beta^\delta}{1+\beta} \right) \Bigg] \Bigg\}. \tag{9}$$

When $\alpha = 4$ and $\beta = 1$, the above approximation can be simplified to

$$\epsilon_p \approx 1 - \exp\left\{ -\sqrt{\theta_p} r_p^2 \left(\lambda_p \frac{\pi^2}{2} + \lambda_c \frac{\pi^2}{4} \sqrt{\frac{\mu_c}{\mu_p}} \exp\left[-\lambda_p \pi r_p^2 \sqrt{\theta_p \left(\frac{\mu_c}{\mu_p} \right)} \right] \right) \right\}. \tag{10}$$

B. Interference to Cognitive Users

Similar to the case of estimating interference to the primary users, the interference to a cognitive user is composed of two parts: the interference to a cognitive receiver from the primary transmitters, denoted as I_{pc}, and the interference to a cognitive receiver from other cognitive transmitters, denoted as I_{cc}.

First let us consider the interference from the primary transmitters. Since a cognitive transmitter is at least at distance D from a primary receiver, and the distance between a primary transmitter-receiver pair is r_p, the distance between a primary transmitter and a cognitive transmitter is at least $D - r_p$. Furthermore, the distance between a cognitive transmitter and its corresponding cognitive receiver is r_c, so the distance to the nearest primary transmitter for a cognitive receiver is at least $\bar{D} = D - r_p - r_c$ ($\bar{D} > 0$ since $D > r_p + r_c$ as described in Section II). Denote by $\hat{I}_{pc}$ the random variable whose Laplace transform is $\mathcal{L}_1(\lambda_p, \mu_p, \bar{D}, s)$. Since the location of the transmitter is not at the center of the exclusion region, the interference I_{pc} to a cognitive receiver from the primary transmitters is stochastically dominated by the random variable $\hat{I}_{pc}$ with Laplace transform

$$\mathcal{L}_{\hat{I}_{pc}}(s) = \mathcal{L}_1(\lambda_p, \mu_p, \bar{D}, s). \tag{11}$$

Now let us consider the interference from the other cognitive transmitters. Let $\hat{I}_{cc}$ be the interference generated by the process Φ_c. Since $\Phi_a \subset \Phi_c$, I_{cc} is stochastically dominated by $\hat{I}_{cc}$. Since Φ_c is a PPP, the Laplace transform of $\hat{I}_{cc}$, denoted as $\mathcal{L}_{\hat{I}_{cc}}(s)$, is

$$\mathcal{L}_{\hat{I}_{cc}}(s) = \mathcal{L}_0(\lambda_c, \mu_c, s), \tag{12}$$

which follows from Lemma 1.

The following theorem gives a upper bound for the outage probability of the cognitive users.

Theorem 2. *Let* $\xi = \frac{\theta_c \mu_p}{\mu_c}\left[\left(\frac{\theta_p \beta \mu_c}{\mu_p}\right)^{\frac{1}{\alpha}}\left(\frac{r_p}{r_c}\right) - \frac{r_p}{r_c} - 1\right]^{-\alpha}$. *The outage probability of the cognitive users* ϵ_c *is upper-bounded as*

$$\begin{aligned}\epsilon_c < 1 - \exp\Bigg\{ -\lambda_p \pi \left(\theta_c^{\delta}\left(\frac{\mu_p}{\mu_c}\right)^{\delta} r_c^2 \mathbb{E}_h\left[h^{\delta}\gamma\left(1-\delta, \xi h\right)\right] - r_p^2\left[\left(\frac{\theta_p \beta \mu_c}{\mu_p}\right)^{\frac{1}{\alpha}} - \frac{r_c}{r_p} - 1\right]^2 \left(\frac{\xi}{1+\xi}\right)\right) \\ - \lambda_c \frac{\pi^2 \delta}{\sin(\pi\delta)} \theta_c^{\delta} r_c^2 \Bigg\}. \end{aligned} \tag{13}$$

Proof: The success transmission probability of the cognitive users is the Laplace transform evaluated at $\theta_c \mu_c^{-1} r_c^{\alpha}$. Since the interference from the primary transmitters and the interference from the cognitive transmitters are independent, the outage probability ϵ_c is upper-bounded by $\hat{\epsilon}_c = 1 - \mathcal{L}_{\hat{I}_{pc}}\left(\theta_c \mu_c^{-1} r_c^{\alpha}\right) \cdot \mathcal{L}_{\hat{I}_{cc}}\left(\theta_c \mu_c^{-1} r_c^{\alpha}\right)$. ■

For $\alpha = 4$, the upper bound for the outage probability of the primary users ϵ_c can be simplified to

$$\begin{aligned}\epsilon_c < 1 - \exp\Bigg\{ -\lambda_p \pi \Bigg[\sqrt{\theta_c\left(\frac{\mu_p}{\mu_c}\right)} r_c^2 \left(\frac{\pi}{2} - \arctan\left(\frac{1}{\sqrt{\xi}}\right) + \frac{\sqrt{\xi}}{\xi+1}\right) \\ - r_p^2 \left[\left(\frac{\theta_p \beta \mu_c}{\mu_p}\right)^{\frac{1}{4}} - \frac{r_c}{r_p} - 1\right]^2 \left(\frac{\xi}{1+\xi}\right)\Bigg] - \lambda_c \frac{\pi^2}{2}\sqrt{\theta_c} r_c^2 \Bigg\}. \end{aligned} \tag{14}$$

C. Numerical Examples

Fig. 2 shows the simulation results and the upper bounds of the outage probabilities of the primary and cognitive users for different θ_p and θ_c. It also shows the approximation of the primary user outage probability and the simulation results for the primary user-only network. The simulation parameters are: $\lambda_p = 0.1$, $\lambda_c = 1$, $\mu_p = 1$, $\mu_c = 0.2$, $r_p = 0.5$, $r_c = 0.1$, $\beta = 81$, and $\alpha = 4$. D is determined using (1). We observe that for large θ_p the primary user outage is dominated by the interference from the primary users, since a large θ_p implies a large exclusion region radius D, which means that few secondary users are active. Fig. 2 also shows that the approximation of the location distribution of the cognitive users outside the exclusion regions with a PPP of the same intensity using (9) is very good.

D. Asymptotic Regions of ϵ_p and ϵ_c

Besides of showing the tightness of the bounds, it is interesting to explore some asymptotic regions of ϵ_p and ϵ_c. First we check the invariant properties. If λ_p and λ_c are scaled by some factor c and both r_p and r_c by $c^{-\frac{1}{2}}$, the same result will be obtained. Thus, as a function of the ratio λ_c/λ_p, the results should look the same as as a function of $\sqrt{r_p/r_c}$. The result will also be the same if both μ_p and μ_c are scaled by c. However, ϵ_p and ϵ_c are not only a function of the ratio λ_c/λ_p, but also a function of the densities λ_p and λ_c themselves. Fig. 3 shows ϵ_p and ϵ_c as a function of λ_c/λ_p under different λ_p using (6) and (13). These two bounds also imply that (a) ϵ_p becomes smaller with the decrease in r_p, λ_p, λ_c, and μ_c and the increase in μ_p, and (b) ϵ_c becomes smaller with the decrease in r_c, λ_p, λ_c, and μ_p and the increase in r_p and μ_c. Thus, it is easy to obtain the following results: (a) If $r_p \to \infty$ (then $D \to \infty$), there will be no interference from CU. Under fixed μ_p, however, $\mu_p r_p^{-\alpha} \to 0$. Therefore, $\epsilon_p \to 1$. For the same reason, if $r_c \to \infty$, then $\epsilon_c \to 1$ since μ_c is fixed. (b) If $r_p \to 0$, then $\epsilon_p \to 0$; similarly, if $r_c \to 0$, then $\epsilon_c \to 0$. (c) If $\lambda_p \to \infty$ or $\lambda_c \to \infty$, then $\epsilon_p \to 1$ and $\epsilon_c \to 1$ since the total interference sums to infinity. (d) Obviously, if $\theta_p \to 0$, then $\epsilon_p \to 0$; if $\theta_p \to \infty$, then $\epsilon_p \to 1$. The same results apply to θ_c.

IV. Variations on the Bipolar Model

In Section III, bounds of the outage probabilities for the exclusion regions around the primary receivers for the bipolar model have been derived. In this section, some variations, i.e., exclusion regions around the primary transmitters, exclusion regions around both the transmitters and the receivers, and the case when primary users employ a CSMA-type MAC, are considered.

A. Exclusion Regions around Primary Transmitters

Detecting primary receivers is very difficult if the receivers are passive. In this case, setting the exclusion regions according to the primary transmitters is a reasonable and practical compromise [20], [21]. Under this setup, the interference $I_{pp,\mathrm{PT}}$ and the interference $I_{cc,\mathrm{PT}}$ (the subscript "PT" denotes the case of exclusion regions around the primary transmitters) remain the same as I_{pp} and I_{cc}, respectively, in the primary receiver exclusion region case. This is because the exclusion regions do not apply to the primary users, so the interference between the primary users is not affected by the change of exclusion regions. The interference between the cognitive users is the same since no matter whether the exclusion regions are around the primary transmitters or around the primary receivers, the fraction of the cognitive users that are active is the same. For the interference $I_{cp,\mathrm{PT}}$ to a primary receiver from the cognitive transmitters and the interference $I_{pc,\mathrm{PT}}$ to a cognitive receiver from the primary transmitters, bounds can be obtained as follows.

The cognitive transmitters must be at distance at least D from the primary transmitters, so the distance between a primary receiver and a cognitive transmitter is at least $D - r_p$ and the distance between a primary transmitter and a cognitive receiver is at least $D - r_c$. Plugging this into (3), it is easy to find that $I_{cp,\mathrm{PT}}$ is stochastically dominated by the random variable $\hat{I}_{cp,\mathrm{PT}}$ with Laplace transform

$$\mathcal{L}_{\hat{I}_{cp,\mathrm{PT}}}(s) = \mathcal{L}_1(\lambda_c, \mu_c, D - r_p, s), \tag{15}$$

and $I_{pc,\mathrm{PT}}$ is stochastically dominated by the random variable $\hat{I}_{pc,\mathrm{PT}}$ with Laplace transform

$$\mathcal{L}_{\hat{I}_{pc,\mathrm{PT}}}(s) = \mathcal{L}_1(\lambda_p, \mu_p, D - r_c, s). \tag{16}$$

The outage probability of the primary users $\epsilon_{p,\mathrm{PT}}$ and the outage probability of the cognitive users $\epsilon_{c,\mathrm{PT}}$ when the exclusion regions are around the primary transmitters are upper-bounded respectively by

$$\begin{aligned}
\hat{\epsilon}_{p,\mathrm{PT}} &= 1 - \mathcal{L}_{I_{pp}}\left(\theta_p \mu_p^{-1} r_p^{\alpha}\right) \cdot \mathcal{L}_{\hat{I}_{cp,\mathrm{PT}}}\left(\theta_p \mu_p^{-1} r_p^{\alpha}\right) && (17)\\
&= 1 - \mathcal{L}_0\left(\lambda_p, \mu_p, \theta_p \mu_p^{-1} r_p^{\alpha}\right) \cdot \mathcal{L}_1\left(\lambda_c, \mu_c, D - r_p, \theta_p \mu_p^{-1} r_p^{\alpha}\right), && (18)
\end{aligned}$$

and

$$\hat{\epsilon}_{c,\mathrm{PT}} = 1 - \mathcal{L}_{\hat{I}_{pc,\mathrm{PT}}}\left(\theta_c \mu_c^{-1} r_c^{\alpha}\right) \cdot \mathcal{L}_{\hat{I}_{cc}}\left(\theta_c \mu_c^{-1} r_c^{\alpha}\right) \tag{19}$$

$$= 1 - \mathcal{L}_1\left(\lambda_p, \mu_p, D - r_c, \theta_c \mu_c^{-1} r_c^{\alpha}\right) \cdot \mathcal{L}_0\left(\lambda_c, \mu_c, \theta_c \mu_c^{-1} r_c^{\alpha}\right). \tag{20}$$

B. Exclusion Regions around both Primary Transmitters and Receivers

In practical scenarios, traffic is often bi-directional due to acknowledgments (ACK). The roles of transmitters and receivers change frequently, and so do the exclusion regions. However, a cognitive user might not be able to react in such a short time, and the consequence of failing to do so is significant. One possible solution is to set the exclusion regions based on both the primary transmitters and the receivers. In this case, the density of active cognitive users is lower compared to the single-exclusion region setup. Hence, the interference from the cognitive transmitters (to either primary or other cognitive receivers) is stochastically dominated by the interference in the single-exclusion region setup, which is bounded according to (5) and (12). Since the exclusion region setup does not affect the relationship between the primary users, the interference between the primary users is the same as I_{pp}. The interference from the primary users to a cognitive user is bounded by (16) due to the exclusion regions around the primary transmitters. Note that the bounds become less tight due to the silencing of extra cognitive users.

C. Primary User MAC

Until now, only the case of controlling the interference from the cognitive users is discussed. However, as shown in Fig. 2, the interference from other primary users might dominate since primary interferers may be arbitrarily close. It is therefore reasonable to apply a MAC scheme among primary users. When a CSMA-type MAC is employed, the primary transmitters form a *hard-core process*, in which no any two primary transmitters are allowed to be closer than a distance D_p (the radius of a guard zone).

When the primary users employ the CSMA-type MAC, the interference $I_{pp,\mathrm{CSMA}}$ to a primary receiver from the other primary transmitters is stochastically dominated by the random variable $\hat{I}_{pp,\mathrm{CSMA}}$ with Laplace transform $\mathcal{L}_{\hat{I}_{pp,\mathrm{CSMA}}}(s) = \mathcal{L}_1(\lambda_p, \mu_p, D_p, s)$, which follows directly from Lemma 2, but now the interference is smaller due to the CSMA-type MAC for every primary transmitter.

Since the active primary transmitters form a hard-core process, the density of the active primary transmitters when the primary users apply CSMA-type MAC with sensing range D_p is $\lambda'_p = \frac{1-\exp(-\lambda_p \pi D_p^2)}{\pi D_p^2}$ [18]. An approximation of $\mathcal{L}_{I_{pp,\mathrm{CSMA}}}(s)$ is thus $\mathcal{L}_{\tilde{I}_{pp,\mathrm{CSMA}}}(s) = \mathcal{L}_0(\lambda'_p, \mu_p, s)$.

Eqn. (5) can be used to bound the interference $I_{cp,\mathrm{CSMA}}$ from the cognitive transmitters to the primary receivers. The bound is tighter in the case with primary user MAC than without primary user MAC. The reason is the following. Let $\lambda'_a = \exp(-\pi\lambda'_p D^2)$ and $\lambda_a = \exp(-\pi\lambda_p D^2)$ be the densities of the active cognitive users with and without primary user MAC, respectively. Since $\lambda'_p < \lambda_p$, it follows that $\lambda_a < \lambda'_a < \lambda_c$. Since $\mathcal{L}_0(\nu, \eta, s)$ is a monotonically decreasing function of the variable ν, the bound is tighter in the case with the primary user MAC.

Eqn. (11) and Eqn. (16) can be used to give bounds for the interference $I_{pc,\mathrm{CSMA}}$ from the primary transmitters to a cognitive receiver when the exclusion regions are around the primary receivers and around the primary transmitters respectively. The interference $I_{pc,\mathrm{CSMA}}$ to a cognitive receiver from the primary transmitters is approximated by the random variable $\tilde{I}_{pc,\mathrm{CSMA}}$ with Laplace transform $\mathcal{L}_{\tilde{I}_{pc,\mathrm{CSMA}}}(s) = \mathcal{L}_1(\lambda'_p, \mu_p, D - r_p - r_c, s)$ and $\mathcal{L}_{\tilde{I}_{pc,\mathrm{CSMA}}}(s) = \mathcal{L}_1(\lambda'_p, \mu_p, D - r_c, s)$ if the exclusion regions are around the primary receivers and around the primary transmitters, respectively. Eqn. (12) can also be used to bound the interference $I_{cc,\mathrm{CSMA}}$ from the cognitive transmitters to the primary receivers. Note that again this bound is tighter than in the case without primary user MAC because $\lambda'_a > \lambda_a$.

V. Interference Modeling using Poisson Cluster Processes

It turns out that the interference between the cognitive users is the hardest to calculate or bound. In this section, a novel approach will be pursued: modeling the interference using a different point process model. We start by defining the Poisson hole process:

Definition 6. (*Poisson hole process*) Let Φ_1 and Φ_2 be independent PPPs of intensities $\lambda_2 > \lambda_1$. For each $x \in \Phi_1$, remove all the points in $\Phi_2 \cap b(x, D)$, where $b(x, D)$ is a ball centered at x with radius D. All the removed points of Φ_2 form the hole-0 process and the remaining points form the hole-1 process, as introduced in [6]. Here we denote the hole-1 process as the *Poisson hole process*.

Then we make the following observation of the process of active cognitive users.

Proposition 1. Φ_a *is a Poisson hole process.*

Proof: Let Φ_p be Φ_1, λ_p be λ_1, Φ_c be Φ_2, and λ_c be λ_2 in Def. 6. Then it follows that the point process formed by the active cognitive users is indeed a Poisson hole process. ■

The Poisson hole process behaves like a Poisson cluster process. The reason is that forming "holes" (due to the exclusion regions in our case) forces nodes to concentrate in some areas. This kind of node distribution looks as if the nodes are "clustered" by nature. Fig. 4 compares the Poisson hole process and the Thomas cluster process, with the same parameters given in Section III ($\lambda_p = 0.1$, $\lambda_c = 1$, $\mu_p = 1$, $\mu_c = 0.2$, $r_p = 0.5$, $\beta = 81$, $\alpha = 4$, $\theta_p = 10$, and $D = 1.7838$). It is easy to observe that both processes are very different from a PPP.

A. *Fitting a Poisson Cluster Process*

Since the Poisson hole process is analytically intractable (in particular, its probability generating functional is unknown), we approximate it with a Poisson cluster process by matching first- and second-order statistics. The first-order statistic is the intensity, so

$$\lambda_c \exp\left(-\lambda_p \pi D^2\right) = \lambda_l \bar{c}, \tag{21}$$

where the left hand side is the intensity of the active cognitive users; λ_l at the right hand side is the density of parent points of the cluster process, and $\bar{c}$ is the average number of points in a cluster. For motion-invariant processes, the second-order statistics are fully described by the pair-correlation function $g(r)$ [18]. Here two kinds of Poisson cluster processes, the Matern cluster process and the Thomas cluster process, are considered.

Let R be the cluster radius in the Matern cluster process. The g-function of the Matern cluster process is [18]

$$g_M(r) = \begin{cases} 1 + \frac{2}{\lambda_l \pi^2 R^2}\left[\arccos\left(\frac{r}{2R}\right) - \frac{r}{2R}\sqrt{1-\frac{r^2}{4R^2}}\right] & \text{if } 0 < r < 2R, \\ 1 & \text{if } r \geq 2R. \end{cases} \tag{22}$$

λ_l and R can be determined using curve-fitting to the g-function of the Poisson hole process. $\bar{c}$ is then determined using (21).

The g-function of the Thomas cluster process is [18]

$$g_T(r) = 1 + \frac{1}{4\pi\lambda_l\sigma^2}\exp\left(-\frac{r^2}{4\sigma^2}\right). \tag{23}$$

Again, λ_l and σ are obtained using curve-fitting and $\bar{c}$ is then determined using (21).

To illustrate the fitting, we use the same example in Section III ($\lambda_p = 0.1$, $\lambda_c = 1$, $\mu_p = 1$, $\mu_c = 0.2$, $r_p = 0.5$, $\beta = 81$, and $\alpha = 4$) and let $\theta_p = 10$, then D is 1.7838. By using the **nlinfit** function (nonlinear least-squares fit) in Matlab, we get $\lambda_l = 0.0825$, $\bar{c} = 4.4623$, and $R = 1.5305$ for the Matern cluster process, and $\lambda_l = 0.0809$, $\bar{c} = 4.5497$, and $\sigma = 0.8206$ for the Thomas cluster process. Fig. 5(a) shows the g-functions of the Poisson hole process, Thomas cluster process, Matern cluster process, and PPP obtained by simulations. Following the same procedure, the Poisson hole process resulting from the primary user MAC (a hard-core process) can also be modeled, as shown in Fig. 5(b), where $D_p = 2$. The parameters for fitting are $\lambda_l = 0.1722$, $\bar{c} = 2.1370$, and $R = 1.4033$ for the Matern cluster process and $\lambda_l = 0.1673$, $\bar{c} = 2.1997$, and $\sigma = 0.7664$ for the Thomas cluster process. The results show that the Poisson hole process can be closely approximated by the Thomas and the Matern cluster processes, no matter whether the primary users employ a CSMA-type MAC or not.

Note that the difference between the Poisson hole process and the Poisson cluster process (as an approximation to the Poisson hole process) is the higher-order statistics. Although we are able to fit the first- and the second-order statistic of the Poisson hole process using the Poisson cluster process, the higher-order statistics might be different. For the interference modeling, however, the first- and the second-order statistics prove sufficient, as shown in the following subsection.

B. Interference Modeling using Poisson Cluster Processes

As explained earlier, it is possible to approximate the Poisson hole process using a Poisson cluster process; now we will show how the Poisson cluster process models the interference in the cognitive network. The focus will be on the interference to a cognitive receiver from the other cognitive transmitters for the following reasons. The Laplace transform of the interference to a primary receiver from the other primary transmitters is given in (4), and the Laplace transform of the interference to a cognitive receiver from the primary transmitters is tightly upper-bounded using (11). For the interference to a primary receiver from the cognitive transmitters, the higher-order statistics of the point process formed by the active cognitive transmitters is less relevant as long as the exclusion region is large enough (see (8)). Whether the cognitive transmitters behave as a Poisson hole process or a PPP will introduce approximately the same interference to the primary receivers (as shown in (8) and Fig. 2).

Fig. 6 shows the simulation results of the complementary cumulative density function (CCDF) of the interference among active cognitive users (Poisson hole process) and among the nodes in the Matern and Thomas cluster processes. The simulation uses the same parameters as before: $\lambda_p = 0.1$, $\lambda_c = 1$, $\mu_p = 1$, $\mu_c = 0.2$, $r_p = 0.5$, $\beta = 81$, $\alpha = 4$, $\theta_p = 10$, and $D = 1.7838$. From the simulation, the interference distributions in the Poisson cluster process and the Poisson hole process are essentially the same.

The way to obtain the outage probability of the cognitive users for Poisson-type cognitive networks from the known results of the Poisson cluster process is the following. First we find the parameters of the Poisson cluster process which give the first- and second-order statistic that match the Poisson hole process. The formula for calculating the outage probability is then adapted from [5], as shown later. By plugging the parameters into the formula, the outage probability is obtained.

Let $\mathcal{L}_{I_{\text{PCP}}}(s,z)$ be the Laplace transform of the interference in the Poisson cluster process, where $z \in \mathbb{R}^2$ is the location of the receiver under consideration. We have [5][6]

$$\mathcal{L}_{I_{\text{PCP}}}(s,z) = \exp\left\{-\lambda_l \int_{\mathbb{R}^2} [1 - \exp(-\bar{c}\varphi(s,z,y))] \, \mathrm{d}y\right\} \times \int_{\mathbb{R}^2} \exp(-\bar{c}\varphi(s,z,y)) f(y) \mathrm{d}y, \quad (24)$$

where

$$\varphi(s,z,y) = \int_{\mathbb{R}^2} \frac{g(x-y-z)}{s^{-1} + g(x-y-z)} f(x) \mathrm{d}x. \quad (25)$$

$f(x)$ is the PDF of the node distribution around its parent point. For the Thomas process,

$$f(x) = \frac{1}{2\pi\sigma^2} \exp\left\{-\frac{\|x\|^2}{2\sigma^2}\right\}, \quad (26)$$

and for the Matern process,

$$f(x) = \begin{cases} \frac{1}{\pi R^2} & \text{if } \|x\| < R, \\ 0 & \text{otherwise.} \end{cases} \quad (27)$$

[6]Note that this equation is different from (35) in [5] due to a different setup. In [5], the transmitter corresponding to the conditioned receiver is at the origin but in our setup, the transmitter is at a fixed distance away from the receiver.

Note that the interference is location-dependent, since the Palm distributions of the cluster and the hole processes are not stationary.

The Laplace transform of the interference among the cognitive receivers can then be approximated as

$$\mathcal{L}_{\tilde{I}_{cc}}(s) \approx \int_{z\in\mathbb{R}^2} \mathcal{L}_{I_{\mathrm{PCP}}}(s,z) f(z)\mathrm{d}z, \tag{28}$$

which is obtained by averaging over all the possible locations of the cognitive receivers. Furthermore, since every cognitive receiver is part of the cluster process (recall that $r_c \ll \lambda_c^{-\frac{1}{2}}$), it must belong to one of the clusters. That means only the locations within one cluster need to be considered.

VI. Conclusions

The interference in the cognitive radio network is hard to analyze due to the interaction between the primary and the cognitive users: the Poisson point process of the primary users and the Poisson hole process of the cognitive users are not independent. Two approaches have been taken in this paper: bounding and approximation. First, we have bounded the four types of interference for the bipolar model: the interference from the primary transmitters to the primary receivers, from the cognitive transmitters to the primary receivers, from the primary transmitters to the cognitive receivers, and from the cognitive transmitters to the cognitive receivers. The outage probabilities for the primary and the cognitive users are also bounded. Different exclusion region setups have been discussed, including exclusion regions around the primary receivers, primary transmitters, and both. Second, we have shown that the Poisson cluster process can model the Poisson hole process accurately, and a good estimate of the interference can be obtained. Consequently, the known results of the Poisson cluster process can be applied to the Poisson hole process formed by the active cognitive users.

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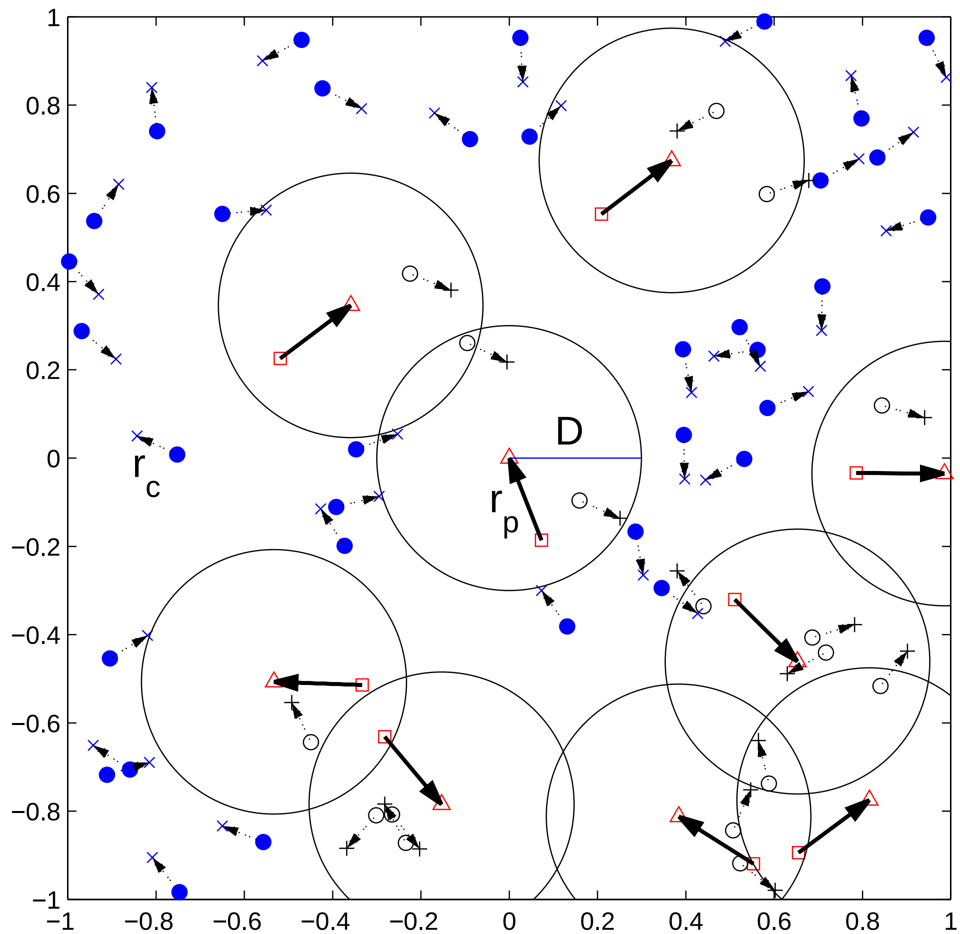


Figure 1. The bipolar network model. The squares are the primary transmitters and the triangles are the primary receivers, and the transmitter-receiver pairs are represented by thick lines with the arrows pointing to the receivers. The distance between a primary transmitter-receiver pair is r_p. The big circles are the exclusion regions with radius D. The filled circles are the cognitive transmitters and the x's are the cognitive receivers. The hollow circles and the +'s are the cognitive transmitters and receivers that are inactive due to the exclusion regions. The cognitive transmitter-receiver pairs are represented by thin lines with the arrows pointing to the receivers, and the distance between a cognitive transmitter-receiver pair is r_c.

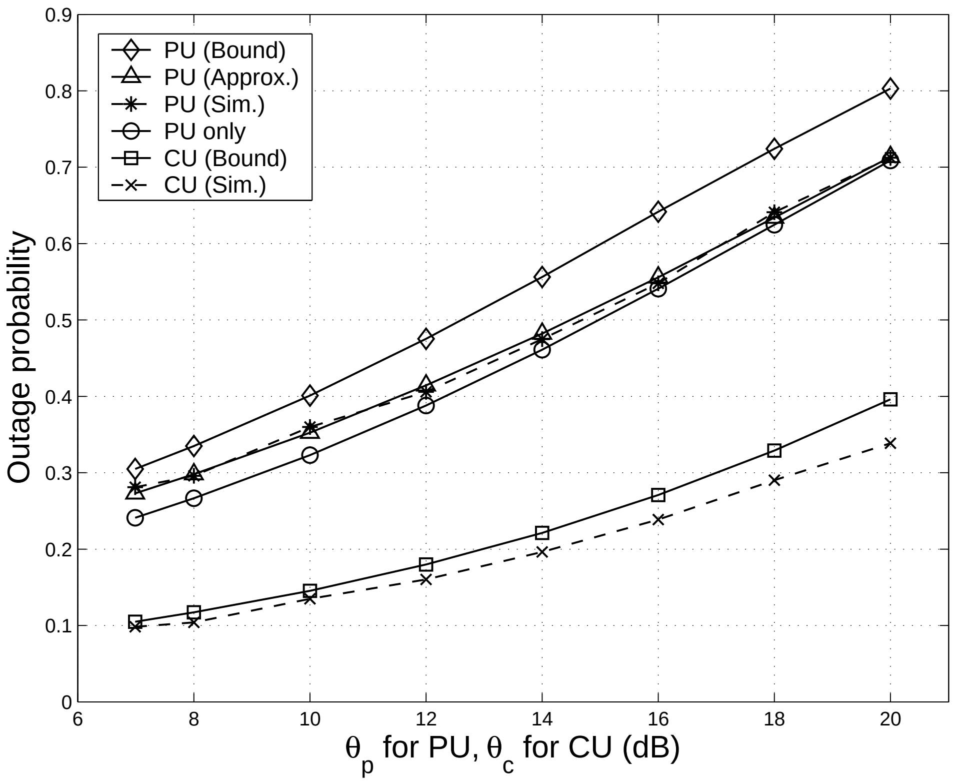


Figure 2. Bounds and simulation results of the outage probabilities of the primary and the cognitive users. For comparison, the outage probability in the primary network without the presence of cognitive users ("PU only" in the figure) is also shown. The simulation parameters are: $\lambda_p = 0.1$, $\lambda_c = 1$, $\mu_p = 1$, $\mu_c = 0.2$, $r_p = 0.5$, $r_c = 0.1$, $\beta = 81$, and $\alpha = 4$. D is determined using (1). When calculating the outage probability of the cognitive users, θ_p is set to 10.

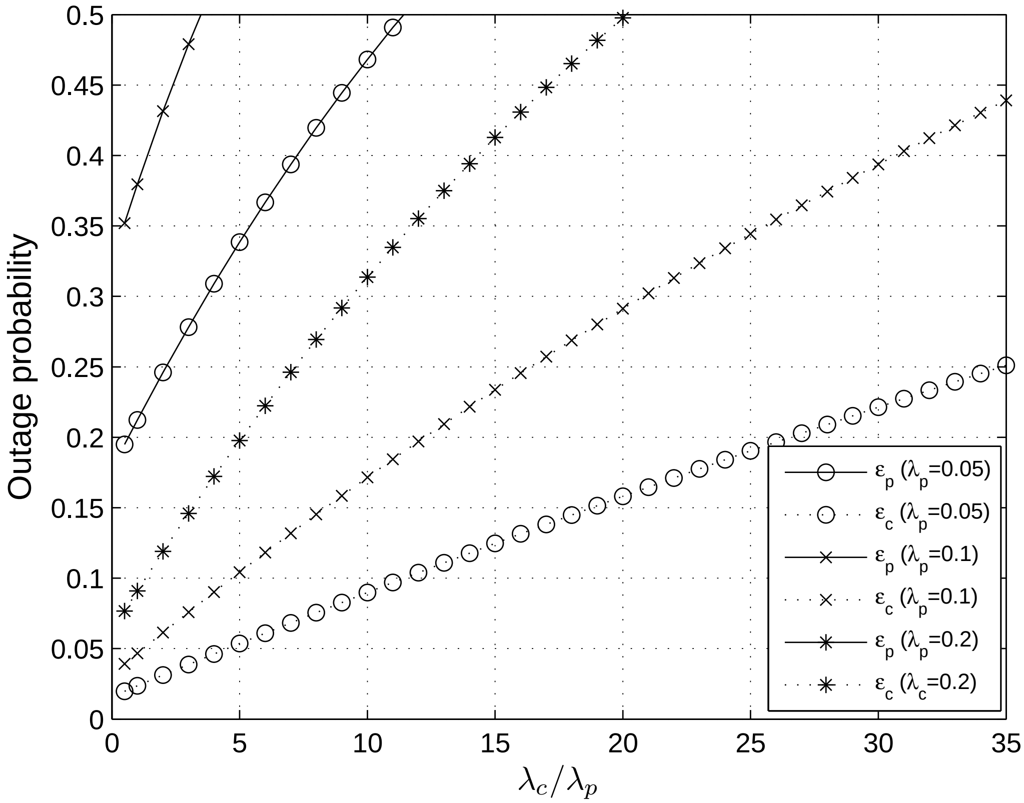


Figure 3. ϵ_p and ϵ_c as a function of λ_c/λ_p under different λ_p using (6) and (13), respectively. $\mu_p = 1$, $\mu_c = 0.2$, $r_p = 0.5$, $r_c = 0.1$, $\theta_p = 10$, and $\theta_c = 10$.

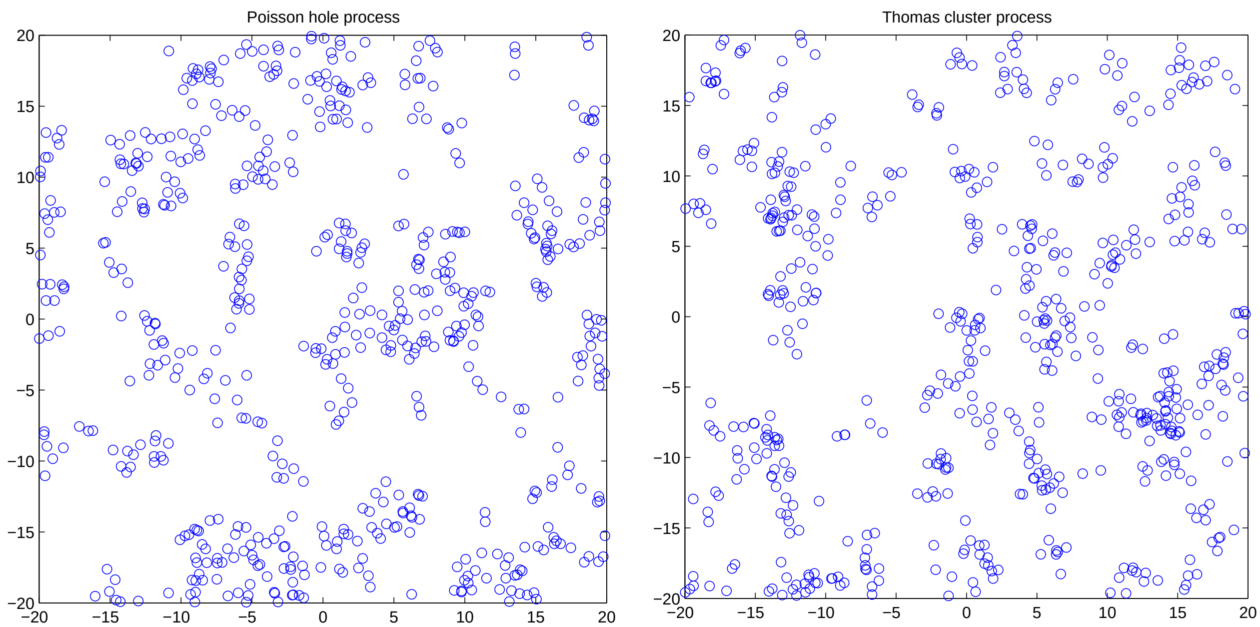
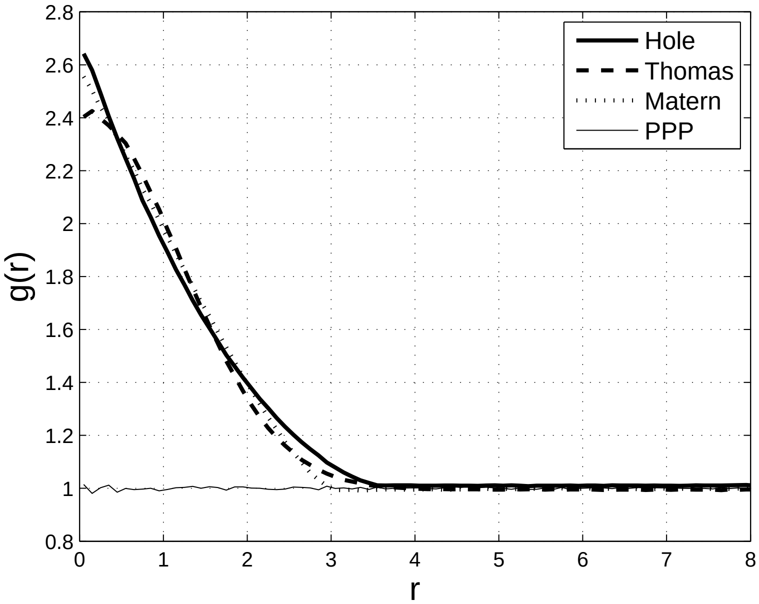
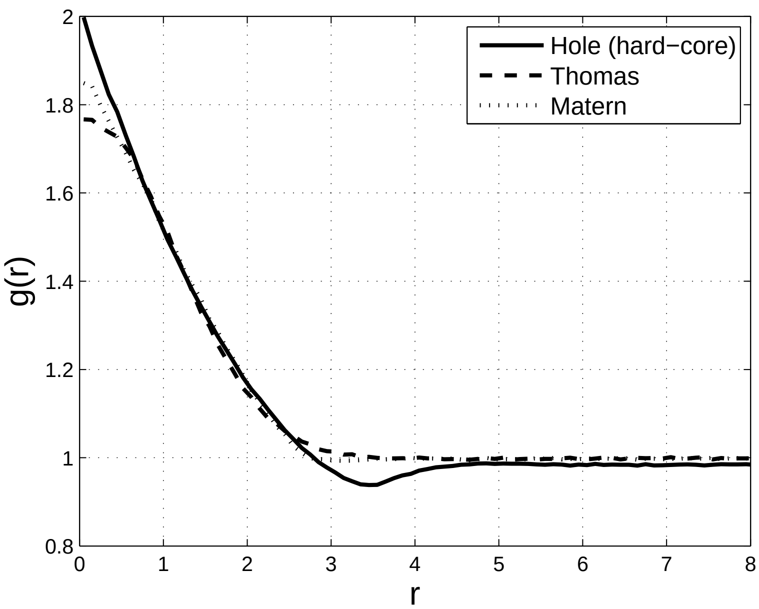


Figure 4. Comparison of the Poisson hole process (left) and the Thomas cluster process (right). $\lambda_p = 0.1$, $\lambda_c = 1$, $\mu_p = 1$, $\mu_c = 0.2$, $r_p = 0.5$, $\beta = 81$, $\alpha = 4$, $\theta_p = 10$, and $D = 1.7838$.



(a)



(b)

Figure 5. (a) Comparison of g-functions of the Poisson hole process, the Thomas cluster process, the Matern cluster process, and PPP. (b) Comparison of g-functions of the Poisson hole process resulting from the primary user hard-core process, the Thomas cluster process, and the Matern cluster process. In both cases, $\lambda_p = 0.1$, $\lambda_c = 1$, $\mu_p = 1$, $\mu_c = 0.2$, $r_p = 0.5$, $\beta = 81$, $\alpha = 4$, $\theta_p = 10$, and $D = 1.7838$. For (b), $D_p = 2$.

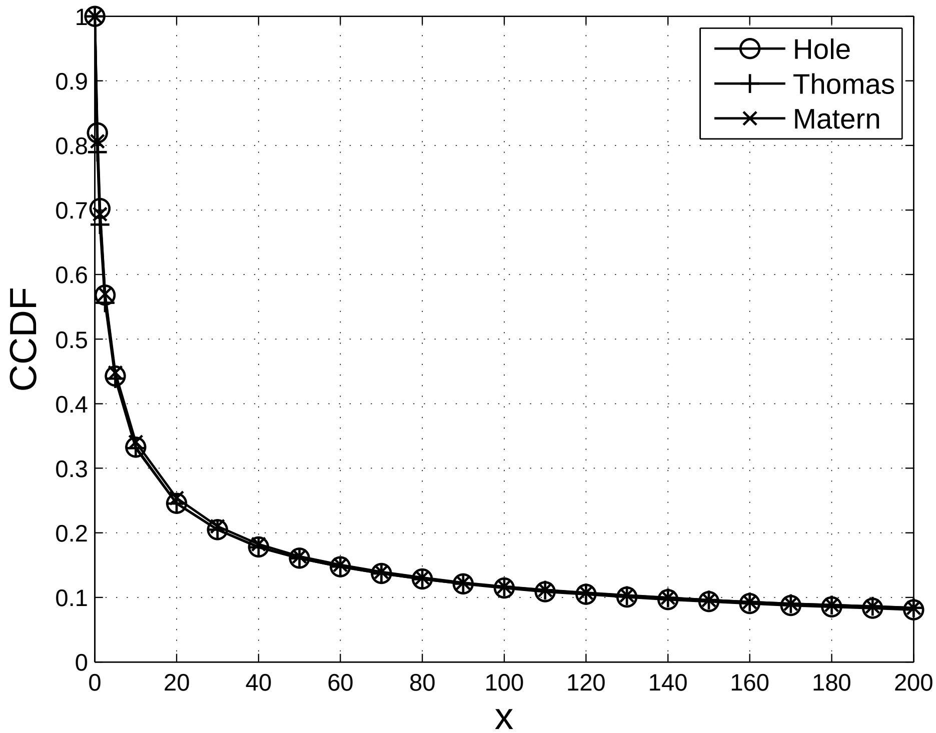


Figure 6. The CCDF $\mathbb{P}(I > x)$ of the interference among active cognitive users (Poisson hole process) and among the nodes in the Matern and Thomas cluster processes. The simulation parameters are $\lambda_p = 0.1$, $\lambda_c = 1$, $\mu_p = 1$, $\mu_c = 0.2$, $r_p = 0.5$, $\beta = 81$, $\alpha = 4$, $\theta_p = 10$, and $D = 1.7838$.