Math 20750
Spring, 2016

Assignment 11, due April 8

Reread §9.2 and read §§9.3,9.4 and 9.6 in Polking, Boggess and Arnold.

Do:
§9.2 #44,46,52,54
§9.3 #10,11,12,16,20,22
§9.4 #19,26

Additional Problem — required

(a) Let A be an $n \times n$ real matrix. Prove that for every $\mathbf{x}, \mathbf{y} \in \mathbf{R}^n$,

$$\langle A\mathbf{x}, \mathbf{y}\rangle = \langle \mathbf{x}, A^T\mathbf{y}\rangle$$

where A^T is the transpose of A and if $\mathbf{v} = (v_1, \ldots, v_n)^T, \mathbf{w} = (w_1, \ldots, w_n)^T$ with $v_j, w_k \in \mathbf{C},\ j, k = 1...n$, then $\langle\ ,\ \rangle$ is the inner product (the dot product if $\mathbf{v}, \mathbf{w} \in \mathbf{R}^n$), $\langle \mathbf{v}, \mathbf{w}\rangle = \sum_{j=1}^{n} v_j \overline{w}_k$.

(b) Prove that if in addition A is symmetric, i.e., $A = A^T$ and λ is an eigenvalue of A with eigenvector $\mathbf{v} \in \mathbf{C}^n$ then

$$\lambda\langle \mathbf{v}, \mathbf{v}\rangle = \overline{\lambda}\langle \mathbf{v}, \mathbf{v}\rangle$$

so $\lambda = \overline{\lambda}$ and all the eigenvalues of A are real.

(c) Under the hypotheses of (b), prove that if λ_1, λ_2 are eigenvalues of A, with corresponding eigenvectors $\mathbf{v}_1, \mathbf{v}_2$ and $\lambda_1 \neq \lambda_2$ then $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal, i.e., $\langle \mathbf{v}_1, \mathbf{v}_2\rangle = 0$

Reread chapters 14 and 15 in *Differential Equations with MATLAB®*.

Do as a MATLAB group:
Problem Set F #1 second and third systems. Do not use **pplane**.

Make sure the names of all members of your MATLAB group are on MATLAB assignment before turning it in.

Hint for Problem Set F #1

In (b), be sure to draw the eigenvectors if relevant and indicate the direction of increasing time on the trajectories.