Math 20750, Spring 2016

Review for Final — Outline of topics we covered

- What is an ODE and what is a solution
  - Geometric meaning of ODE
    - direction field
    - solution curves
- Special types of problems
  - Autonomous
  - Linear
    - Homogeneous linear
    - Nonhomogeneous linear
- Existence and uniqueness theorem
  - Applies to
    - First order equation
    - Systems
    - Second order equation
  - Stronger existence result in linear case
  - geometric interpretation of uniqueness
    - First order
    - Solution curves of autonomous systems
  - Dependence of solutions on initial conditions
- Linear problems
  - Solutions of a homogeneous linear problem form a vector space
    - Linear independence
    - Fundamental set of solutions
  - Two solutions of a nonhomogeneous linear problem differ by solution of corresponding homogeneous problem
    - Applies in particular to nonhomogeneous linear
      - first order ODE
      - systems
      - second order ODE
- Analytic solution

  - First order ODEs
    - Separable equations
    - Linear equations
      - can be solved using integrating factor or variation of parameters
    - Exact equations
  - Constant coefficient systems
    - Homogeneous
      - Technique for finding a fundamental set of solutions, using eigenvalues, eigenvectors, generalized eigenvectors
      - Matrix exponential
    - Nonhomogeneous
      - Variation of parameters
      - Undetermined coefficients
  - Constant coefficient second order ODEs
    - Homogeneous
    - Nonhomogeneous
      - Undetermined coefficients
      - Variation of parameters

- Qualitative analysis
  - First order autonomous equations
    - Using direction field
    - Phase line
    - Equilibrium points and solutions
      - Classification — unstable or asymptotically stable
    - Using phase line to sketch solutions
  - Autonomous systems
    - Stability
    - Phase plane (more generally, phase space), phase portrait
    - Linear planar systems
      - Classification of equilibrium point by type (e.g., nodal sink) in linear case
    - Nonlinear autonomous systems
      - Linearization
      - Classification of equilibrium point by type and stability from linearization when possible
      - Phase portraits
      - Nullclines

- Applications
  - Key idea — derivative is rate of change
  - Motion
  - Mixing problems (single and multiple tanks)
  - Population models
    - Malthusian model
    - logistic model
  - Personal finance
  - Spring systems, harmonic motion
    - Beats
    - Resonance
- Numerical solutions of ODEs
  - Euler's method
  - Runge–Kutta method
  - **ode45**
  - Errors, controlling error, reliability of numerical methods