Math 30750
Spring, 2017

Assignment 12, due April 28

Read: §5.6 (again), §§5.7-5.8

Do:
§5.6 #3,6,11,14 *Hint*: In 14(a), to prove the triangle inequality first show that $f(x) = \frac{x}{1+x}$ is increasing.
§5.7 #3,4,9 In #9 change $(0, b)$ to $(0, b]$.

Additional problem:

(i) Show that if T is defined by

$$T(f)(x) = 1 + \int_0^x f(t)\,dt$$

then $T : \mathcal{C}([0, \frac{1}{2}]) \to \mathcal{C}([0, \frac{1}{2}])$.

(ii) Show that T is a contraction where the metric is ρ_∞ with

$$\rho_\infty(f, g) = ||f - g||_\infty = \sup_{[0,\frac{1}{2}]} |f(x) - g(x)|.$$

(iii) Find the Picard iterates of the constant function 0, that is, let

$$f_1 = T(0),\ f_2 = T(f_1), \ldots, f_{n+1} = T(f_n), \ldots.$$

Find a formula for f_n.

(iv) Find $f_0 = \lim_{n\to\infty} f_n$ and show directly that f_0 is a fixed point of T.

Assignment 13, due Wednesday, May 3

Reread: §5.8

Do: §5.8 #1(a),(b),3,4