Math 40760, Fall 2009

Summary

Curves

- Local theory of parametrized curves
  - Parametrization by arc length
  - Frenet frame $\{t(s), n(s), b(s)\}$
  - Curvature, torsion and the Frenet equations
  - Fundamental Theorem of the Local Theory of Curves
  - Local canonical form (Taylor expansion of $\alpha(s)$ in terms of Frenet frame)
- Global theory of curves
  - Total twist
    - The total twist of a curve on the sphere is 0.
  - Fenchel's Theorem (Theorem 3 on p. 399)
  - Isoperimetric inequality

Surfaces—differentiable properties

- Regular surfaces, local parametrizations or local coordinates
- Differentiable functions on surfaces
- Tangent plane
- Differentiable maps between surfaces and their differentials
- Orientability

Surfaces—local geometry

- First fundamental form
  - Arc length of a curve on a surface
  - Area of a surface
- Gauss map and second fundamental form
  - Normal curvature
  - Principal curvatures, principal directions
  - Asymptotic directions, asymptotic curves

- Mean curvature
- Gaussian curvature
  - Types of points: elliptic, hyperbolic, parabolic, planar
  - How the surface sits in relation to tangent space (elliptic and hyperbolic points only)
- Vector fields on surfaces
- Types of special coordinates
  - Coordinate lines are orthogonal
  - Coordinate curves are asymptotic curves (hyperbolic case only)
  - Coordinate curves are lines of curvature (only guaranteed possible in neighborhood of nonumbilical point)
  - Isothermal coordinates
- Minimal surfaces
  - Criterion for minimal surface in isothermal coordinates

Intrinsic geometry of surfaces

- Intrinsic: only depends on I
  - Preserved by isometries
- Moving frames
  - Gauss equations
  - Weingarten equations
  - Cartan's structure equations
  - Codazzi equation
- Theorema Egregium
- Geodesic curvature
  - Geodesics
- Euler characteristic
- Gauss–Bonnet Theorem
- Index of a vector field at a singularity
- Poincaré's theorem