Math 40760
Fall, 2009

Paper or Project, Due November 23

Your paper or project should be a well-written explanation of the topic and should convince me that you have a good understanding of it. It should not be something copied from a book, article, web site, ... Include references and a list of any computer programs you used. Here are some possible topics. If you want to do a topic not on this list, please discuss it with me ahead of time.

1. Learn more about the total twist of a closed curve. For example, learn about Scherrer's theorem: If the total twist of every closed curve on the surface S is 0 then S is a sphere. Or, learn about Banchoff and White's theorem that the total twist of a closed curve is invariant under an inversion. See Banchoff and White, "The behavior of the total twist and self-linking number of a closed space curve under inversions," *Math. Scand.* 36 (1975), no. 2, 254–262. (*Note*: The online version of the first page of the article is tacked on to the end of the previous article.)

2. Learn about the geometry of soap bubbles. Read Almgren and Taylor, "The geometry of soap films and soap bubbles" in *Scientific American*, July, 1976.

3. Learn about differential geometry and DNA and the mathematics of supercoiling of DNA. (Pohl, "Differential Geometry and DNA" in *The Mathematical Intelligencer*, March, 1980, and Bauer, Crick and White, "Supercoiling of DNA" in *Scientific American*, July, 1980)

4. Learn about the proof of equality in Fenchel's Theorem and about the Fary–Milnor Theorem. See Chern's article in *Studies in Global Geometry and Analysis* (on reserve in the Mathematics Library) and §5.7 in Do Carmo for two different approaches.

5. Learn about the radius of curvature according to Huygens or about curvature and elastic force (Lodder, "Curvature in the calculus curriculum," *American Mathematical Monthly*, Aug.-Sept., 2003)

6. Learn more about minimal surfaces and make some models.

7. Learn about hyperbolic geometry, including the Poincaré half plane model and the Poincaré disk model and their geodesics. How is the disk model related to some famous Escher prints such as *Circle Limit 1*? (See Dunham's article, article "Hyperbolic Art and the Poster Pattern" for information about Escher and hyperbolic geometry.)

8. Learn about the four vertex theorem. Compare the proof in Do Carmo with Osserman's proof of "The four–or–more vertex theorem" in the American Mathematical Monthly 92 (1985), 332-337.

9. Create some computer tools for visualizing some of the topics. One possibility is to use Prof. Banchoff's applets to create new applets. These could include illustrating concepts he doesn't have applets for or improving his applets or both. Other possibilities are creating easy-to-use Maplets or MATLAB GUIs to illustrate some things

in differential geometry. Possible examples include one to compute the Frenet frame, curvature and torsion of a parametrized space curve, one to recover a plane curve from its curvature and torsion, one to compute geodesics on a closed surface.

10. Learn about the problem "Can one hear the shape of a drum?" (For this project, you need some knowledge of partial differential equations.)

11. Read and write a report on *Poetry of the Universe* by Robert Osserman. The report should include a discussion of a topic in the book from the point of view of differential geometry.

12. The Ricci curvature flow was used by Perelman in his proof of the Poincaré conjecture, one of the seven Clay Mathematics Institute Millenium Problems. There is a $1,000,000 prize for solving one of these problems. Learn a little about the Poincaré conjecture and how the Ricci flow comes in by reading Milnor's article. Then learn about the much simpler case of curvature flow in the plane from one or more of the following papers:

> Gage, "An isoperimetric inequality with applications to curve shortening" in the Duke Mathematical Journal 50 (1983), 1225-1229,
>
> Gage, "Curve shortening makes convex curves circular" in *Inventiones mathematicae* 76 (1984), 357-364,
>
> Gage and Hamilton, "The heat equation shrinking convex plane curves" in the *Journal of Differential Geometry* 23 (1986), 69-96.

13. Watch the *Nova* series *The Elegant Universe*. Write a report about it. Learn more about and include a discussion of something from the series from the point of view of differential geometry.