Models for Measuring and Hedging Risks in a Network Plan

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Robust Network Planning

Design a network and place in sufficient link and node capacity to satisfy the expected demand for point-to-point connections in a cost-effective manner,

while hedging against the prominent operating risks.
Planning Decisions

Scheduling the expansion of link and node capacity during planning horizon

- Routes for pt-to-pt connections
- Timing / sizing of (equipment) purchases
- Technology decisions
- Adjustments to network topology
Demand Uncertainty Risk

- Demand may not arise as anticipated.
- Demand levels naturally fluctuate throughout the planning horizon – “Churn”.
- Prior data determines a distribution of potential values.
- Correlation / elasticity of demand for services across locations are hard to identify.

Unexpected demands may not be served, resulting in a loss of potential revenue.
Connection Risk

Some link or node (equipment) may fail during network operation, resulting in:

- Temporary disruption of some services
- Loss of customer revenue
- Financial penalties, lawsuits
- Exposure of customers to danger, e.g., loss of 911 service, home heat, electricity
- Potential defection of customers
Utilization Risk

Cost of lost opportunity associated with:
- Over-estimating potential demand
- Purchasing/placing capacity too early
- Cost / Revenue uncertainty
- Providing traffic protection to customers not paying for it
- Dedicating protection capacity to unlikely or benign failure scenarios

Address this risk to keep plan economical
Hedging Connection Risk

If the network is 2-connected and has accessible extra capacity, then network is “Survivable”: demands can still be satisfied even if some link or node is rendered useless.

[Diagrams showing Working Routes and Protection Routes]
Hedging Strategies

- **Route Diversity:** Split traffic over multiple (diverse) paths to satisfy demand for a pt-to-pt connection.
  
  (Low Cost; Low Effectiveness)

- **Protection:** Back-up paths with capacity to be used only during a network failure. May be dedicated to specific connections or to specific links.
  
  (High Cost; High Effectiveness)

- **Restoration:** Use smart switching to access available capacity to recover traffic lost to a network failure.
  
  (Cost and effectiveness dependent on traffic distribution at failure time and on the quality of capacity decisions)
“Infinite Severity” Model

Connection Risk Assumptions:

- Any loss of traffic due to a link or node failure is unacceptable.
- The relative probability of the potential network failure events is irrelevant.

Hedging Strategy: “100% Survivability”

All of the network traffic can be recovered despite any link or node failure in the network.
Infinite Severity Model

- Providing 100% Survivability may be impossible or prohibitively expensive.
- Cost-benefit analyses are irrelevant.
  
  (Math Prog. Model: Min Cost Objective with Survivability Constraints)

- Some customers’ traffic may not warrant the expense of protection/restoration.
- Some failure scenarios are more likely / more severe than others.
Model Formulation

Let $z$ represent the capacity assigned to the links.
Let $x(k)$ represent the allocation of connection paths to satisfy demand $k$.

*Network capacity should be sufficient to accommodate all demands under normal conditions:*

$$
\sum x_k = d_k \quad \text{all demands } k \\
\sum_k P_k x_k \leq z
$$

Path-arc incidence matrix for connections satisfying $k$
Formulation

For each potential network failure $f$:

$$x_k^f = I^f x_k$$

Some working path assignments for demand $k$ are lost.

Let $y(k,f)$ represent the use of some paths to recover some of demand $k$ after failure $f$.

$$\sum (x_k^f + y_k^f) = d_k - \text{loss}_k^f \quad \text{all } k$$

$$\sum_k P_k^f (x_k^f + y_k^f) \leq D^f z$$

Unmet demand

Modified path-arc incidence matrix for demand $k$ during failure $f$

Some network capacity is lost to the failure
Formulation

Controlling the level of survivability:

Objective function: \( \text{Minimize } c^T z \)

100% Survivability Constraint:

\[ \text{loss}_k^f = 0 \quad \text{all } f, \text{ all } k \]
Flexible Alternatives

**Objective: Effective use of capital**

- Allow lower levels of Survivability
- Apply targeted survivability constraints
  - Specific customers’ demand or failure scenarios
- Perform cost / benefit analyses for risk hedging
  - Models for marginal cost of restoration capacity
  - Valid (dollar-based) measures for connection risk
- Integrated models for hedging both demand uncertainty and connection risk
Lower Survivability Levels

What does it mean if a network is 90% Survivable?

- At least 90% of the total traffic survives any failure.
- Each customer is guaranteed that at least 90% of their traffic survives any failure.
- In 90% of the possible failure scenarios, all of the traffic survives.
- The probability that any unit of traffic will be lost to some failure is less than 10%.
- The expected proportion of traffic to survive, over the possible failure events is 90%.

For 100% all of these meanings are equivalent!
Pictorial Model – Demand k

A: Average protection of demand k from potential failures
B: Worst-case = Survivability guarantee for k
C: Prob. that demand k is insulated from some random failure
The Severity of a Failure

D: Proportion of total traffic surviving the failure.
E: Minimum survivability guarantee provided to customers.
Modified Formulations

Controlling the level of survivability:

Through the objective function:

\[
\text{Minimize } \mathbf{c}^T \mathbf{z} + \lambda \times p(\text{loss})
\]

Capacity expansion cost \hspace{1cm} Penalty for losing service

and/or through the constraints:

\[
\text{loss}_k^f \leq s(d_k)
\]

Functions to set appropriate limits on loss
Modified Formulations

Constraints on loss of demand $k$: \[
\sum_f \frac{\text{loss}^f_k}{F} \leq (0.1)d_k \quad \text{(Area A in chart is 90%)}
\]

\[
\text{loss}^f_k \leq Mw^f_k, \quad \text{where } w^f_k \in \{0,1\}
\]

\[
\sum_f w^f_k / F < (0.5) \quad \text{(Length C in chart is 50%)}
\]

Constraint on severity of failure $f$: \[
\sum_k \text{loss}^f_k \leq (0.1)\sum_k d_k \quad \text{(Area D in chart is 90%)}
\]

These can be applied flexibly to specific demand/failures.
Costs/Benefits of Hedges

- Penalty function must capture (true) costs of disrupted connections – beyond revenue loss.
- The parameter $\lambda$ allows planner to control tradeoff between survivability and expansion costs.

Note: The MILP problem as formulated is quite large and complex
Integrated Models

Restoration (protection) capacity also provide a hedge against demand uncertainty.

- Develop planning models that measure the combined effectiveness of hedging strategies
- Survivability measures when demand is uncertain
- Measuring the marginal costs associated with hedging capacity – which capacity is extra?!
- MILP formulation is even more complex!
Integrated Evaluations

- At least 90% survivability is provided (to the offered services) in every demand scenario.
- The average network survivability over the demand scenarios is at least 90%.
- In at least 90% of the demand scenarios there is 100% survivability (for the offered services).
- At least 90% of the demands are offered 100% survivability in all of the demand scenarios.
Hedging Uncertainty Risk

If accessible capacity is placed throughout the network at sufficient levels, the network might accommodate a variety of potential demand scenarios.

**Deterministic Approach:** Apply a solution model based on the *expected* pt-to-pt demand levels (or some higher percentile).

**Stochastic Programming Approach:** Maximize some probabilistic profit function based on the service provided *over a set* of demand scenarios.
Simulation-based Approach

- For each demand scenario, determine a routing over the network topology.
- For each node / link, collect statistics about the capacity requirements, e.g.,

<table>
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<th>Capacity</th>
<th>Prob.</th>
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<tr>
<td>50</td>
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</tr>
<tr>
<td>60</td>
<td>20%</td>
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<td>50%</td>
</tr>
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<td>80%</td>
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<tr>
<td>90</td>
<td>95%</td>
</tr>
<tr>
<td>100</td>
<td>100%</td>
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</tbody>
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Percentage of scenarios in which the capacity is sufficient to accommodate all of the demands.
Approach, cont’d

Determine a capacity level:
(Note similarity to “Newsvendor model”)

Revenue index +
shortage penalty = 10 *
Cost index +
overage penalty

Note: Shortage penalty ratio would have
to be set above 20 for the min-risk
capacity to be the most profitable.