Unifying Dynamical Systems and Complex Networks Theories

~ A Proposal of “Generative Network Automata (GNA)” ~

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What’s Been Used in Complex Systems Research (1)

- **Nonlinear dynamical systems**
  - Ordinary / partial differential equations
  - Iterative maps
  - Artificial neural networks
  - Random Boolean networks
  - Cellular automata

- **Describe dynamics in a phase space with fixed dimensions; no topological changes**
What’s Been Used in Complex Systems Research (2)

- Complex network growth models
  - Recent advances of “network science”
  - Small-world / scale-free networks

- Describe statistical properties of complex networks resulting from exogenous topological transformations
  - Limited attention paid to dynamical state transitions; no autonomous topological transformations
What’s Missing in Them?

- Many real-world complex systems change their states and topologies simultaneously due to their own dynamical rules

<table>
<thead>
<tr>
<th>System</th>
<th>Nodes</th>
<th>Edges</th>
<th>States of nodes</th>
<th>Topological changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Organism</td>
<td>Cells</td>
<td>Intercellular communication channels</td>
<td>Gene/protein activities</td>
<td>Fission and death of cells during development</td>
</tr>
<tr>
<td>Ecological community</td>
<td>Species</td>
<td>Interspecific relationships</td>
<td>Population</td>
<td>Speciation, invasion, extinction of species</td>
</tr>
<tr>
<td>Human society</td>
<td>Individual</td>
<td>Conversations, social relationships</td>
<td>Social, professional, economical, political, cultural statuses</td>
<td>Changes in social relationships, entry and withdrawal of individuals</td>
</tr>
<tr>
<td>Communication network</td>
<td>Terminals, hubs</td>
<td>Cables, wireless connections</td>
<td>Information stored and transacted</td>
<td>Addition and removal of terminal or hub nodes</td>
</tr>
</tbody>
</table>
Objective

- To develop a novel modeling framework “Generative Network Automata (GNA)” that can effectively describe the dynamics of a broader class of complex systems whose states and topologies both keep changing due to their own dynamical rules.
  
  - This is uniquely achieved by integrating:
    - Nonlinear dynamical systems (automata)
    - Complex network growth models
    - Generative graph grammars
Configuration

- Defined as a directed graph where each node has a state $s_i \in S$

- Config. at time $t$ is given by $<V_t, C_t, L_t>$:
  - $V_t$ A set of nodes
  - $C_t: V_t \rightarrow S$ States of nodes
  - $L_t: V_t \rightarrow V_t^*$ Ordered lists of neighbors
Dynamics

- Evolution of GNA is described as a repetitive rewriting process

- Defined by $<E, R, I>$:
  - $E$: Extraction mechanism
    - Selects part of GNA (subGNA) for rewriting
  - $R$: Replacement mechanism
    - Produces a new subGNA as a replacement
    - Specifies how to embed it into the rest of the GNA
  - $I$: Initial configuration
A Rewriting Event

(a) $E$ selects part of the GNA. (b) $R$ produces a new subGNA as a replacement of the old subGNA and also specifies the correspondence of nodes between old and new subGNAs. The “bridge” edges that used to exist between the old subGNA and the rest of the GNA remain unconnected and open. (c) The new subGNA is embedded into the rest of the GNA according to the node correspondence. (d) The updated configuration after this rewriting event.
Handling Multiple Rewritings

- Rewriting is typically defined as **asynchronous** processes
  - Synchronous scheme may cause conflicts between multiple rewriting events

- GNA may be updated **synchronously** if
  - rewriting rules are all context-free (i.e., $E$ always selects just one node), or
  - GNA is used to simulate conventional dynamical networks, etc.
Simulation Software

- Preliminary simulator implemented as an add-on package in Wolfram Research Mathematica
  - Still under development but available upon request

- Development of full-scale simulation software planned
Generality of GNA

- GNA can uniformly represent in $<E, R, I>$:
  
  - **Conventional dynamical systems models**
    - If $R$ always conserves local network topologies and modifies states of nodes only
    - E.g. CA, ANNs, RBNs
  
  - **Complex network growth models**
    - If $R$ causes no change in local states of nodes and modifies topologies of networks only
    - E.g. small-world, scale-free networks
Summary

- Proposed GNA, a novel modeling framework of complex systems
  - Describes both state transitions and topology transformations via repetitive rewritings
  - Provides a common “format” of modeling applicable to diverse fields
- Demonstrated its generality and explored possible dynamics in its simplest form
- Poster/preprints available
What’s Next

- Develop appropriate metrics for local rewriting rules, resulting global networks, and their quantitative relationships
- Evaluate the designability/controllability of GNA evolution
- Develop efficient simulation/analysis tools
- Apply GNA to real-world problems
  - Modeling morphogenesis, social dynamics, self-organization of collective intelligence