Endogenous Money and Exchange Rates

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1 A set of exchange rate puzzles

1. The PPP Puzzle:

    - Why is the real exchange rate both highly persistent and volatile?
    - Why is the nominal exchange rate so highly correlated with the real exchange rate?
    - A successful model will explain these facts. Chari, Kehoe, McGrattan do with price stickiness lasting 4 quarters and relative risk aversion coefficient of 5.
    - Flexible price models have difficulty explaining volatility (without high risk aversion). Sticky price models have difficulty explaining persistence (without unrealistically sticky prices).

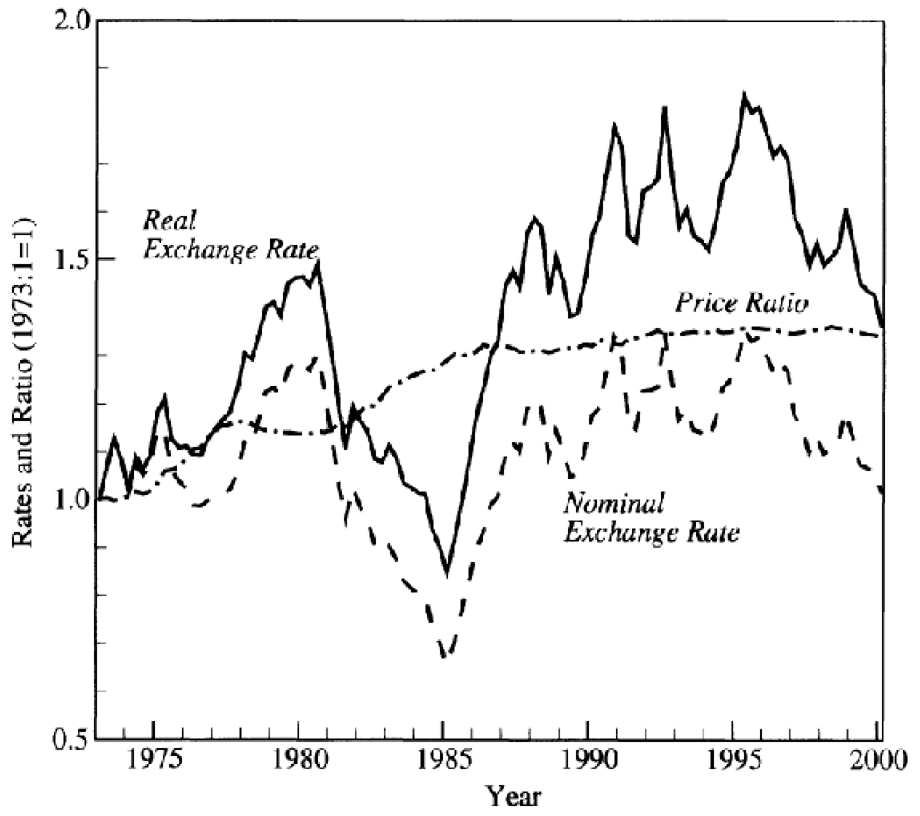


FIGURE 1

Exchange rates and the price ratio between the U.S. and Europe.

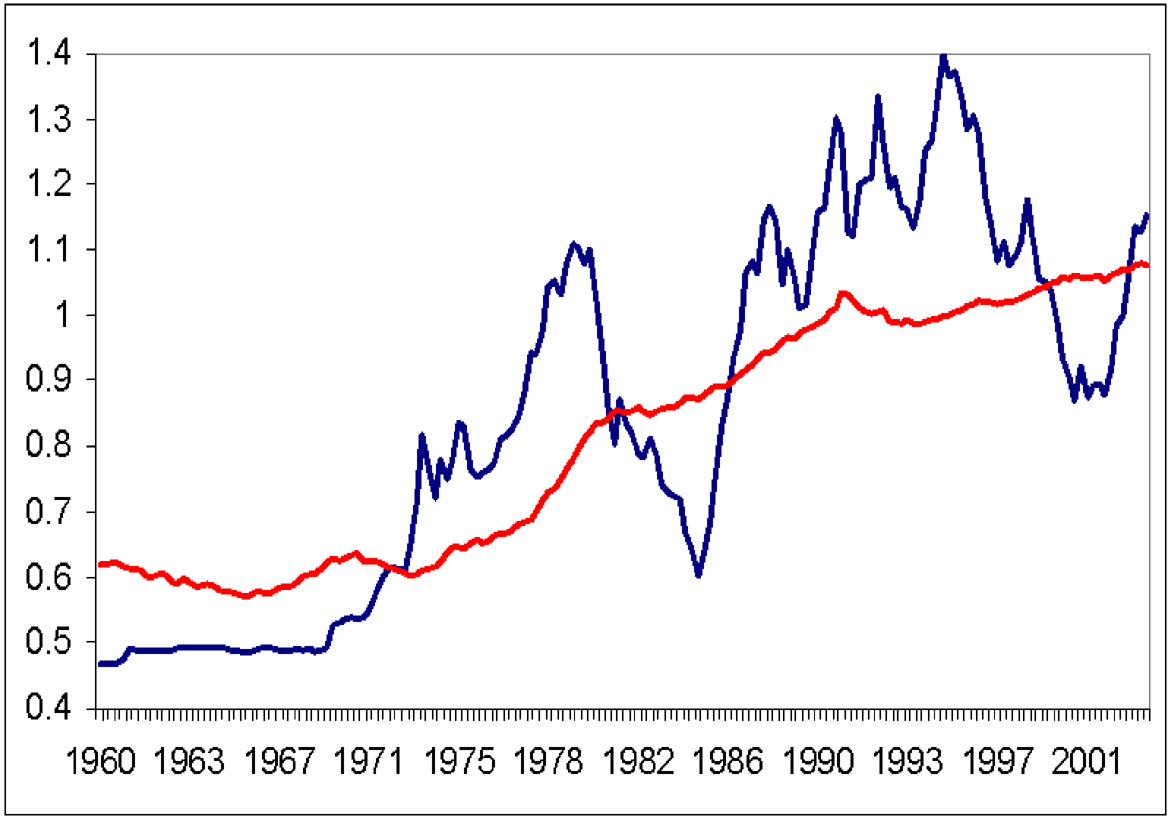
Note: the real exchange rate is eP^*/P, where the nominal exchange rate e is the U.S. dollar price of a basket of European currencies, P^* is an aggregate of European CPIs, and P is the U.S. CPI. The price ratio is P^*/P

From Chari, Kehoe and McGrattan 2002, RE Stud.

2. Traditional monetary approachs

- The PPP theory: Higher inflation should lead to dollar weakening.

$$s = p - p^* \rightarrow \Delta s = \pi - \pi^*$$



- Monetary models
  - Lucas JME 1982

$$s = (m - m^*) - \mu (y - y^*)$$

  - Obstfeld–Rogoff JPE 1995.

$$s = (m - m^*) - \frac{1}{\epsilon} (c - c^*)$$

  - Original monetary model.

$$\begin{aligned} m_t - p_t &= \alpha + \gamma y_t - \lambda i_t + v_t \\ m_t^* - p_t^* &= \alpha + \gamma y_t^* - \lambda i_t^* + v_t^* \\ q_t &= s_t + p_t^* - p_t \\ i_t - i_t^* &= E_t (s_{t+1}) - s_t \end{aligned}$$

- Substitute into UIP for the difference equation

$$
\begin{aligned}
s_t &= \frac{1}{1+\lambda}\left(\underbrace{(m_t - m_t^*) - \gamma\,(y_t - y_t^*) + q_t}_{f_t} - \underbrace{(v_t - v_t^*)}_{z_t}\right) + \frac{\lambda}{1+\lambda}E_t\,(s_{t+1}) \\
f_t &= (m_t - m_t^*) - \gamma\,(y_t - y_t^*) + q_t
\end{aligned}
$$

- Iterate forward,

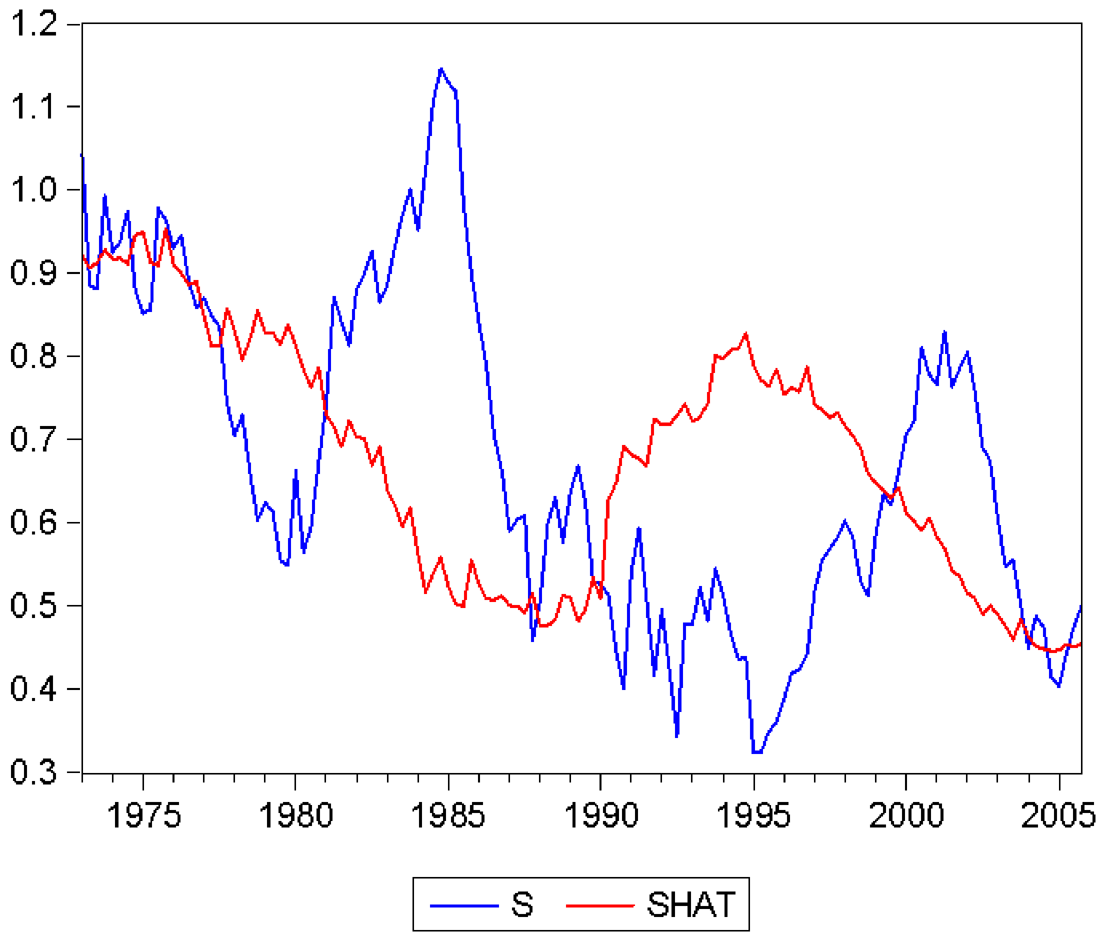
$$s_t = \frac{1}{1+\lambda}E_t\sum_{j=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^j\left(f_{t+j} + z_{t+j}\right)$$

- If f_t is AR(1) with coefficient ρ,

$$s_t = \frac{1}{1+\lambda\,(1-\rho)}\left[(m_t - m_t^*) - \gamma\,(y_t - y_t^*)\right]$$

- Low-brow econometric results.

(a)



3. Relatively new empirical evidence on news and the exchange rate.

- Real-time (5 minute sampling) exchange rate and announcement news (errors). Anderson et. al. AER 2003.

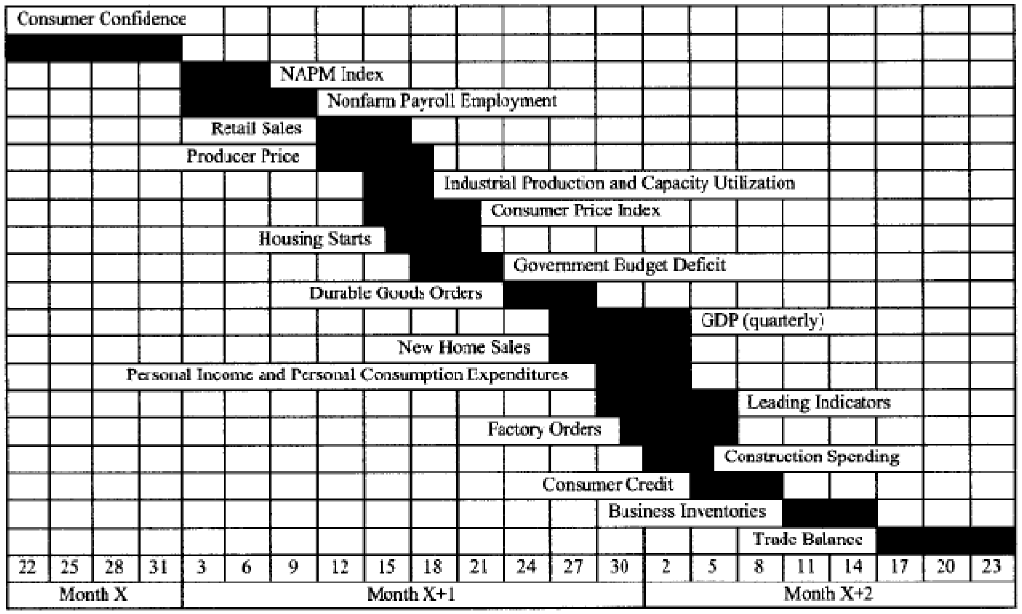


FIGURE 2. U.S. MACROECONOMIC ANNOUNCEMENT RELEASE DATES DATA FOR MONTH X

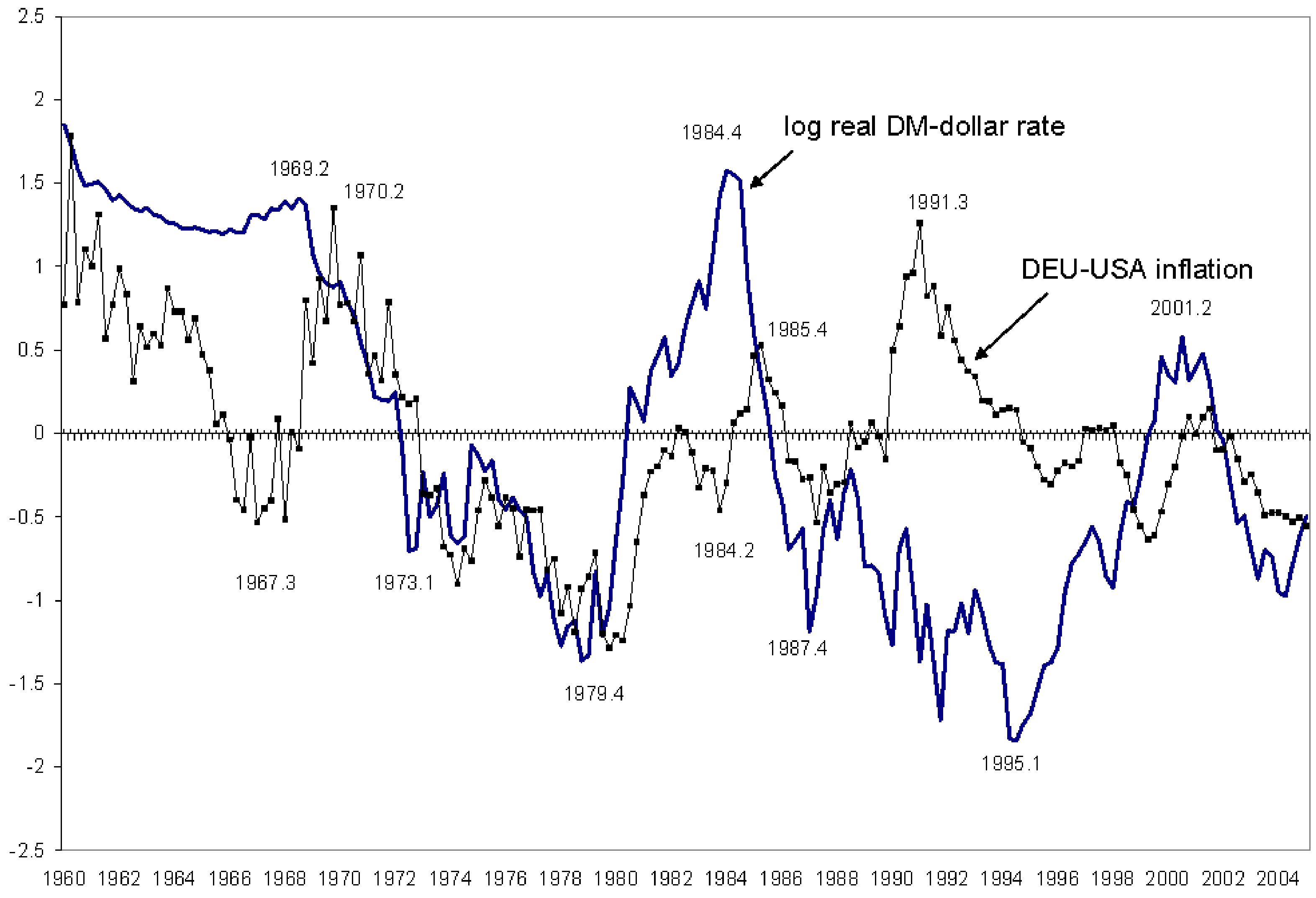
Notes: We show the sequence of announcement dates corresponding to data for month X, for most of the economic indicators used in the paper. For example, March (month X) consumer credit data are announced between May (month X − 2) 5 and May 10. GDP data are special, because they are released only quarterly. Hence, the GDP data released in a given month are either advance, preliminary, or final depending on whether the month is the first, second, or third of the quarter. For example, first quarter Q1 GDP advance data are announced between April (month X − 1) 27 and May 4, first quarter GDP preliminary data are announced between May (month X + 2) 27 and June 4, and first quarter GDP final data are announced between June (month X + 3) 27 and July 4. The table is based on 2001 Schedule of Release Dates for Principal Federal Economic Indicators, produced by the U.S. Office of Management and Budget and available at http://clinton4.nara.gov/textonly/OMB/pubpress/pei2001.html.

We model the 5-minute spot exchange rate, R_t, as a linear function of I lagged values of itself, and J lags of news on each of K fundamentals:

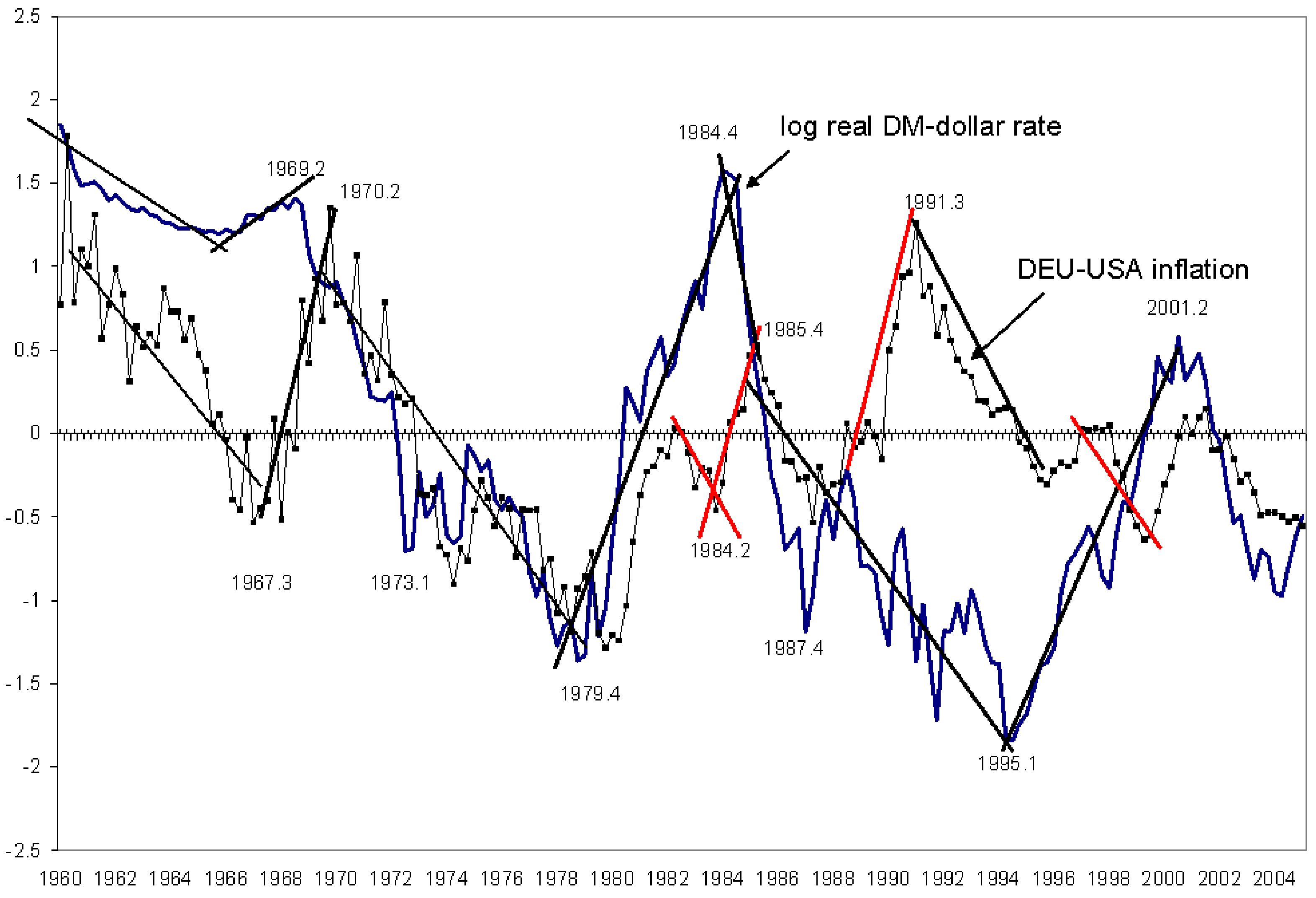
$$(1)\qquad R_t = \beta_0 + \sum_{i=1}^{I} \beta_i R_{t-i} + \sum_{k=1}^{K} \sum_{j=0}^{J} \beta_{kj} S_{k,t-j} + \varepsilon_t, \qquad t = 1, \dots, T.$$

- Their findings:
  - Only unanticipated shocks to the fundamentals have a statistically significant effect on the exchange rate.
  - Good real news good news for the dollar: Unexpectedly strong real indicator is followed by a stengthening of the dollar. Consistent with most theory.
  - Bad news about inflation is good news for the dollar: Unexpectedly high inflation is followed by a strengthening of the dollar. This is contrary to most theory.

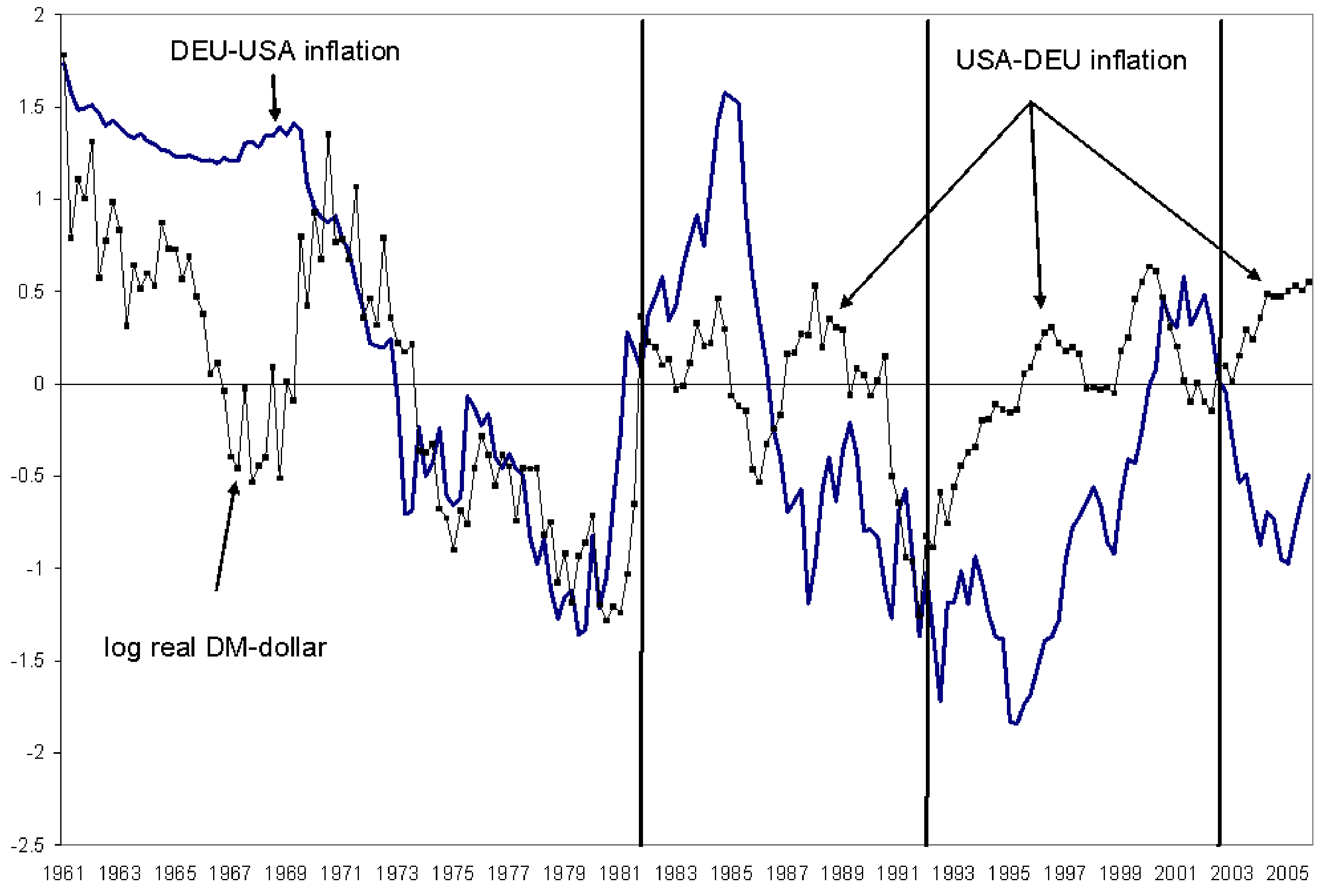
Inflation Differentials and Real Dollar–DM Rate



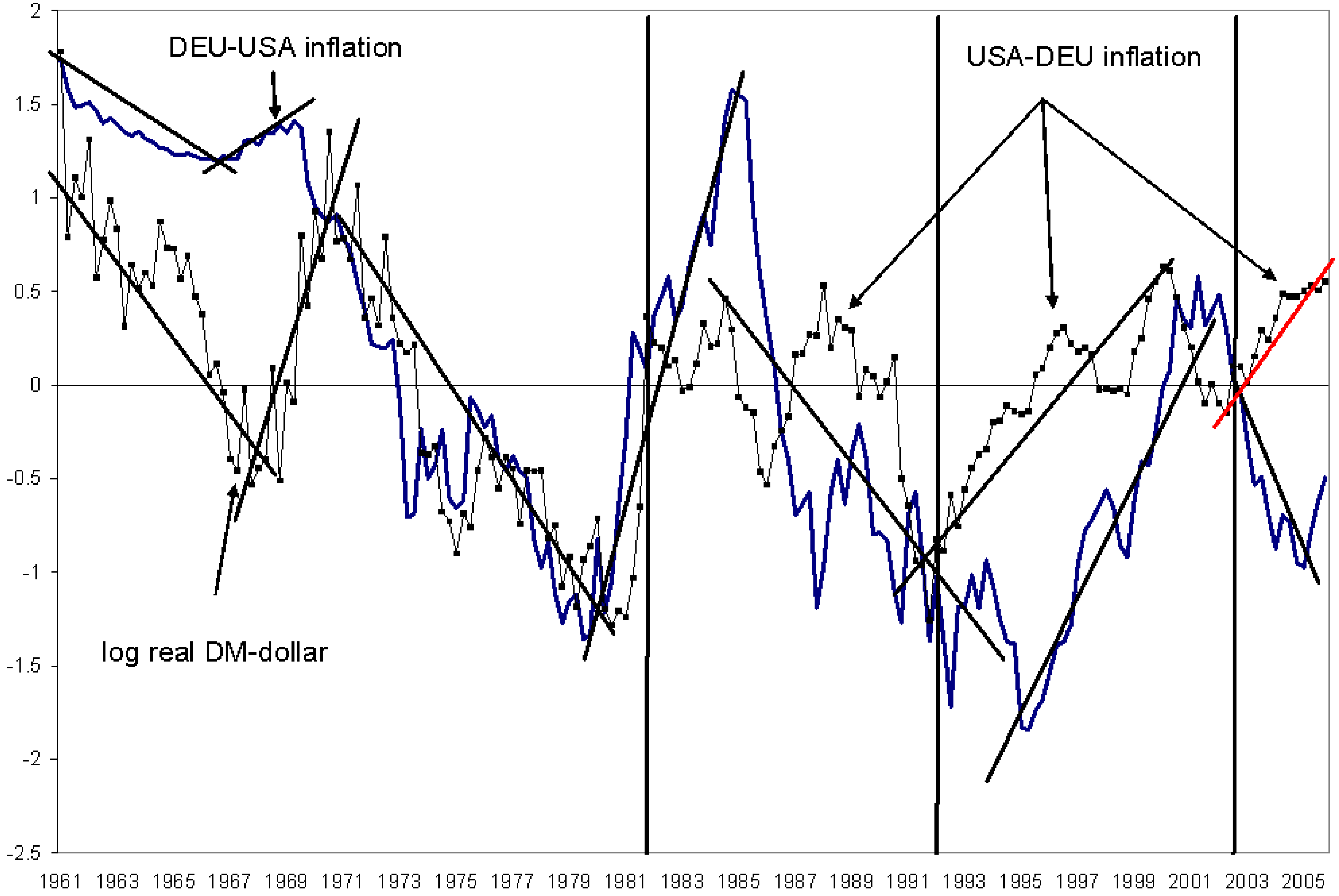
Inflation Differentials and Real Dollar–DM Rate



Inflation Differentials and Real Dollar–DM Rate



Inflation Differentials wutg Structural Break and Real Dollar–DM Rate



4. Simplified Taylor-rule model with uncovered interest parity. Taylor rule (without interest rate smoothing).

   - Let i_t be the target rate,
   - $\bar{\pi}$ be targeted inflation,
   - $x_t = y_t - y_t^p$ be the output gap,
   - $\bar{\imath}$ be the natural nominal rate,
   - $\bar{s}_t = p_t - p_t^*$ be an exchange rate target. Then the deviation from the targeted exchange rate is $q_t = s_t + p_t^* - p_t$.

- In Germany,

$$i_t = \gamma_\pi \left(E_t \pi_{t+1} - \bar{\pi}\right) + \gamma_x x_t + \gamma_q q_t$$

- In the U.S., no exchange rate feedback

$$i_t^* = \gamma_\pi \left(E_t \pi_{t+1}^* - \bar{\pi}^*\right) + \gamma_x x_t^*$$

  Important for $\gamma_\pi > 1$. This is called the Taylor Principle.

  – Take Germany to be the home country. Ignore constant terms. Assume identical targets and coefficients across countries.

– Exploit UIP

$$\begin{aligned} E_t s_{t+1} - s_t &= i_t - i_t^* \\ E_t s_{t+1} - E_t\left(p_{t+1} - p_{t+1}^*\right) - s_t - (p_t - p_t^*) &= i_t - i_t^* - E_t\left(\pi_{t+1} - \pi_{t+1}^*\right) \end{aligned}$$

$$E_t q_{t+1} - q_t = (\gamma_\pi - 1) E_t\left(\pi_{t+1} - \pi_{t+1}^*\right) + \gamma_x (x_t - x_t^*) + \gamma_q q_t$$

$$\begin{aligned} q_t &= \frac{(1-\gamma_\pi)}{1+\gamma_q} \sum_{j=0}^{\infty} \left(\frac{1}{1+\gamma_q}\right)^j E_t\left(\pi_{t+1+j} - \pi_{t+1+j}^*\right) \\ &\quad - \frac{\gamma_x}{1+\gamma_q} \sum_{j=0}^{\infty} \left(\frac{1}{1+\gamma_q}\right)^j E_t\left(x_{t+j} - x_{t+j}^*\right) \end{aligned}$$

- Assume inflation differentials and output gap follow AR(1) processes

  .

$$q_t = \frac{(1-\gamma_\pi)\,\rho_\pi}{1+\gamma_q-\rho_\pi}(\pi_t - \pi_t^*) - \frac{\gamma_x \rho_x}{1+\gamma_q-\rho_x}(x_t - x_t^*)$$

  * Significantly different character of fundamentals.

  * Good news for home output is good news for the exchange rate.

  * Bad news about inflation means what about the exchange rate?
    Depends on γ_π

2 The existence of interest rate rules

2.1 Clarida, Gali, Gertler QJE: For the Fed.

$$(1) \qquad r_t^* = r^* + \beta\,(E\{\pi_{t,k}|\Omega_t\} - \pi^*) + \gamma\, E\{x_{t,q}|\Omega_t\},$$

r is the nominal rate (FFR)

$$(2) \qquad rr_t^* = rr^* + (\beta - 1)\,(E\{\pi_{t,k}|\Omega_t\} - \pi^*) + \gamma E\{x_{t,q}|\Omega_t\},$$

$$rr^* = r^* - \pi^*$$

$$(3) \qquad r_t = \rho(L)\, r_{t-1} + (1 - \rho)\, r_t^*,$$

$$(4) \qquad r_t = (1 - \rho)\,\{rr^* - (\beta - 1)\pi^* + \beta\pi_{t,k} + \gamma x_{t,q}\} + \rho(L)\, r_{t-1} + \varepsilon_t,$$

where $\varepsilon_t \equiv -(1 - \rho)\,\{\beta(\pi_{t,k} - E\{\pi_{t,k}|\Omega_t\}) + \gamma(x_{t,q} - E\{x_{t,q}|\Omega_t\})\}$. No-

TABLE II

BASELINE ESTIMATES

	π^*	β	γ	ρ	p
Pre-Volcker	4.24	0.83	0.27	0.68	0.834
	(1.09)	(0.07)	(0.08)	(0.05)	
Volcker-Greenspan	3.58	2.15	0.93	0.79	0.316
	(0.50)	(0.40)	(0.42)	(0.04)	

Standard errors are reported in parentheses. The set of instruments includes four lags of inflation: output gap, the federal funds rate, the short-long spread, and commodity price inflation.

TABLE III

ALTERNATIVE VARIABLES

	π^*	β	γ	ρ	p
Detrended output					
Pre-Volcker	4.17	0.75	0.29	0.67	0.801
	(0.68)	(0.07)	(0.08)	(0.05)	
Volcker-Greenspan	4.52	1.97	0.55	0.76	0.289
	(0.58)	(0.32)	(0.30)	(0.05)	
Unemployment rate					
Pre-Volcker	3.80	0.84	0.60	0.63	0.635
	(0.87)	(0.05)	(0.11)	(0.04)	
Volcker-Greenspan	4.42	2.01	0.56	0.73	0.308
	(0.44)	(0.28)	(0.41)	(0.05)	
CPI					
Pre-Volcker	4.56	0.68	0.28	0.65	0.431
	(0.53)	(0.06)	(0.08)	(0.05)	
Volcker-Greenspan	3.47	2.14	1.49	0.88	0.138
	(0.79)	(0.52)	(0.87)	(0.03)	

Standard errors are reported in parentheses. The set of instruments includes four lags of inflation, output gap, the federal funds rate, the short-long spread, and commodity price inflation.

2.2 Other central banks

Germany, Japan, US, UK, France, Italy. CGG 1998 EER. Data 1979-1993

Table 1
Bundesbank reaction functions

	β	γ	ρ	α
Baseline	1.31	0.25	0.91	3.14
	(0.09)	(0.04)	(0.01)	(0.28)

Table 2
Bank of Japan reaction functions

	β	γ	ρ	α
Baseline	2.04	0.08	0.93	1.21
	(0.19)	(0.03)	(0.01)	(0.44)

Table 3
FED reaction functions

	β	γ	$\rho_1 + \rho_2$	α
Baseline	1.79	0.07	0.92	0.26
	(0.18)	(0.06)	(0.03)	(0.85)
post 82:10	1.83	0.56	0.97	– 0.10
	(0.45)	(0.16)	(0.03)	(1.54)

Table 4
Bank of England reaction functions

	β	γ	ρ	α
Baseline	0.98	0.19	0.92	5.76
	(0.09)	(0.04)	(0.01)	(0.69)

3 Taylor rules in New Keynesian (New Open Economy) Models

Benigno, JME

PPP puzzle: Flexible price models have difficulty explaining volatility (without high risk aversion). Sticky price models have difficulty explaining persistence (without unrealistically sticky prices). Benigno breaks the link between real exchange rate persistence and the speed of nominal price adjustment.

Table 1
Properties of real exchange rates

RS rate	$\rho(RS)$(CKM)	$\sigma(RS)/\sigma(y^{H})$(CKM)	$\rho(RS)$(BF)	$\sigma(RS)/\sigma(y^{H})$(BF)
U.S./Fr	0.84	4.44	0.78	7.57
U.S./Ger	0.82	4.50	0.74	4.35
U.S./Ca	NA	NA	0.88	2.05
U.S./Japan	NA	NA	0.81	5.87
U.S./Spain	0.86	4.70	NA	NA
U.S./Nor	0.77	3.39	NA	NA
U.S./Italy	0.83	4.35	0.79	4.26
U.S./UK	0.81	4.40	0.79	4.79
Average	0.82	4.3	0.8	4.81

CKM refers to the data reported by Chari et al. (2002). Their statistics are based on logged and Hodrick–Prescott filtered data. Their data are quarterly series from International Monetary Fund and the Organisation for Economics Co-operation and Development, 1973:1 2000:1. BF refers to the data reported by Bergin and Feenstra (2001). All their data are quarterly series from International Financial Statistics, 1973:1–1997:3. Series are logged and Hodrick–Prescott filtered. For both contributions, $\rho(RS)$ represents the one-quarter autocorrelation of the real exchange rate while $\sigma(RS)/\sigma(y^{H})$ represents the standard deviation of the real exchange rate, $\sigma(RS)$, relative to the standard deviation of output, $\sigma(y^{H})$.

Some useful calculations: Typical household problem in new Keynesian international macroeconomics

Household wants to maximize lifetime utility defined over consumption, leisure, and real money balances. Assume that period utility is separable.

$$E_t \sum_{j=t}^{\infty} \beta^j \left[U\left(C_{t+j}\right) + N\left(\frac{M_{t+j}}{P_{t+j}}\right) - V\left(L_{t+j}\right) \right]$$

where home goods are indexed as $i \in [0, n)$ and foreign goods are $i \in [n, 1]$ with consumption indices,

$$
\begin{aligned}
C &= \left(a_1 x^{\mu} + a_2 y^{\mu}\right)^{\frac{1}{\mu}} \\
x &= (b_1)^{\frac{\rho(1-\theta)}{\theta}} \left(\int_0^n c\,(i)^{\theta}\, di\right)^{\frac{\rho}{\theta}} \\
y &= (b_2)^{\frac{\rho(1-\theta)}{\theta}} \left(\int_n^1 c\,(i)^{\theta}\, di\right)^{\frac{\rho}{\theta}}
\end{aligned}
$$

and $\frac{1}{1-\mu}$ is the elasticity of substitution between home and foreign good indices, $\frac{1}{1-\theta}$ is the elasticity of substitution between within country varieties.

Obtaining the price index: Cost minimization problem I.

For a two-good CES consumption index

$$C = (a_1 x^\mu + a_1 y^\mu)^{\frac{1}{\mu}}$$

the elasticity of substitution between x and y is $\frac{1}{1-\mu}$. The corresponding price index is

$$P = \left(a_1^{\frac{1}{1-\mu}} (p_1)^{\frac{\mu}{\mu-1}} + a_2^{\frac{1}{1-\mu}} p_2^{\frac{\mu}{\mu-1}} \right)^{\frac{\mu-1}{\mu}}$$

Proof: For a given expenditure PC, we seek the best way to allocate it across x and y. Form the Lagrangian

$$L = p_x x + p_y y + \lambda \left[C - (a_1 x^\mu + a_1 y^\mu)^{\frac{1}{\mu}} \right]$$

- First-order conditions

$$
\begin{aligned}
p_1 &= \lambda a_1 x^{\mu-1} \left(a_1 x^{\mu} + a_2 y^{\mu}\right)^{\frac{1}{\mu}-1} \\
p_2 &= \lambda a_2 y^{\mu-1} \left(a_1 x^{\mu} + a_2 y^{\mu}\right)^{\frac{1}{\mu}-1}
\end{aligned}
$$

Eliminate the multiplier

$$\frac{p_1}{p_2} = \frac{a_1}{a_2}\left(\frac{x}{y}\right)^{\mu-1}$$

From this relation, we can write

$$x = \left(\left(\frac{p_1}{p_2}\right)\frac{a_2}{a_1}\right)^{\frac{1}{\mu-1}} y$$

or

$$y = \left(\left(\frac{p_2}{p_1}\right)\frac{a_1}{a_2}\right)^{\frac{1}{\mu-1}} x$$

- Take the expression for x and plug into the consumption index,

$$
\begin{aligned}
C &= (a_1 x^{\mu} + a_1 y^{\mu})^{\frac{1}{\mu}} \\
C &= \left(a_1 \left(\left(\left(\frac{p_1}{p_2}\right)^{\frac{\mu}{\mu-1}} \left(\frac{a_2}{a_1}\right)^{\frac{\mu}{\mu-1}}\right) y^{\mu}\right) + a_2 y^{\mu}\right)^{\frac{1}{\mu}} \\
&= a_2^{\frac{1}{\mu-1}} p_2^{\frac{1}{1-\mu}} y \left[\left(a_1^{\frac{1}{1-\mu}} (p_1)^{\frac{\mu}{\mu-1}} + (a_2)^{\frac{1}{1-\mu}} p_2^{\frac{\mu}{\mu-1}}\right)\right]^{\frac{1}{\mu}}
\end{aligned}
$$

By symmetry of the consumption index, it must also be the case that $C = a_1^{\frac{1}{\mu-1}} p_1^{\frac{1}{1-\mu}} x \left[\left(a_1^{\frac{1}{1-\mu}} (p_1)^{\frac{\mu}{\mu-1}} + (a_2)^{\frac{1}{1-\mu}} p_2^{\frac{\mu}{\mu-1}}\right)\right]^{\frac{1}{\mu}}$. Thus, we have

$$
\begin{aligned}
C &= a_2^{\frac{1}{\mu-1}} p_2^{\frac{1}{1-\mu}} y \left[\left(a_1^{\frac{1}{1-\mu}} (p_1)^{\frac{\mu}{\mu-1}} + (a_2)^{\frac{1}{1-\mu}} p_2^{\frac{\mu}{\mu-1}}\right)\right]^{\frac{1}{\mu}} \\
C &= a_1^{\frac{1}{\mu-1}} p_1^{\frac{1}{1-\mu}} x \left[\left(a_1^{\frac{1}{1-\mu}} (p_1)^{\frac{\mu}{\mu-1}} + (a_2)^{\frac{1}{1-\mu}} p_2^{\frac{\mu}{\mu-1}}\right)\right]^{\frac{1}{\mu}}
\end{aligned}
$$

which gives

$$\begin{aligned} x &= C a_1^{\frac{1}{1-\mu}} p_1^{\frac{1}{\mu-1}} \left[\left(a_1^{\frac{1}{1-\mu}} (p_1)^{\frac{\mu}{\mu-1}} + (a_2)^{\frac{1}{1-\mu}} p_2^{\frac{\mu}{\mu-1}} \right) \right]^{-\frac{1}{\mu}} \\ y &= C a_2^{\frac{1}{1-\mu}} p_2^{\frac{1}{\mu-1}} \left[\left(a_1^{\frac{1}{1-\mu}} (p_1)^{\frac{\mu}{\mu-1}} + (a_2)^{\frac{1}{1-\mu}} p_2^{\frac{\mu}{\mu-1}} \right) \right]^{-\frac{1}{\mu}} \end{aligned}$$

- Use these expressions of x and y in the budget constraint and after some simplification, one obtains,

$$p_1 x + p_2 y = \left(a_1^{\frac{1}{1-\mu}} (p_1)^{\frac{\mu}{\mu-1}} + a_2^{\frac{1}{1-\mu}} p_2^{\frac{\mu}{\mu-1}} \right)^{\frac{\mu-1}{\mu}} C$$

Therefore

$$P = \left(a_1^{\frac{1}{1-\mu}} (p_1)^{\frac{\mu}{\mu-1}} + a_2^{\frac{1}{1-\mu}} p_2^{\frac{\mu}{\mu-1}} \right)^{\frac{\mu-1}{\mu}}$$

- Demand functions:

$$
\begin{aligned}
x &= C a_1^{\frac{1}{1-\mu}} p_1^{\frac{1}{\mu-1}} \left[\left(a_1^{\frac{1}{1-\mu}} (p_1)^{\frac{\mu}{\mu-1}} + (a_2)^{\frac{1}{1-\mu}} p_2^{\frac{\mu}{\mu-1}} \right) \right]^{-\frac{1}{\mu}} = a_1^{\frac{1}{1-\mu}} \left(\frac{P}{p_1} \right)^{\frac{1}{1-\mu}} C \\
y &= a_2^{\frac{1}{1-\mu}} \left(\frac{P}{p_2} \right)^{\frac{1}{1-\mu}} C
\end{aligned}
$$

Obtaining the price sub-indices for home and foreign goods: Cost minimization problem II.

For the home goods consumption index

$$C_h = b_1^{\frac{(1-\theta)}{\theta}} \left(\int_0^n c\,(i)^\theta \, di \right)^{\frac{1}{\theta}}$$

the price sub-index is

$$P_h = b_1^{\frac{(\theta-1)}{\theta}} \left(\int_0^n (p(i))^{\frac{\theta}{\theta-1}} \, di\right)^{1-\frac{1}{\theta}}$$

- Proof: The problem is how to allocate a given expenditure PC across $i \in [0, n)$, where

$$C_h = b_1^{\frac{(1-\theta)}{\theta}} \left(\int_0^n c(i)^{\theta} \, di\right)^{\frac{1}{\theta}}$$

  – Note here that $b_1^{\frac{(1-\theta)}{\theta}} \left(\int_0^n c(i)^{\theta} \, di\right)^{\frac{1}{\theta}}$ is shorthand for $\left(\sum_i^{\infty} c(i)^{\theta}\right)^{\frac{1}{\theta}}$. Since $\frac{\partial\left(\sum_i^{\infty} c(i)^{\theta}\right)^{\frac{1}{\theta}}}{\partial c(i)} = c(i)^{(\theta-1)} \left(\sum_i^{\infty} c(i)^{\theta}\right)^{\frac{1-\theta}{1}}$, by analogy, we have the differentian rule,

$$\frac{\partial b_1^{\frac{(1-\theta)}{\theta}} \left(\int_0^n c(i)^{\theta} \, di\right)^{\frac{1}{\theta}}}{\partial c(i)} = b_1^{\frac{(1-\theta)}{\theta}} c(i)^{(\theta-1)} \left(\int_0^n c(i)^{\theta} \, di\right)^{1-\theta} = c(i)^{(\theta-1)} \, b_1^{(1-\theta)} C_h^{(1-\theta}$$

– Form the Lagrangian

$$L = \int_0^n p(i)\, c(i)\, di + \lambda \left(C_h - b_1^{\frac{(1-\theta)}{\theta}} \left(\int_0^n c(i)^\theta \, di \right)^{\frac{1}{\theta}} \right)$$

First-order conditions are

$$p(i) = \lambda c(i)^{(\theta-1)}\, b_1^{(1-\theta)} C_h^{(1-\theta)}$$

which we use to write

$$\begin{aligned} \frac{p(i)}{p(j)} &= \left(\frac{c(i)}{c(j)} \right)^{\theta-1} \\ c(i) &= \left(\frac{p(i)}{p(j)} \right)^{\frac{1}{\theta-1}} c(j) \end{aligned}$$

– Use the expression for $c(i)$ in the sub-index,

$$\begin{aligned}
C_h &= b_1^{\frac{(1-\theta)}{\theta}} \left(\int_0^n c(i)^\theta \, di\right)^{\frac{1}{\theta}} = b_1^{\frac{(1-\theta)}{\theta}} \left(\int_0^n \left(\frac{p(i)}{p(j)}\right)^{\frac{\theta}{\theta-1}} c(j)^\theta \, di\right)^{\frac{1}{\theta}} \\
&= b_1^{\frac{(1-\theta)}{\theta}} p(j)^{\frac{1}{1-\theta}} c(j) \left(\int_0^n (p(i))^{\frac{\theta}{\theta-1}} \, di\right)^{\frac{1}{\theta}} \\
&\rightarrow \; c(j) = C_h b_1^{\frac{(\theta-1)}{\theta}} p(j)^{\frac{1}{\theta-1}} \left(\int_0^n (p(i))^{\frac{\theta}{\theta-1}} \, di\right)^{\frac{-1}{\theta}}
\end{aligned}$$

– Now use the expression for $c(j)$ in the budget constraint.

$$\begin{aligned}
P_h C_h &= \int_0^n p(j)\, c(j) \, dj = \left(\int_0^n (p(i))^{\frac{\theta}{\theta-1}} \, di\right)^{\frac{-1}{\theta}} C_h b_1^{\frac{(\theta-1)}{\theta}} \int_0^n p(j)^{\frac{\theta}{\theta-1}} \, dj \\
P_h &= b_1^{\frac{(\theta-1)}{\theta}} \left(\int_0^n (p(i))^{\frac{\theta}{\theta-1}} \, di\right)^{\frac{-1}{\theta}} \int_0^n p(j)^{\frac{\theta}{\theta-1}} \, dj \\
&\rightarrow \; P_h = b_1^{\frac{(\theta-1)}{\theta}} \left(\int_0^n (p(i))^{\frac{\theta}{\theta-1}} \, di\right)^{1-\frac{1}{\theta}}
\end{aligned}$$

– Use the price index back in the $c\left(j\right)$ expression to write the good's demand

$$
\begin{aligned}
c\left(j\right) &= p\left(j\right)^{\frac{1}{\theta-1}} b_1^{\frac{(\theta-1)}{\theta}} \left(\int_0^n \left(p\left(i\right)\right)^{\frac{\theta}{\theta-1}} di\right)^{1-\frac{1}{\theta}} \left(\int_0^n \left(p\left(i\right)\right)^{\frac{\theta}{\theta-1}} di\right)^{-1} C_h \\
&= p\left(j\right)^{\frac{1}{\theta-1}} \frac{P_h}{b_1 \left(P_h\right)^{\frac{\theta}{\theta-1}}} C_h \\
&= p\left(j\right)^{\frac{1}{\theta-1}} \frac{P_h}{b_1 \left(P_h\right)^{\frac{\theta}{\theta-1}}} a_1^{\frac{1}{1-\mu}} \left(\frac{P}{P_h}\right)^{\frac{1}{1-\mu}} C \\
\rightarrow &\quad c\left(j\right) = \frac{a_1^{\frac{1}{1-\mu}}}{b_1} \left(\frac{P_h}{p\left(j\right)}\right)^{\frac{1}{1-\theta}} \left(\frac{P}{P_h}\right)^{\frac{1}{1-\mu}} C
\end{aligned}
$$

Reparameterization for Benigno's model

In Benigno's JME paper, home consumer's preferences for $h \in [0, n]$ are

$$U_t^h = E_t \sum_{j=t}^{\infty} \beta^{(j-t)} \left[U\left(C_{t+j}^h\right) + N\left(\frac{M_{t+j}^h}{P_{t+j}}\right) - V\left(L_{t+j}^h\right) \right]$$

where for elasticity of substitution between foreign and home ζ and elasticity of substitution across within country goods σ,

$$\begin{aligned}
C &= \left[n^{1/\zeta} C_H^{(\zeta-1)/\zeta} - (1-n)^{1/\zeta} C_F^{(\zeta-1)/\zeta}\right]^{\zeta/(\zeta-1)}, \quad \zeta > 0 \\
C_H &= \left[\left(\frac{1}{n}\right)^{1/\sigma} \int_0^n c(z)^{(\sigma-1)/\sigma} \, dz\right]^{\sigma/(\sigma-1)} \\
C_F &= \left[\left(\frac{1}{1-n}\right)^{1/\sigma} \int_n^1 c(z)^{(\sigma-1)/\sigma} \, dz\right]^{\sigma/(\sigma-1)} \\
P &= \left[n P_H^{1-\zeta} + (1-n) P_F^{1-\zeta}\right]^{1/(1-\zeta)} \\
P_H &= \left[\left(\frac{1}{n}\right) \int_0^n p(z)^{1-\sigma} \, dz\right]^{1/(1-\sigma)} \\
P_F &= \left[\left(\frac{1}{1-n}\right) \int_n^1 p(z)^{1-\sigma} \, dz\right]^{1/(1-\sigma)}
\end{aligned}$$

Parameter correspondence:

$$\begin{aligned}
\mu &= \frac{\zeta-1}{\zeta} \\
&\rightarrow \zeta = \frac{1}{1-\mu} \\
&\rightarrow \frac{\mu}{\mu-1} = 1-\zeta \rightarrow \frac{1}{\mu} = \frac{\zeta}{\zeta-1} \\
a_1 &= n^{\frac{1}{\zeta}}
\end{aligned}$$

For the general indices,

$$\begin{aligned}
C &= \left(a_1 x^{\mu} + a_2 y^{\mu}\right)^{\frac{1}{\mu}} \rightarrow C = \left[n^{1/\zeta} x^{(\zeta-1)/\zeta} + (1-n)^{1/\zeta}\, y^{(\zeta-1)/\zeta}\right]^{\zeta/(\zeta-1)} \\
P &= \left[a_1^{\frac{1}{1-\mu}} p_x^{\frac{\mu}{\mu-1}} + a_2^{\frac{1}{1-\mu}} p_y^{\frac{\mu}{\mu-1}}\right]^{\frac{\mu-1}{\mu}} \rightarrow P = \left[n p_1^{1-\zeta} + (1-n)\, p_y^{1-\zeta}\right]^{1/(\zeta-1)}
\end{aligned}$$

For sub-price indices,

$$\theta = \frac{(\sigma - 1)}{\sigma} \longrightarrow \sigma = \frac{1}{(1-\theta)}, \longrightarrow \frac{1}{\sigma} = (1 - \theta.) \longrightarrow \frac{1}{(\sigma - 1)} = \frac{(1-\theta)}{\theta}$$

$$\begin{aligned} C_h &= b_1^{\frac{(1-\theta)}{\theta}} \left(\int_0^n c\,(i)^\theta \, di \right)^{\frac{1}{\theta}} \longrightarrow C_H = \left[\left(\frac{1}{n} \right)^{1/\sigma} \int_0^n c\,(z)^{(\sigma-1)/\sigma} \, dz \right]^{\sigma/(\sigma-1)} \\ P_h &= b_1^{\frac{(\theta-1)}{\theta}} \left(\int_0^n (p\,(i))^{\frac{\theta}{\theta-1}} \, di \right)^{1-\frac{1}{\theta}} \longrightarrow P_H = \left[\left(\frac{1}{n} \right) \int_0^n p\,(z)^{1-\sigma} \, dz \right]^{1/(1-\sigma)} \end{aligned}$$

3.1 Benigno's model

- Households
  - Notation
    * Home: $[0, n]$. Foreign: $(n, 1]$. Continuum of goods and agents. Each agent owns one firm and supplies labor to that firm.
    * h denotes home agent. f denotes foreign agent.
    * s_t an event in period t.
    * $s^t = (s_t, s_{t-1}, ..., s_0)$ is the history of events.
    * $B^h(s_{t+1})$: home agent's holdings of one-period nominal state-contingent bonds.

  * $Q\left(s_{t+1}|s^t\right)$ is the home-currency nominal price of the state-contingent bond.

- Preferences: Home consumer: For $h \in [0, n]$,

$$U_t^h = E_t \sum_{j=t}^{\infty} \beta^{(j-t)} \left[U\left(C_{t+j}^h\right) + N\left(\frac{M_{t+j}^h}{P_{t+j}}\right) - V\left(L_{t+j}^h\right) \right]$$

where for elasticity of substitution between foreign and home ζ and elasticity of substitution across within country goods σ,

$$
\begin{aligned}
C &= \left[n^{1/\zeta} C_H^{(\zeta-1)/\zeta} - (1-n)^{1/\zeta} C_F^{(\zeta-1)/\zeta}\right]^{\zeta/(\zeta-1)}, \quad \zeta > 0 \\
C_H &= \left[\left(\frac{1}{n}\right)^{1/\sigma} \int_0^n c(z)^{(\sigma-1)/\sigma} \, dz\right]^{\sigma/(\sigma-1)} \\
C_F &= \left[\left(\frac{1}{1-n}\right)^{1/\sigma} \int_n^1 c(z)^{(\sigma-1)/\sigma} \, dz\right]^{\sigma/(\sigma-1)} \\
P &= \left[n P_H^{1-\zeta} + (1-n) P_F^{1-\zeta}\right]^{1/(1-\zeta)} \\
P_H &= \left[\left(\frac{1}{n}\right) \int_0^n p(z)^{1-\sigma} \, dz\right]^{1/(1-\sigma)} \\
P_F &= \left[\left(\frac{1}{1-n}\right) \int_n^1 p(z)^{1-\sigma} \, dz\right]^{1/(1-\sigma)}
\end{aligned}
$$

– Budget constraint: Income comes from profits of firms owned by households, payoffs from state-contingent bonds, wages, and transfers from the government.

$$C_t^h + \sum_{s_{t+1}} \frac{Q\left(s_{t+1}|s^t\right) B^h\left(s_{t+1}\right)}{P_t} + \frac{M_t^h}{P_t} = \frac{B^h\left(s_t\right)}{P_t} + \frac{M_{t-1}^h}{P_t} + \frac{W_t^h L_t^h}{P_t} + \frac{\pi_t^h}{P_t} + \frac{Tr_t^h}{P_t}$$

– Government budget constraint: Transfers are helicopter drops of cash.

$$\int_0^n \left(M_t^h - M_{t-1}^h\right) dh = \int_0^n Tr_t^h dh$$

– For the intertemporal allocation problem, write down two terms from the Lagrangian

$$\beta^t \sum_{s_t} \pi\left(s_t, s^{t-1}\right)\left[U\left(C_t^h\left(s^t\right)\right) + N\left(\frac{M_t^h\left(s^t\right)}{P_t\left(s^t\right)}\right) - V\left(L_t^h\left(s^t\right)\right)\right]$$

$$-\sum_{s_t} \lambda_t\left(s^t\right)\left(\begin{array}{c} C_t^h\left(s^t\right) + \sum_{s_{t+1}} \frac{Q(s_{t+1}|s^t)B^h(s_{t+1})}{P_t(p_t)} + \frac{M_t^h(s^t)}{P_t(s^t)} \\ -\left[\frac{B^h(s_t)+M_{t-1}^h(s^{t-1})+W_t^h(s^t)L_t^h(s^t)+\pi_t^h(s^t)+Tr_t^h(s^t)}{P_t(s^t)}\right] \end{array}\right)$$

$$\beta^{t+1} \sum_{s_{t+1}} \pi\left(s^{t+1}\right)\left[U\left(C_{t+1}^h\left(s^{t+1}\right)\right) + N\left(\frac{M_{t+1}^h\left(s^{t+1}\right)}{P_{t+1}\left(s^{t+1}\right)}\right) - V\left(L_{t+1}^h\left(s^{t+1}\right)\right)\right]$$

$$-\sum_{s_{t+1}} \pi\left(s^{t+1}\right)\lambda_{t+1}\left(s^{t+1}\right)\left(\begin{array}{c} C_{t+1}^h\left(s^{t+1}\right) + \sum_{s_{t+2}} \frac{Q(s_{t+2}|s^{t}\ldots}{P_{t+1}\ldots} \\ +\frac{M_{t+1}^h(s^{t+1})}{P_{t+1}(s^{t+1})} - \left[\frac{B^h(s_{t+1})+M_t^h(s^t)+W_{t+1}^h(s^{t+1})L_{t+1}^h\ldots}{P_{t+1}(s^{t}\ldots}\right. \end{array}\right.$$

* Obtain the first-order conditions for consumption,

$$
\begin{aligned}
C_t^h &: \beta^t \pi\left(s_t, s^{t-1}\right) U'\left(C_t^h\left(s^t\right)\right) = \lambda_t\left(s^t\right) \\
B^h\left(s_{t+1}\right) &: \lambda_t\left(s^t\right) \frac{Q\left(s_{t+1}|s^t\right)}{P_t\left(s^t\right)} = \lambda_{t+1}\left(s_{t+1}\right) \frac{1}{P_{t+1}\left(s_{t+1}\right)}
\end{aligned}
$$

Eliminating the multiplier gives the price of state contingent bonds

$$
Q\left(s_{t+1}|s^t\right) = \beta\pi\left(s_{t+1}|s^t\right) \frac{U'\left(C_{t+1}^h\left(s^{t+1}\right)\right)}{U'\left(C_t^h\right)} \frac{P_t}{P_{t+1}}
$$

from which it follows that the price of one-period riskless nominal bonds are

$$
\frac{1}{1+i_t} = \sum_{s_{t+1}} Q\left(s_{t+1}|s^t\right) = \beta E_t \left\{ \frac{U'\left(C_{t+1}^h\right)}{U'\left(C_t^h\right)} \frac{P_t}{P_{t+1}} \right\}
$$

* Euler equations for labor and real money holdings:

$$\begin{aligned} L &: \rightarrow V_L\left(s^t\right) = U'\left(C_t^h\left(s^t\right)\right)\frac{W_t^h\left(s^t\right)}{P_t\left(s^t\right)} \\ M &: \rightarrow N_M\left(\frac{M_t}{P_t}\right) = U'\left(C_t^h\left(s^t\right)\right)\frac{i_t}{1+i_t} \end{aligned}$$

· In equilibrium, we can drop the h superscript on consumption. Due to complete markets, everyone's consumption will be equal.

* Home consumer demand for the home good and the foreign good

$$\begin{aligned} c^h(h) &= \left(\frac{p(h)}{P_H}\right)^{-\sigma}\left[\frac{P_H}{P}\right]^{-\zeta} C \\ c^h(f) &= \left(\frac{p(f)}{PF}\right)^{-\sigma}\left[\frac{P_F}{P}\right]^{-\zeta} C \end{aligned}$$

with similar demand functions for the foreign guy.

- Complete international asset markets. One-period state contingent nominal bonds denominated in the home currency are traded internationally. Home guy's Euler equation has already been found. For foreign guy,

$$
\begin{aligned}
C_t^f &: \quad \beta^t \pi\left(s^t\right) U'\left(C_t^f\left(s^t\right)\right) = \lambda_t^f\left(s^t\right) \\
B^h\left(s_{t+1}\right) &: \quad \lambda_t^f\left(s^t\right) \frac{Q\left(s_{t+1}|s^t\right)}{S_t P_t^*\left(s^t\right)} = \lambda_{t+1}\left(s_{t+1}\right) \frac{1}{S_t P_{t+1}^*\left(s_{t+1}\right)} \\
Q\left(s_{t+1}|s^t\right) &= \beta\pi\left(s_{t+1}|s^t\right) \frac{U'\left(C_{t+1}^f\left(s^{t+1}\right)\right)}{U'\left(C_t^f\left(s^t\right)\right)} \frac{S_t P_t^*\left(s^t\right)}{S_{t+1} P_{t+1}^*\left(s_{t+1}\right)} \\
&= \beta\pi\left(s_{t+1}|s^t\right) \frac{U'\left(C_{t+1}^h\left(s^{t+1}\right)\right)}{U'\left(C_t^h\right)} \frac{P_t}{P_{t+1}}
\end{aligned}
$$

- By the law of one price,

$$\frac{S_t P_t^*}{P_t}\frac{P_{t+1}}{S_{t+1}P_{t+1}^*} = \frac{U'\left(C_{t+1}^h\left(s^{t+1}\right)\right)}{U'\left(C_{t+1}^f\left(s^{t+1}\right)\right)}\frac{U'\left(C_t^f\right)}{U'\left(C_t^h\right)}$$

and after backwards recursive substitution,

$$\frac{U'\left(C_t^f\right)}{U'\left(C_t^h\right)} = \kappa\frac{S_t P_t^*}{P_t} = \kappa R S_t$$

where κ depends on initial conditions. This is the Backus–Smith condition. Benigno achieves real exchange rate dynamics through relative movements in consumption.

- Monetary policy is conducted through interest-rate feedback rules (Taylor rules). Let ε_t^H, ε_t^F be monetary policy shocks, F and F^* be a set of target variables. The general specification of the rules are

$$\begin{aligned} \frac{1+i_t}{1+\bar{\imath}} &= \boldsymbol{\Psi}\left(F, s^t, \varepsilon_t^H\right) \\ \frac{1+i_t^*}{1+\bar{\imath}^*} &= \psi^*\left(F^*, s^t, \varepsilon_t^F\right) \end{aligned}$$

$$1 = (1+i_t)\,\beta E_t\left\{\frac{U'\left(C_{t+1}^h\right)}{U'\left(C_t^h\right)}\frac{P_t}{P_{t+1}}\right\} = (1+i_t^*)\,\beta E_t\left\{\frac{U'\left(C_{t+1}^f\right)}{U'\left(C_t^f\right)}\frac{P_t^*}{P_{t+1}^*}\right\}$$

- Production with local currency pricing. All produced goods are tradable. Deviations from PPP are due to international market segmentation. Prices are sticky in terms of the local currency.

  - Sticky prices through the Calvo mechanism. Say the Calvo-lottery chooses home firm h to set a new price at time t. The firm does so to maximize expected present value of net profits. Let α^s be the probability that the price will remain in effect from $t+1$ through $t+s$. The owner's discount factor is

$$\Phi_{t,t+s} = \beta^s \frac{U'(C_{t+s})}{U'(C_t)} \frac{P_t}{P_{t+s}}$$

    and the production technology

$$c_t^h(h) + c_t^f(h) = y_t(h) = A_t L_t^h$$

    where A_t is a country-specific technology (productivity) shock.

– The objective is to maximize

$$E_t \sum_{j=0}^{\infty} \alpha^j \Phi_{t,t+j} \left[\underbrace{p_{t+j}(h)\, y_{t+j}^h(h)}_{\text{home sales}} + \underbrace{S_{t+j} p_{t+j}^*(h)\, y_{t+j}^f(h)}_{\text{foreign sales}} - \underbrace{W_{t+j}^h L_{t+j}^h}_{\text{wage bill}} \right]$$

subject to

$$\begin{aligned} y_t^h(h) &= \left(\frac{p_t(h)}{P_{Ht}}\right)^{-\sigma} \left[\frac{P_{H_t}}{P_t}\right]^{-\zeta} nC_t = \Lambda_t^h p_t(h)^{-\sigma} \\ y_t^f(h) &= \left(\frac{p_t^*(h)}{P_{H_t}^*}\right)^{-\sigma} \left[\frac{P_{H,t}^*}{P_t^*}\right]^{-\zeta} (1-n)\, C_t^* = \Lambda_t^* p_t^*(h)^{-\sigma} \end{aligned}$$

where

$$\begin{aligned} \Lambda_t^h &= \left(\frac{1}{P_{Ht}}\right)^{-\sigma} \left[\frac{P_{H_t}}{P_t}\right]^{-\zeta} nC_t \\ \Lambda_t^* &= \left(\frac{1}{P_{H_t}^*}\right)^{-\sigma} \left[\frac{P_{H,t}^*}{P_t^*}\right]^{-\zeta} (1-n)\, C_t^* \end{aligned}$$

The firm must set $p_t(h)$ and $p_t^*(h)$.

$$\begin{aligned}
&\Phi_{t,t}\left[p_t(h)\Lambda_t^h p_t(h)^{-\sigma} + S_t p_t^*(h)\Lambda_t^* p_t^*(h)^{-\sigma} - W_t^h L_t^h\right] \\
&+\lambda_t\left(A_t L_t^h - \Lambda_t^h p_t(h)^{-\sigma} - \Lambda_t^* p_t^*(h)^{-\sigma}\right) \\
&+\alpha E_t \Phi_{t,t+1}\left[p_{t+1}(h)\Lambda_{t+1}^h p_{t+1}(h)^{-\sigma} + S_{t+1} p_{t+1}^*(h)\Lambda_{t+1}^* p_{t+1}^*(h)^{-\sigma} - W_{t+1}^h L\right. \\
&+E_t\lambda_{t+1}\left(A_{t+1}L_{t+1}^h - \Lambda_{t+1}^h p_{t+1}(h)^{-\sigma} - \Lambda_{t+1}^* p_{t+1}^*(h)^{-\sigma}\right) + \cdots
\end{aligned}$$

* First, choose $p_t(h)$, with the expectation that it will stay in effect.

$$p_t(h) : (1-\sigma)\, p_t(h)^{-\sigma} E_t \sum_{j=0}^{\infty} \alpha^j \Phi_{t,t+j}\Lambda_{t+j}^h - \sigma p_t(h)^{-(1+\sigma)} E_t \sum_{j=0}^{\infty} \Lambda_{t+j}^h$$

and rearranging gives

$$p_t(h) = \frac{\sigma}{(1-\sigma)} \frac{E_t \sum_{j=0}^{\infty} \Lambda_{t+j}^h}{E_t \sum_{j=0}^{\infty} \alpha^j \Phi_{t,t+j}\Lambda_{t+j}^h}$$

* Similarly, the optimal way to set the foreign price is,

$$p_t^*(h) = \frac{\sigma}{(1-\sigma)} \frac{E_t \sum_{j=0}^{\infty} S_{t+j} \Lambda_{t+j}^*}{E_t \sum_{j=0}^{\infty} \alpha^j \Phi_{t,t+j} \Lambda_{t+j}^*}$$

* Evolution of price sub-indices. A measure α of the firms have to keep the price as before. $(1-\alpha)$ set to the new price $p_t(h)$, therefore,

$$\begin{aligned} P_{H,t} &= \alpha_h P_{H,t-1} + (1-\alpha_h) p_t(h) \\ P_{H,t}^* &= \alpha_h^* P_{H,t-1}^* + (1-\alpha_h^*) p_t^*(h) \end{aligned}$$

with similar equations holding for the foreign firm

$$\begin{aligned} P_{F,t}^* &= \alpha_f^* P_{F,t-1}^* + \left(1-\alpha_f^*\right) p_t^*(f) \\ P_{F,t} &= \alpha_f P_{F,t-1} + \left(1-\alpha_f\right) p_t(f) \end{aligned}$$

Benigno allows for the possibility that the degree of price stickiness may differ across countries.

* Relative price indices of the imported good in terms of the domestically produced good in terms of the local currency

$$
\begin{aligned}
T &= \frac{P_F}{P_H} \\
T^* &= \frac{P_H^*}{P_F^*}
\end{aligned}
$$

- Aggregate fluctuations.
  - Expand around a steady state with zero inflation and exchange rate depreciation, nominal interest rates equal to utility-based discount rate, and a real exchange rate of 1 and equal consumption across countries.
  - Highlights of the log-linearization.
    1. $\pi = \ln(1+\pi)$
    2. $\widehat{RS}_t = \ln(RS_t)$
    3. $\hat{T} = \ln\left(\frac{P_F}{P_H}\right),\ \hat{T}^* = \ln\left(\frac{P_H^*}{P_F^*}\right)$
    4. $\tilde{C}, \tilde{T}$ are flexible-price equilibrium values.
    5. $\eta = \frac{LV_{LL}}{V_L},\ \ \rho = \frac{-CU_{CC}}{U_c}$

– Relative prices and the real exchange rate have unit roots. Obtain an IS-like curve, uncovered interest parity, and an expenditure switching effect.

$$
\begin{aligned}
\hat{T}_t &= \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t} \\
\hat{T}_t^* &= \hat{T}_t^* - \pi_{F,t} + \pi_{H,t} \\
\widehat{RS}_t &= \widehat{RS}_{t-1} + \pi_t^* - \pi_t + \Delta S_t \\
E_t \hat{C}_{t+1} &= \hat{C}_t + \frac{1}{\rho} \left[\hat{\imath}_t - E_t \pi_{t+1} \right] \\
E_t \Delta S_{t+1} &= \hat{\imath}_t - \hat{\imath}_t^* \\
y_{H,t} - y_{F,t} &= \zeta \left(n \left(\hat{T}_t - \tilde{T}_t \right) - (1-n) \left(\hat{T}_t^* - \tilde{T}_t^* \right) \right)
\end{aligned}
$$

- Dynamics under sticky prices
  - $\alpha_h = \alpha_h^* = \alpha_f = \alpha_f^*$: Chari-Kehoe-McGrattan case.
  - $\alpha_h = \alpha_f, \ \alpha_h^* = \alpha_f^*$: Location of consumption determines degree of price stickiness.
  - $\alpha_h = \alpha_h^*, \ \alpha_f = \alpha_f^*$: Price stickiness is firm location specific and faces the same stickiness in the home and foreign market.

- Monetary policy rules reflect interest rate smoothing and exchange rate feedback.

$$\begin{aligned} \hat{\imath}_t &= \gamma_H \hat{\imath}_{t-1} + \phi_H \pi_t + \psi_H y_t^H + \mu_H \Delta S_t + \xi_H \widehat{RS}_t + \varepsilon_t^H \\ \hat{\imath}_t^* &= \gamma_F \hat{\imath}_{t-1}^* + \phi_F \pi_t^* + \psi_F y_t^F - \mu_F \Delta S_t - \xi_F \widehat{RS}_t + \varepsilon_t^F \end{aligned}$$

$$\varepsilon_t^j = \rho_j \varepsilon_{t-1}^j + v_{j,t}$$

- Alternative policy rules

  - Fixed exchange rates. Foreign country equates $i^*$to i with a reaction to deviations of the exchange rate from target. Let $\hat{S} = \ln\left(S/\bar{S}\right)$, where $\bar{S}$ is the target. home country follows Taylor rule

$$\begin{aligned} \hat{\imath}_t^* &= \hat{\imath}_t - \lambda \hat{S}_t \\ \hat{\imath}_t &= \phi \pi_t + \psi y_{H,t} \end{aligned}$$

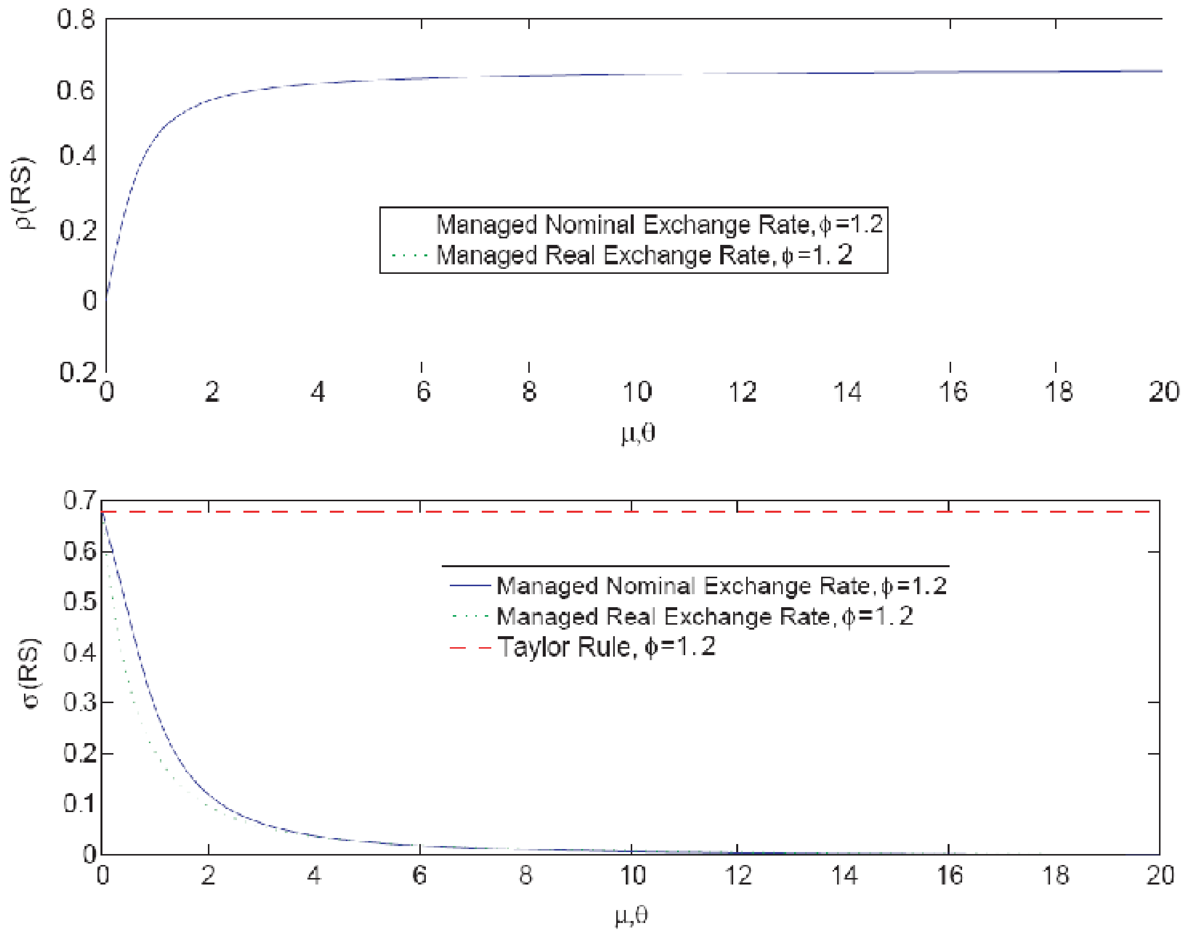
  - Taylor rules

$$\begin{aligned} \hat{\imath}_t &= \phi \pi_t + \psi y_{H,t} + \hat{\varepsilon}_{H,t} \\ \hat{\imath}_t^* &= \phi \pi_t^* + \psi y_{F,t} + \varepsilon_{F,t} \end{aligned}$$

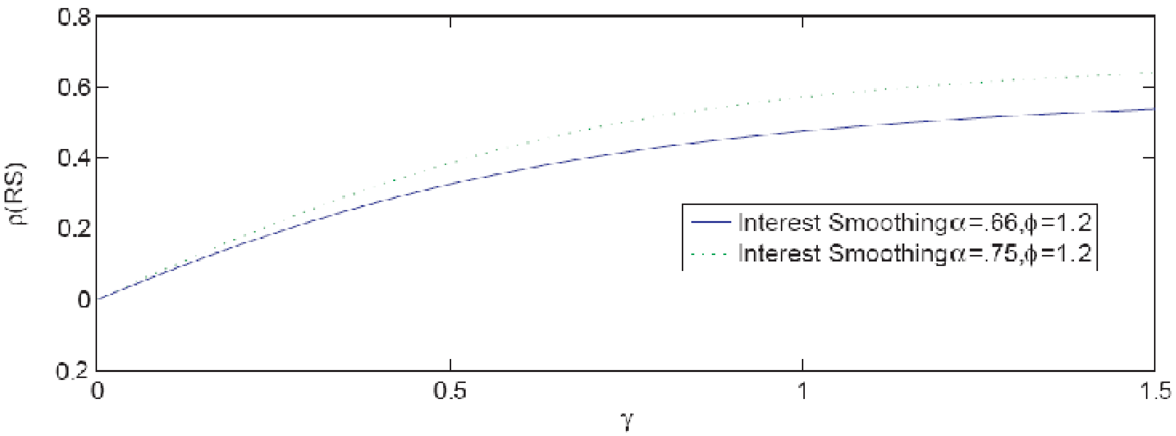
  - Managed float

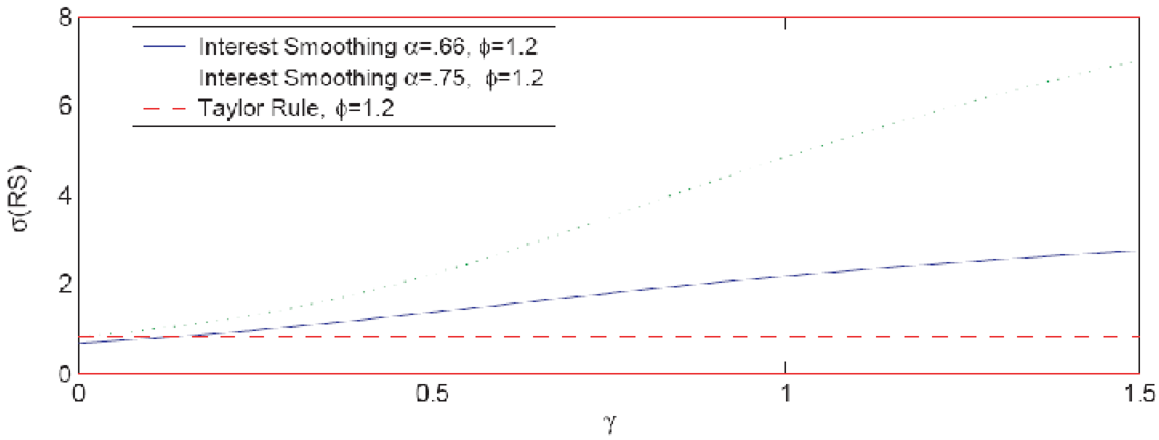
$$\begin{aligned} \hat{\imath}_t &= \phi \pi_t + \psi y_{H,t} \\ \hat{\imath}_t^* &= \phi \pi_t^* + \psi y_{F,t} - \mu \boldsymbol{\Delta} S_t \end{aligned}$$

- Tradeoff between persistence and volatility as response to exchange rate μ increases.



- Interest rate smoothing induces additional inertia. γ is the smootheing coefficient. For symmetric Taylor rules





- Preliminary results
  - Suppose $\alpha_H = \alpha_H^* = \alpha_F = \alpha_F^*$.
    - * Then price dynamics are synchronized and the relative prices $T = P_F/P_H$ and $T^* = P_H^*/P_F^*$ are uncorrelated with monetary policy. Relative prices will be affected only by productivity shocks.
    - * The real exchange rate displays no persistence following a monetary shock under inflation targeting or under the Taylor rule.
      - · After a monetary shock, the nominal and real exchange rates return to equilibrium after one period, as in the Redux model.
      - · There will be persistence in the exchange rates if monetary shocks are serially correlated.
    - * Under inflation targeting, the real exchange rate is isolated from productivity shocks.

- When there is no weight on output stabilization, productivity shocks wil have no effect on the real exchange rate. Also, the inflation differential is independent of relative price changes. The link between productivity shocks and the real exchange rate is broken.

- Plausible examples of persistence generation in the real exchange rate: Calibration and simulations. $\beta = 0.99, \eta = 2, n = 0.5, \rho = 6, \sigma = 10, \zeta = 1.5$. The only shock is from domestic monetary policy with no autoregressive component.

- Interest rate smoothing

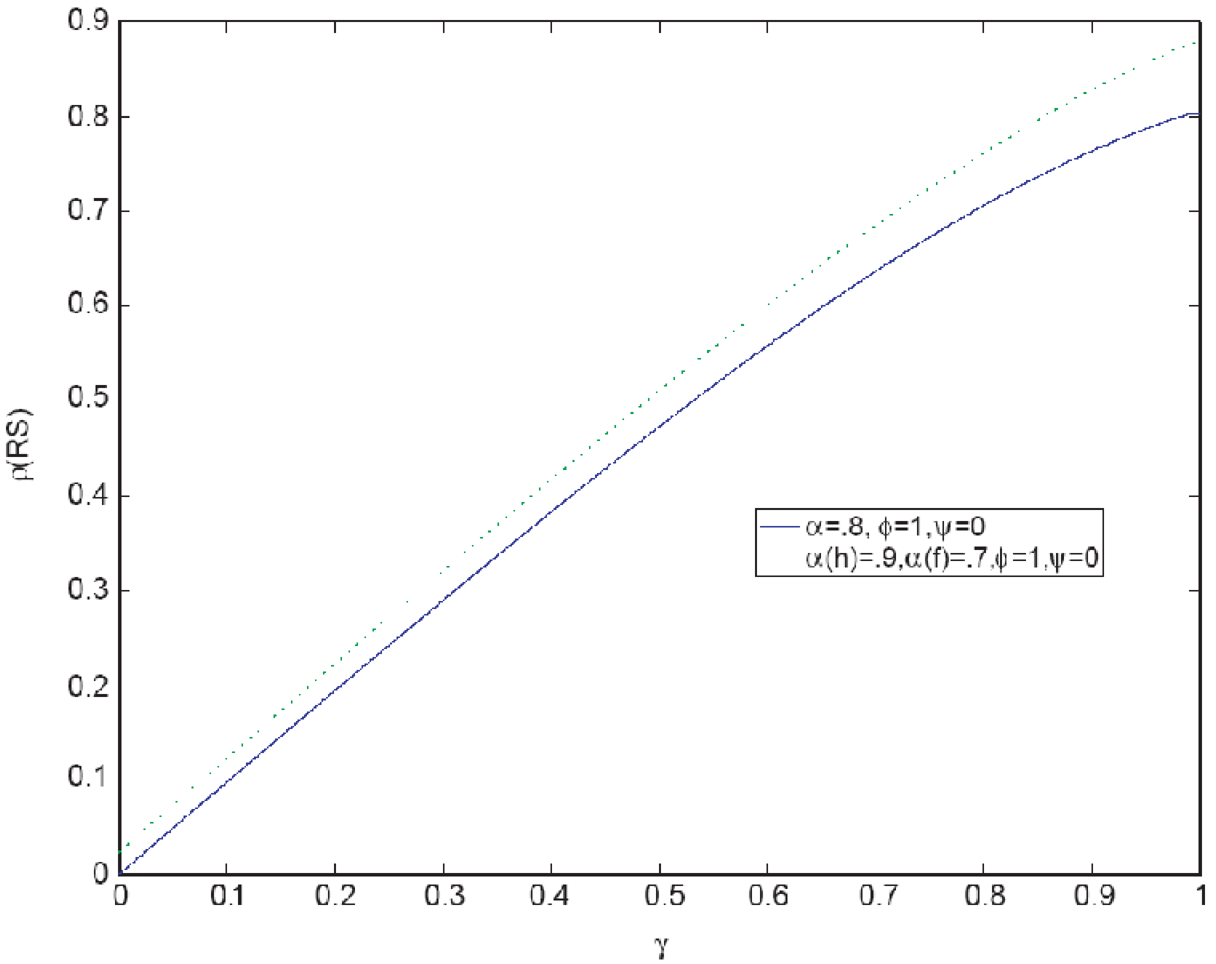


Fig. 5. Persistence with smoothing regime.

– Firm-specific price rigidity $\alpha_H = \alpha_H^* \neq \alpha_F = \alpha_F^*$

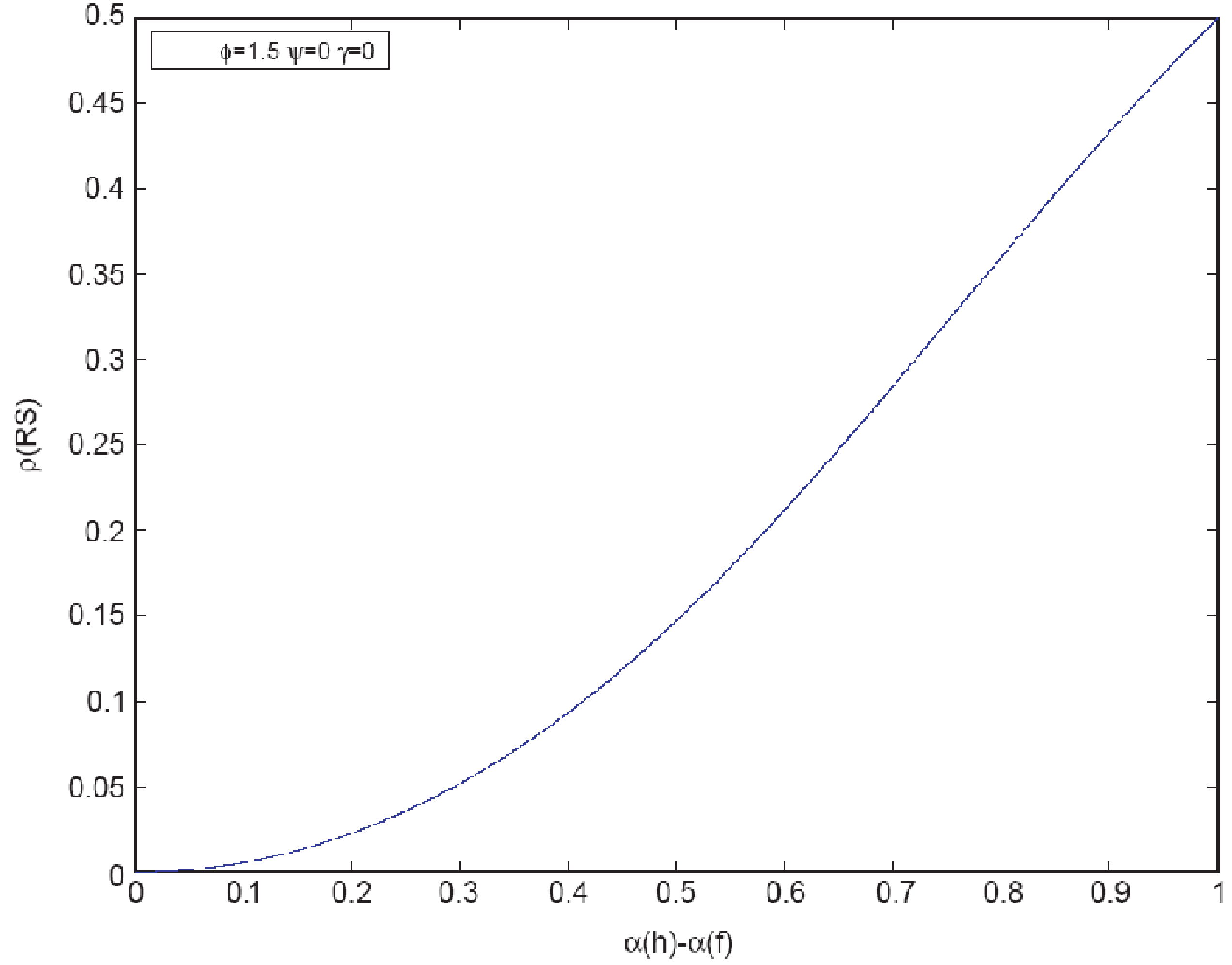
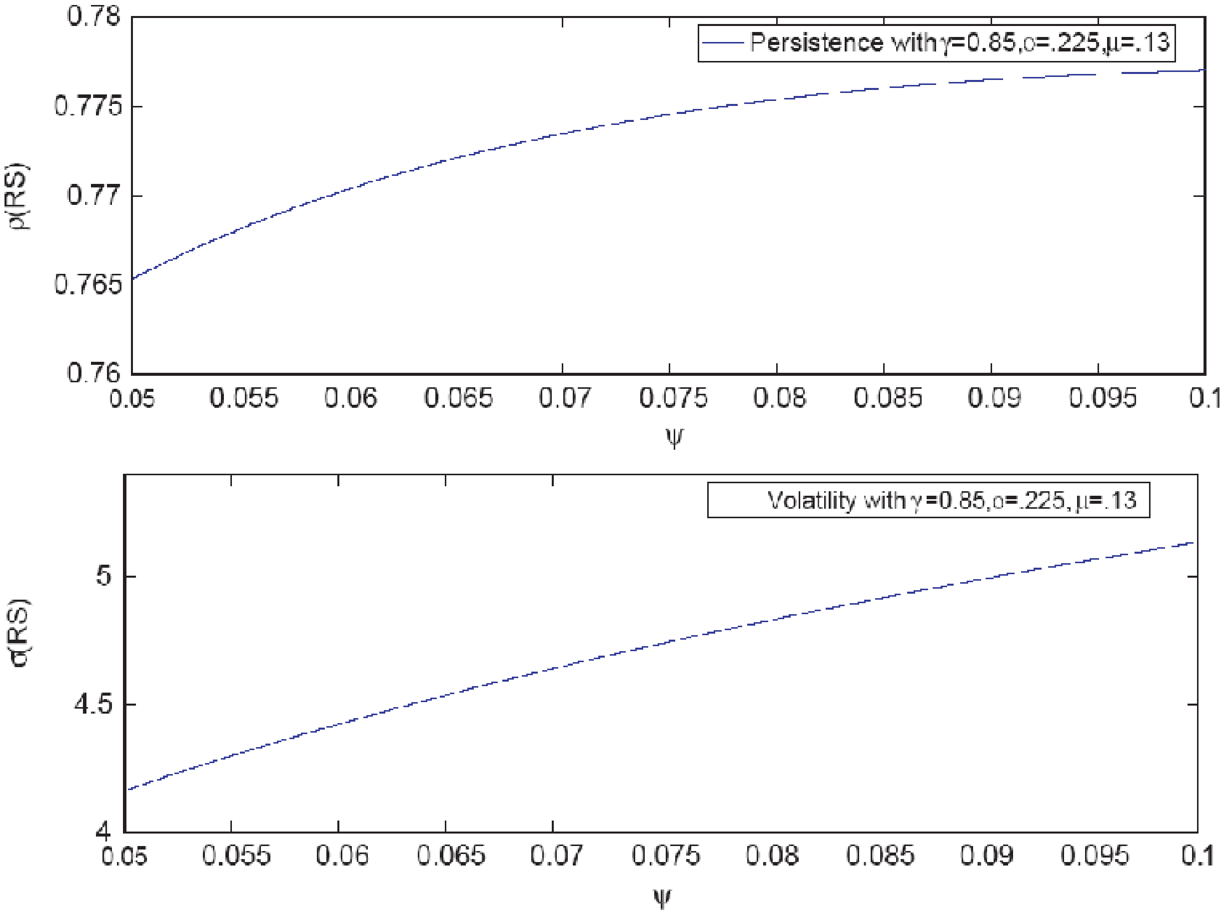


Fig. 6. Persistence of the real exchange rate with different degrees of rigidities.

- $\alpha_H = 0.8, \alpha_H^* = 0.66, \alpha^F = 0.67, \alpha_F^* = 0.4.,\ \gamma = 0.85, \phi = 0.225$



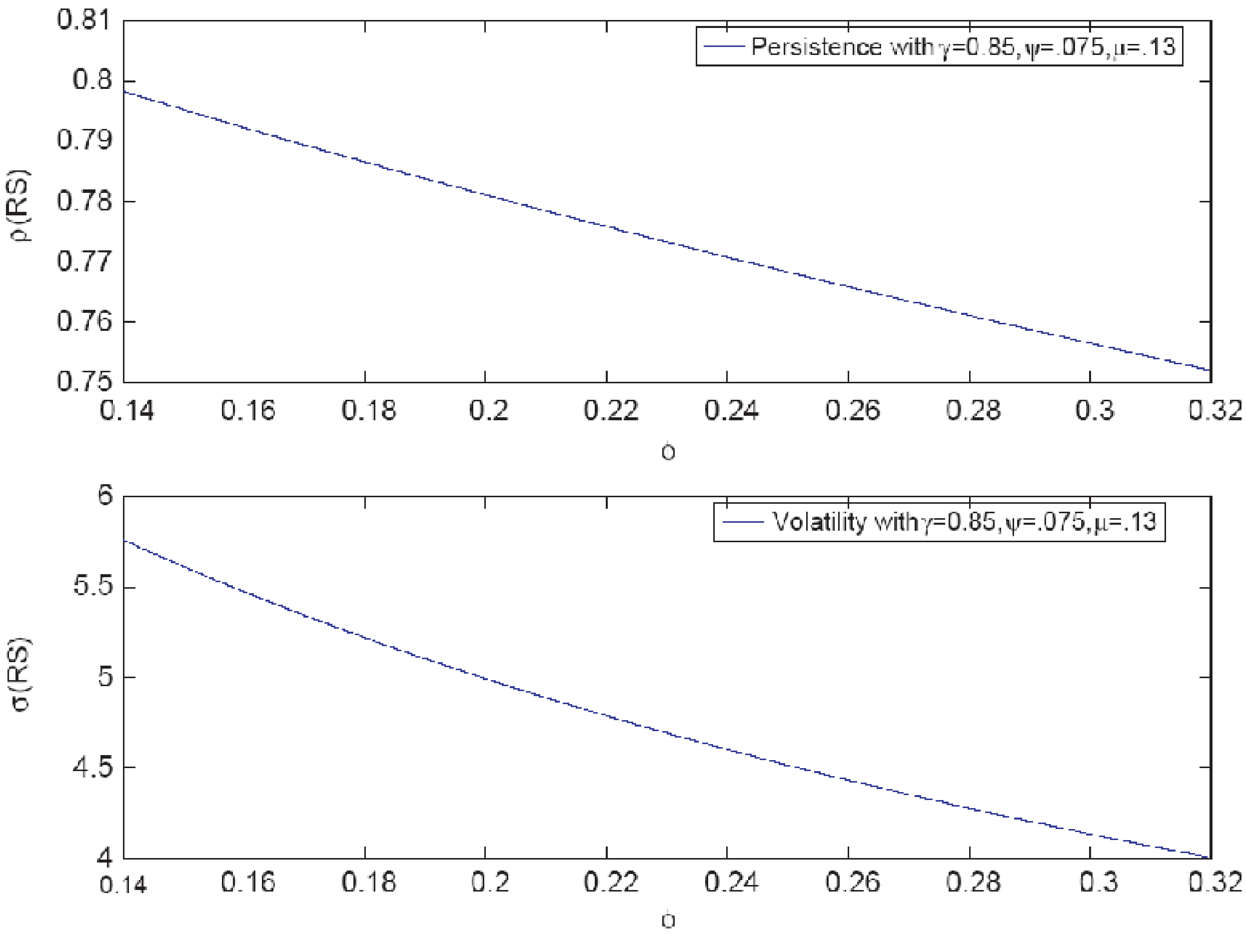


Fig. 8. **Persistence and volatility of the real exchange rate.**

- Takeaway: Previously thought: Persistence of real exchange rate increasing in degree of price stickiness, and unrealistically long period of price stickiness to match the persistence in real exchange rate. Now thought: Price rigidity is not sufficient by itself to generate persistence following a monetary shock.

4 Inflation News and the Exchange rate: Clarida and Waldman

Theoretical model for how bad news about inflation (higher than expected) is good news for the exchange rate (home currency strengthens). Interesting and nicely constructed empirical work.

Questions:

1. What is the correlation between inflation surprises and changes in the nominal exchange rate?

2. Is the sign different for inflation targeters and non-inflation targeters?

- Data: 5-minute nominal spot exchange rate data for USD–JPY,CAD,NOK,SEK,CHF,EUR,GE and NZD, July 2001 through December 2005.

- Construct returns 10-minute percentage changes to capture behavior in a plus/minus 5 minute window of the announcement.

- Positive inflation surprise means higher than expected. Expectations is the median survey response from Bloomberg News Service, which surveys commercial and investment banks on macroeconomic announcements. Use consumer price inflation. Month-over-month (MoM) and year-over-year (YoY) inflation for headline and core inflation.

- Regression. R_t is 10 minute return around the announcement. $R > 0$ means home currency *appreciates*. S_t is the inflation surprise.

$$R_{it} = \alpha + \beta S_{it} + u_{it}$$

– Normalize coefficients to interpret β as an elasticity.

– Pool the data and run as stacked OLS regression.

Table 3: All Countries

	Headline		Core	
	MoM	**YoY**	**MoM**	**YoY**
Coefficient	0.2	0.2	0.5	0.5
T- Statistic	5.9	6.2	9.7	9.2
R-Squared	0.08	0.09	0.27	0.25
# Observations	394	387	257	259

Regression method: stacked OLS.
Percentage change in exchange rate resulting from a one percentage point upward surprise in inflation.
Positive coefficient indicates appreciation of domestic currency.
Countries: Australia, Canada, Euro area, Japan, New Zealand, Norway, Sweden, Switzerland, UK, and US.
Data: July 2001- December 2005. Some countries missing observations.

– Inflation targeters versus non-inflation targeters.

Table 4: Inflation Targeters versus Non-Inflation Targeters

	Inflation Targeters				Non-Inflation Targeters			
	Headline		Core		Headline		Core	
	MoM	YoY	MoM	YoY	MoM	YoY	MoM	YoY
Coefficient	0.3	0.2	0.6	0.5	0.01	-0.08	0.1	0.1
T- Statistic	6.1	6.7	9.4	8.9	0.2	-0.8	1.3	1.1
R-Squared	0.11	0.13	0.37	0.31	0.00	0.01	0.02	0.02
# Observations	286	310	152	182	108	77	105	77

Regression method: stacked OLS.

Percentage change in exchange rate resulting from a one percentage point upward surprise in inflation.

Positive coefficient indicates appreciation of domestic currency.

Inflation targeters includes: Australia, Canada, Euro area, New Zealand, Norway, Sweden, Switzerland, and UK.

Non-inflation targeters includes: Japan and US.

Non-inflation targeters YoY includes only Japan.

Data: July 2001- December 2005. Number of observations may be less than total months due to missing observations.

– Regime changes: Bank of England independence (1997), Norway's formal adoption of inflation targeting (2001). Before the regime change, coefficients are negative (not significant).

Table 6: UK and Norway Pre-Inflation Targeting

	UK			Norway
	Headline		Core	Headline
	MoM	YoY	YoY	YoY
Coefficient	0.006	-0.05	-0.06	-0.08
T- Statistic				
OLS	0.1	-0.5	-0.7	-1.0
White	0.1	-0.7	-0.8	-1.6
Newey-West	0.1	-1.1	-1.4	-1.6
R-Squared	0.00	0.01	0.01	0.02
# Observations	46	46	46	40

Percentage change in exchange rate resulting from a one percentage point upward surprise in inflation.
Positive coefficient indicates appreciation of domestic currency.
Dates: Norway: August 1997- December 2000. UK: March 1993- December 1996.
Number of observations may be less than total months due to missing observations.
White and Newey-West used to correct for potential heteroscedasticity.

5 Calibrated partial equilibrium models of the exchange rate and Taylor rules

Engel and West JMCB. Rational expectations model.

Mark (mimeo). Adaptive learning

- Partial equilibrium setup. We assume
  - The public believes Fed and Bundesbank use some form of the Taylor rule.
  - Public views inflation and output gap are exogenously generated by a VAR(4).
  - Economic model is real interest parity, a stochastic difference equation, whose solution gives the real exchange rate.
  - Public employs least-squares learning rules to form beliefs about model's coefficients since their true values are unknown and may change over time.
- Feed historically observed data on inflation and the output gap into the model. Observe the implied learning equilibrium path of the real exchange rate and compare with historical real exchange rate path.

5.1 Can we assume that the Fed and Bundesbank (ECB) reacts the same to inflation and the output gap?

- German variables subscripted by ‘G’ U.S. variables subscripted by ‘U.’

- German–U.S. differentials have no special notation.

$$\begin{aligned}\pi_t &= \left(\pi_{G,t} - \pi_{U,t}\right)\\ i_t &= \left(i_{G,t} - i_{U,t}\right)\\ x_t &= \left(x_{G,t} - x_{U,t}\right)\end{aligned}$$

are German-U.S. differentials in inflation, short-term nominal interest rates and activity gaps, respectively. Activity gap defined such that the economy operates in excess of its potential when $x_{tj} > 0, j = G, U$. The log real exchange rate is q_t.

- Fed rule for target rate

$$i^T_{U,t} = \bar{i}_U + \gamma_\pi \left(E_t \pi_{U,t+1} - \overline{\pi}_U\right) + \gamma_x x_{U,t}.$$

  Actual interest rate subject to exogenous and i.i.d. policy shock $\eta_{U,t}$ and interest rate smoothing

$$i_{U,t} = (1-\rho) i^T_{U,t} + \rho i_{U,t-1} + \eta_{U,t}.$$

- Bundesbank target rule

$$i^T_{G,t} = \bar{\imath}_G + \gamma_\pi \left(E_t \pi_{G,t+1} - \overline{\pi}_G\right) + \gamma_x x_{G.t} + \gamma_q q_{t-1},$$

- Impose homogeneity of the coefficients (γ_π, γ_x) across countries and write interest differential as

$$i_t = \delta + \rho i_{t-1} + (1-\rho)\left(\gamma_\pi E_t \pi_{t+1} + \gamma_x x_t + \gamma_q q_{t-1},\right) + \eta_t,$$

  - $\delta \equiv (1-\rho)\left(\left(\bar{i}_G - \bar{i}_U\right) - \gamma_\pi \left(\overline{\pi}_G - \overline{\pi}_U\right)\right)$, and $\eta_t \overset{iid}{\sim} (0, \sigma_\eta^2)$.
  - Add and subtract $(1-\rho)\gamma_\pi \widetilde{\pi}_{t+1}$ on the right side of and rearrange

$$i_t = \delta + (1-\rho)\left[\gamma_\pi \pi_{t+1} + \gamma_x x_t + \gamma_s q_{t-1}\right] + \rho i_{t-1} + \eta_t',$$

where

$$\eta_t' = \eta_t - (1-\rho)\gamma_\pi \left[\pi_{t+1} - E_t \pi_{t+1}\right].$$

is uncorrelated with date t information. Estimate by GMM. Instruments are a constant, the current value and three lags of the inflation differential, the current value and three lags of the output (alternatively unemployment) gap differential, four lags of the nominal interest differential, and four lags of the real exchange rate.

Table 2: GMM Estimates of Bundesbank–Fed Relative Interest-Rate Reaction Function with Lagged Real Exchange Rate Feedback. Bold indicates significance at the 5% level

Source output gap						
Sample	δ (t-ratio)	ρ (t-ratio)	γ_π (t-ratio)	γ_x (t-ratio)	γ_q (t-ratio)	J-statis (p-valu
61.2-79.2	**-0.006**	**0.666**	0.034	**0.134**	**0.019**	9.684
	(-6.477)	(10.545)	(0.376)	(5.092)	(4.069)	(0.644
79.3-05.4	-0.001	**0.895**	**1.318**	**0.376**	0.011	4.330
	(-1.194)	(22.450)	(3.063)	(2.318)	(0.880)	(0.977
	Structural change test					
	All coeffs.	Test statistic	**40.037**	**0.000**		
	Inflation coeff.	Test statistic	**8.525**	**0.004**		

HP output gap

	δ (t-ratio)	ρ (t-ratio)	γ_π (t-ratio)	γ_x (t-ratio)	γ_q (t-ratio)	J-statis (p-valu
61.2-79.2	**-0.003**	**0.745**	0.121	0.045	**0.011**	9.370
	-3.589	12.551	1.357	1.166	2.708	0.671
79.3-05.4	0.000	**0.873**	**1.544**	**0.428**	0.004	5.314
	-0.202	24.480	4.406	3.239	0.558	0.947
	Structural change test					
	All coeffs.	Test statistic	**36.747**	**0.000**		
	Inflation coeff.	Test statistic	**15.501**	**0.000**		

HP unemployment gap	δ (t-ratio)	ρ (t-ratio)	γ_π (t-ratio)	γ_x (t-ratio)	γ_q (t-ratio)	J-statis (p-valu
61.2-79.2	**-0.003**	**0.579**	0.034	**-0.644**	0.005	7.746
	-3.077	8.394	0.554	-4.847	1.873	0.805
79.3-05.4	0.000	**0.860**	**1.154**	**-0.828**	0.004	5.083
	-0.163	22.028	2.812	-3.171	0.443	0.955
	Structural change test					
	All coeffs.	Test statistic	**25.042**	**0.015**		
	Inflation coeff.	Test statistic	**7.280**	**0.007**		

6 Modeling real exchange rate dynamics with learning

- Economic model is uncovered interest parity. For the log nominal exchange rate s_t (DM/\$)

$$s_t = E_t s_{t+1} - i_t.$$

where i is the German–US interest differential.

- Add and subtract $E_t \pi_{t+1}$ from the right side, rearrange

$$\begin{aligned} q_t &= E_t q_{t+1} - i_t + E_t \pi_{t+1} \\ i_t &= \delta + \rho i_{t-1} + (1-\rho)\left(\gamma_\pi E_t \pi_{t+1} + \gamma_x x_t + \gamma_q q_{t-1},\right) + \eta_t \end{aligned}$$

- Let π and x be exogenously given by VAR(4). Let $Y_t' = (\pi_t, \ldots, \pi_{t-3}, x_t, \ldots, x_{t-3})$, and $Z_{1,t}' = Z_{2,t}' = (Y_t', 1)$. Regression form of the VAR is

$$\begin{aligned} \pi_t &= B_1' Z_{1,t-1}' + v_{1t}, \\ x_t &= B_2' Z_{2,t-1}' + v_{2t}, \end{aligned}$$

  B_1 and B_2 are 9 x 1 least-squares coefficient vectors.

- Companion representation

$$Y_t = \alpha + AY_{t-1} + v_t,$$

$$\begin{aligned} e_1 &= (1, 0, 0, 0, 0, 0, 0, 0) \\ \pi_t &= e_1 Y_t \\ e_2 &= (0, 0, 0, 0, 1, 0, 0, 0) \\ x_t &= e_2 Y_t \end{aligned}$$

$$E_t \pi_{t+1} = e_1 (\alpha + AY_t).$$

- The above gives a second-order stochastic difference equation in q_t,

$$\begin{aligned} q_t &= \delta\,((1-\rho)\,\gamma_\pi - 1)\,e_1\alpha + (1-\rho)\,\gamma_q q_{t-1} + \rho i_{t-1} \\ &\quad + ((1-\rho)\,\gamma_x e_2 + ((1-\rho)\,\gamma_\pi - 1)\,e_1 A)\,Y_t + \eta_t + E_t q_{t+1} \end{aligned}$$

- *Rational expectations solution*

$$q_t = a_0 + a_1 i_{t-1} + a_2 q_{t-1} + a_3 \eta_t + bY_t$$

$$\begin{aligned} a_2 &= \frac{1}{2}(1-\rho) \pm \frac{\sqrt{(1-\rho)^2 - (1-\rho)\,4\,\gamma_q}}{2}, \\ a_1 &= \frac{-\rho a_2}{(1-\rho)\,\gamma_q}, \\ a_0 &= \left(\frac{((-1 + ((\rho-1)\,a_1 + (1-\rho))\,\gamma_\pi)\,e_1 - b)}{a_2}\right)\alpha - \frac{(a_1 - 1\,)\,\delta}{a_2}, \\ a_3 &= \frac{1-a_1}{1-a_2}, \\ b &= (((1 + (a_1 - 1)\,(1-\rho)\,\gamma_\pi)\,e_1)\,A + (a_1 - 1)\,(1-\rho)\,\gamma_x e_2)\,((1-a_2)\,I - A)^{-1}\,. \end{aligned}$$

- Non-uniqueness. Choose solution with positive a_2
- b depends on inflation response coefficient γ_π.
  * $\gamma_\pi < 1$ in pre-Volker sample: Decline in $E_t\pi_{t+1}$ leadds public to expect increase in the r and real dollar depreciation.
  * $\gamma_\pi > 1$ in the post-1979 sample: Decline in $E_t\pi_{t+1}$ leads to real dollar appreciation

- *Learning the rational expectations equilibrium.* Solve real interest parity condition using expectations formed from perceived law of motion.

  – At t, coefficient vectors $B_{1,t-1}, B_{2,t-1}$ are given (estimated in $t-1$). From regression form,

  $$\begin{aligned} \pi_t &= B'_{1,t-1} Z_{1,t} + u_{1,t}, \\ x_t &= B'_{2,t-1} Z_{2,t} + u_{2,t}, \end{aligned}$$

  form companion form

  $$Y_t = \alpha_{t-1} + A_{t-1} Y_{t-1} + v_t,$$

  Construct expected inflation

  $$E_t \pi_{t+1} = e_1 \left(\alpha_{t-1} + A_{t-1} Y_t \right).$$

  – Believe the rational expectation solution and use it for perceived law of motion

  $$q_t = a_{0,t-1} + a_{1,t-1} i_{t-1} + a_{2,t-1} q_{t-1} + a_{3,t-1} \eta_t + b_{t-1} Y_t \equiv B'_{3,t-1} Z_{3,t}$$

- Observe the policy shock η_t from perceived law of motion for the interest differential

$$\begin{aligned} i_t &= \delta_{t-1} + \rho_{t-1} i_{t-1} + \theta_{\pi,t-1} e_1 \left(\alpha_{t-1} + A_{t-1} Y_t\right) + \theta_{x,t-1} e_2 Y_t + \theta_{q,t-1} q_{t-1} + \eta \\ &\equiv B'_{4,t-1} Z_{4,t} + \eta_t. \end{aligned}$$

- The expected exchange rate

$$E_t q_{t+1} = a_{0,t-1} + a_{1,t-1} i_t + a_{2,t-1} q_t + b_{t-1} \left(\alpha_{t-1} + A_{t-1} Y_t\right).$$

- Plug inflation forecast and q forecast into real UIP gives the actual law of motion,

$$q_t = \frac{1}{\left(1 - a_{3,t-1}\right)} \left[a_{0,t-1} + \left(e_1 + b_{t-1}\right)\left(\alpha_{t-1} + A_{t-1} Y_t\right) + \left(a_{2,t-1} - 1\right) i_t\right].$$

- For next period, least-squares update the coefficients Let $y_{1,t} = \pi_t$, $y_{2,t} = x_t$, $y_{3,t} = q_t$, and $y_{4,t} = i_t$. For given $R_{j,t-1}$ $(j = 1, ..., 4)$, and a fixed gain g, the updating formulae are

$$R_{j,t} = R_{j,t-1} + g\left(Z_{j,t-1}Z'_{j,t-1} - R_{j,t-1}\right), \qquad (1)$$

$$B_{j,t} = B_{j,t-1} + gR^{-1}_{j,t}Z_{j,t-1}(y_{j,t} - B'_{j,t-1}Z_{j,t-1}). \qquad (2)$$

The learning path and coefficient updating is generated using observations of π_t, x_t, and i_t from the data, but not with exchange rate data. The learning values of q_t generated by the actual law of motion and employed in coefficient updating are generated solely as functions of π, x, and i.

- Calibrated Learning and Rational Exchange Rate Paths

The observations are standardized to highlight comovements between the series.

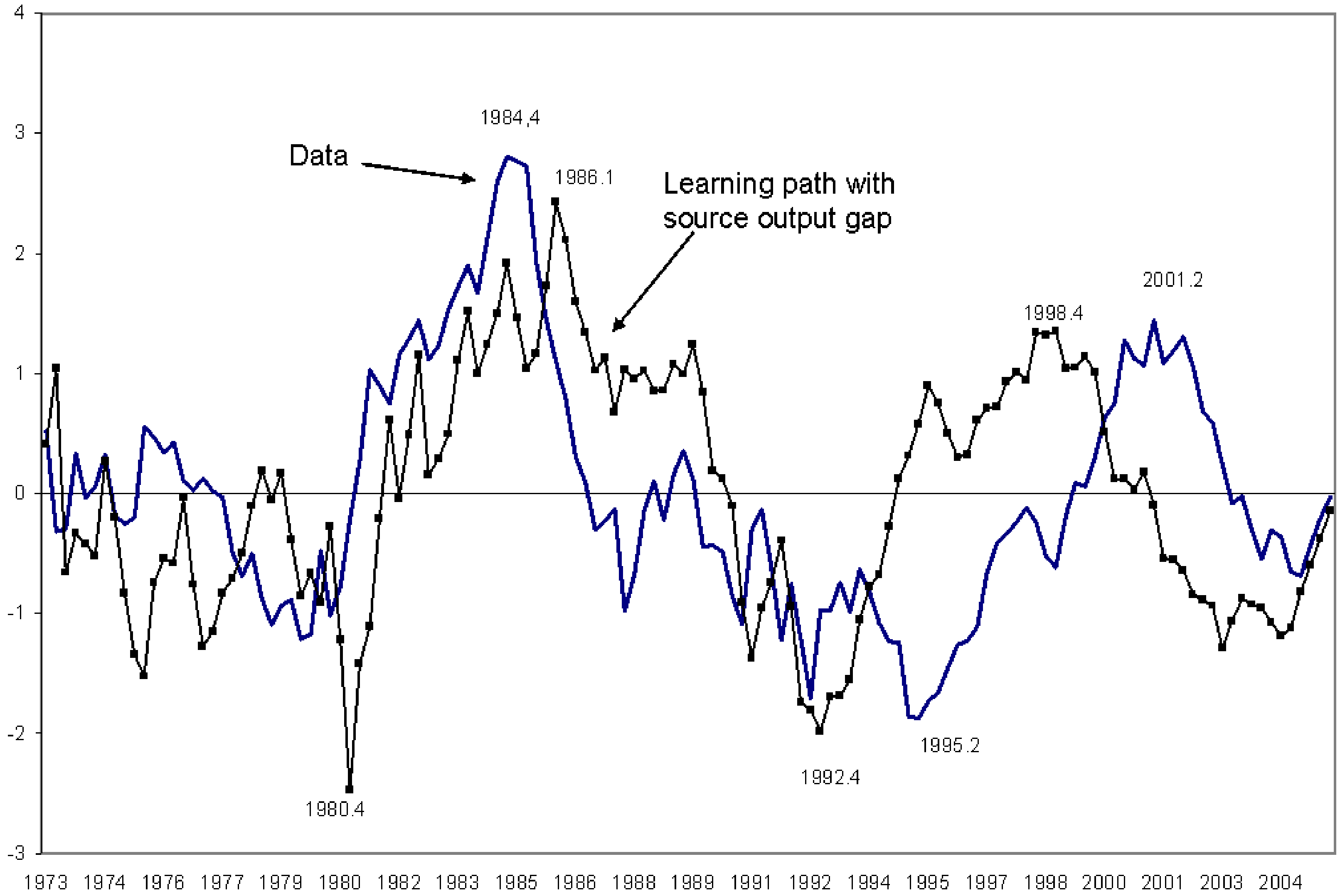


Figure 5. Learning path with source output gap.

The learning path shown in Figure 5 generally captures closer comovements with the data and does a good job of capturing the dollar cycle of the 1980s. The learning path exhibits the dollar appreciation and subsequent depreciation in the latter part of the sample although timing of the turning points are off a bit with the phase of the learning cycle leading the data.

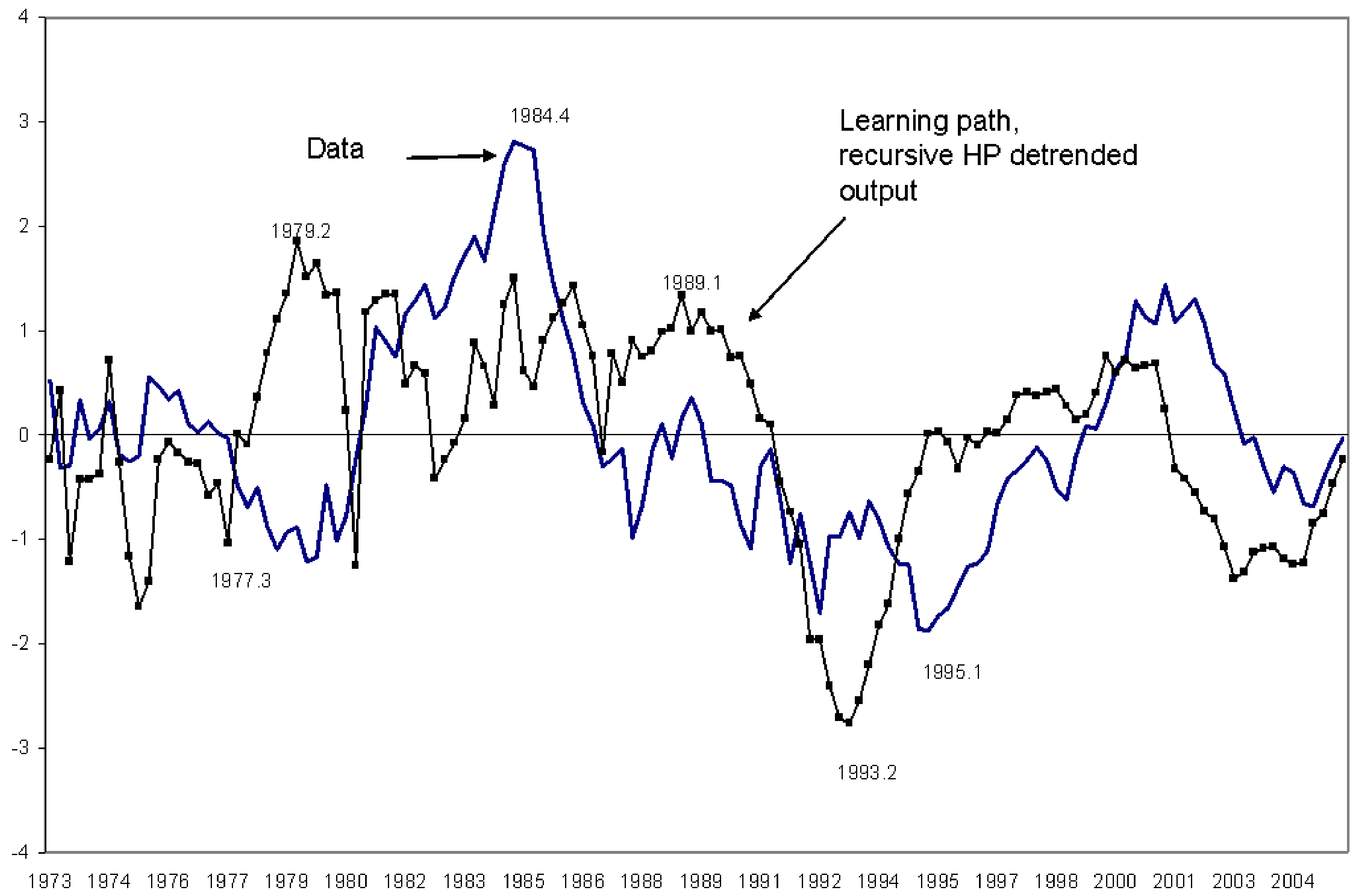


Figure 7. Learning path with HP output gap.

The learning path in Figure 7 shows the dollar prematurely appreciating in 1979.2 and it does not generate quite the strength attained in the data by 1984.4. The learning path also leads the turning points in the dollar appreciation and depreciation beginning in 1995.2 but otherwise comoves with the data.

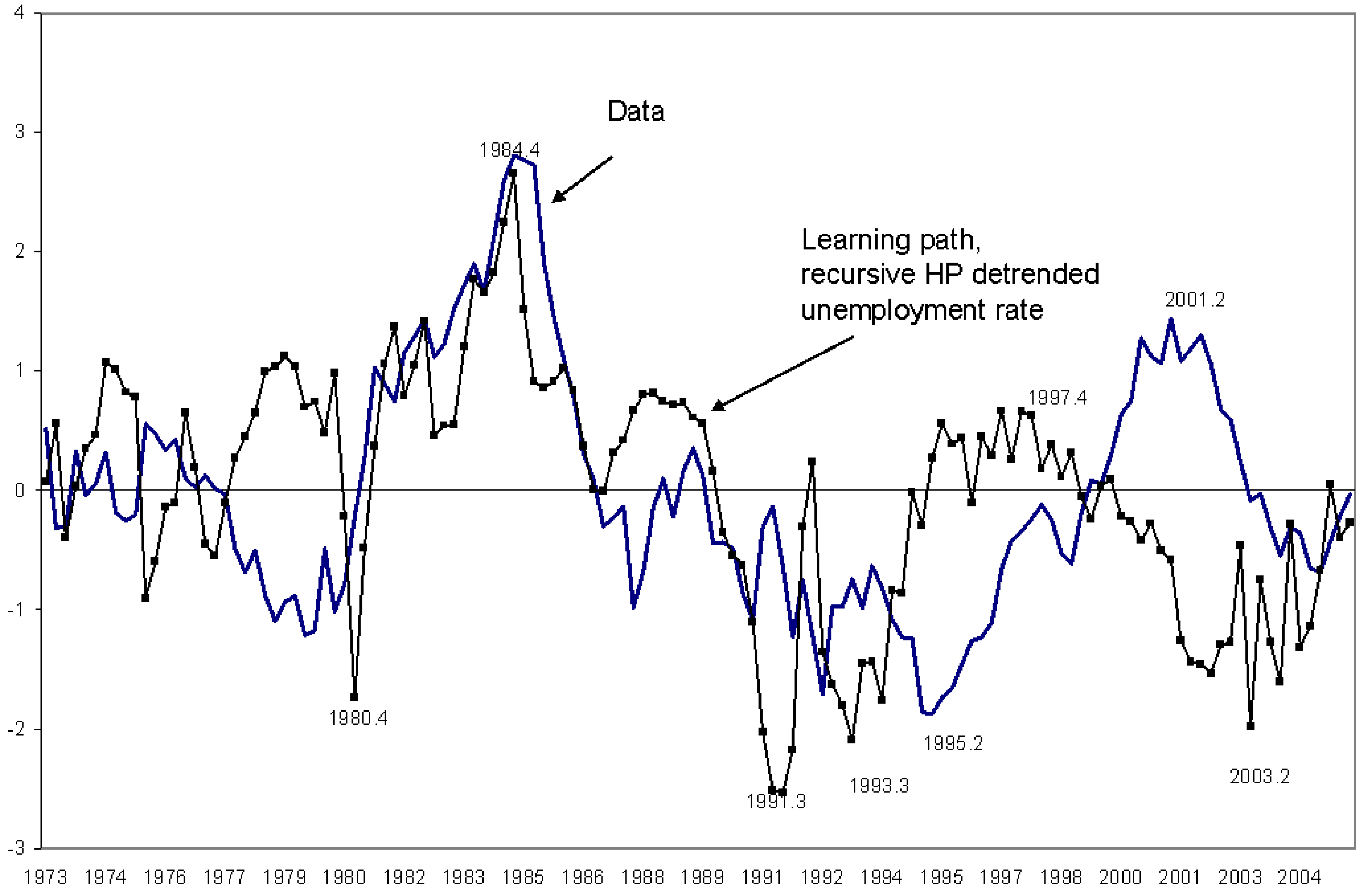


Figure 9. Learning path with HP unemployment gap.

Figure 9, it is seen that the comovmements of the learning path with the data are also quite good. The learning path matches the timing of the 1980s dollar cycle very well. Except for a phase shift that leads the data, it also captures the cycle from 1995 to 2005

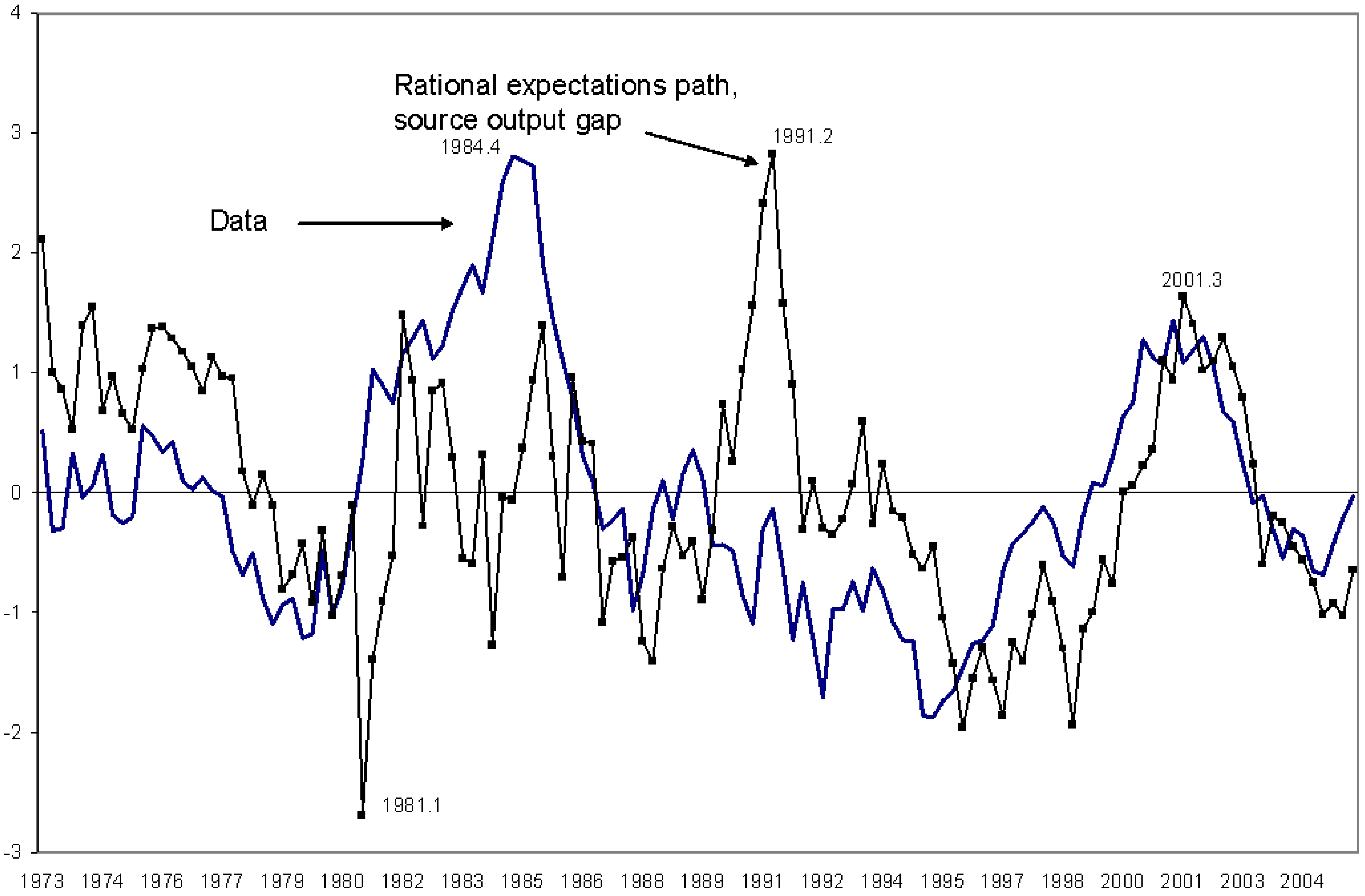


Figure 4: Rational path with source output gap.

Using the source output gap, the rational path shown in Figure 4 misses a good deal of the real dollar appreciation from 1980.4 to 1984.4 and falsely predicts a dollar appreciation from 1989.3 to 1991.2. It also erroneously predicts a large dollar depreciation in 1981.1 Apart from these episodes, there is a close connection between the implied rational expectations path and the data. The rational expectations path does a good job of explaining the real dollar appreciation from 1996.2 to 2002.2 and the subsequent real dollar depreciation.

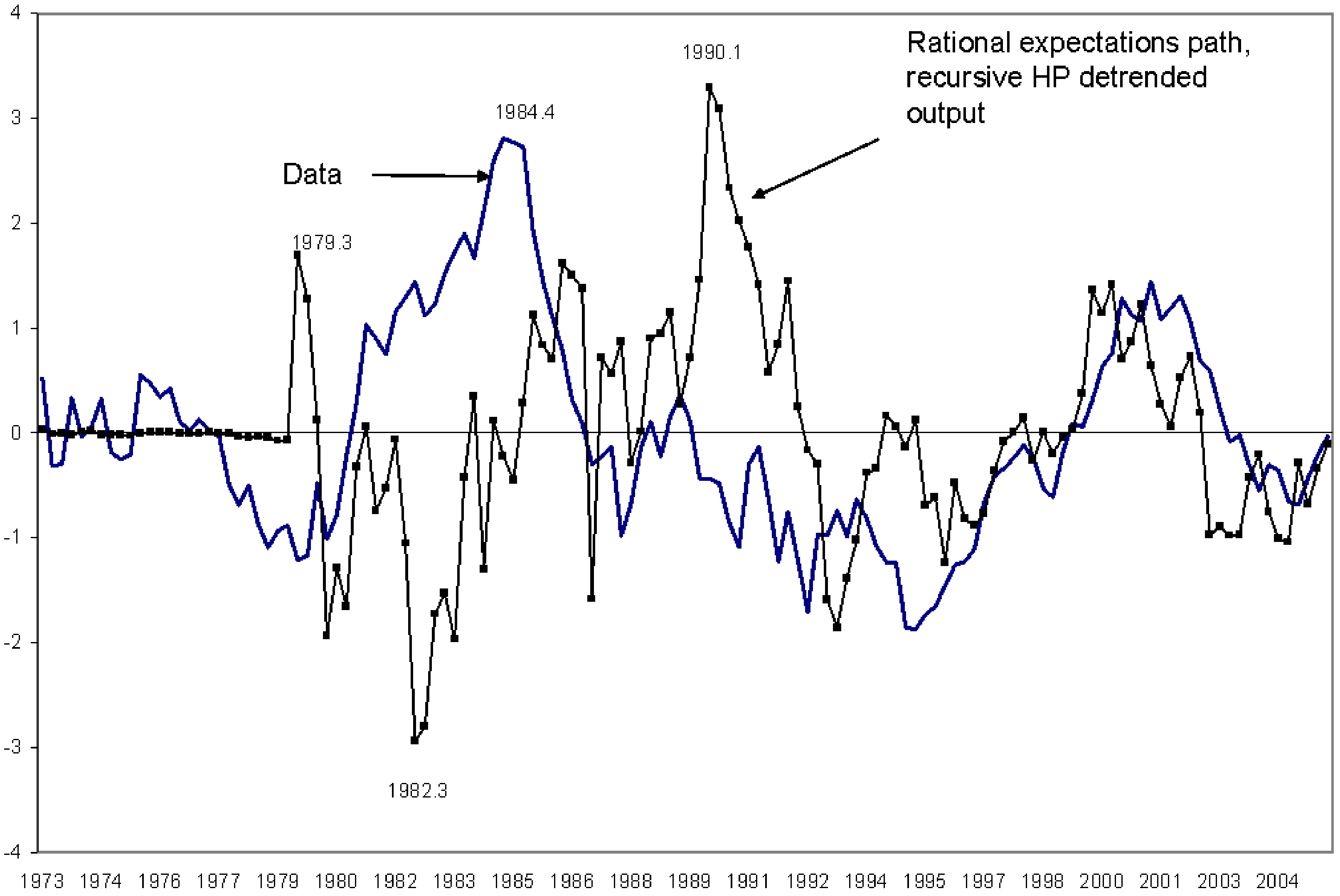


Figure 6: Rational path with HP output gap.

In Figure 6, the rational path generated with the HP output gap is quite similar to the path generated with the source output gap. Here, the rational path also misses the real dollar appreciation of the 1980s and signals a false appreciation in the early 1990s.

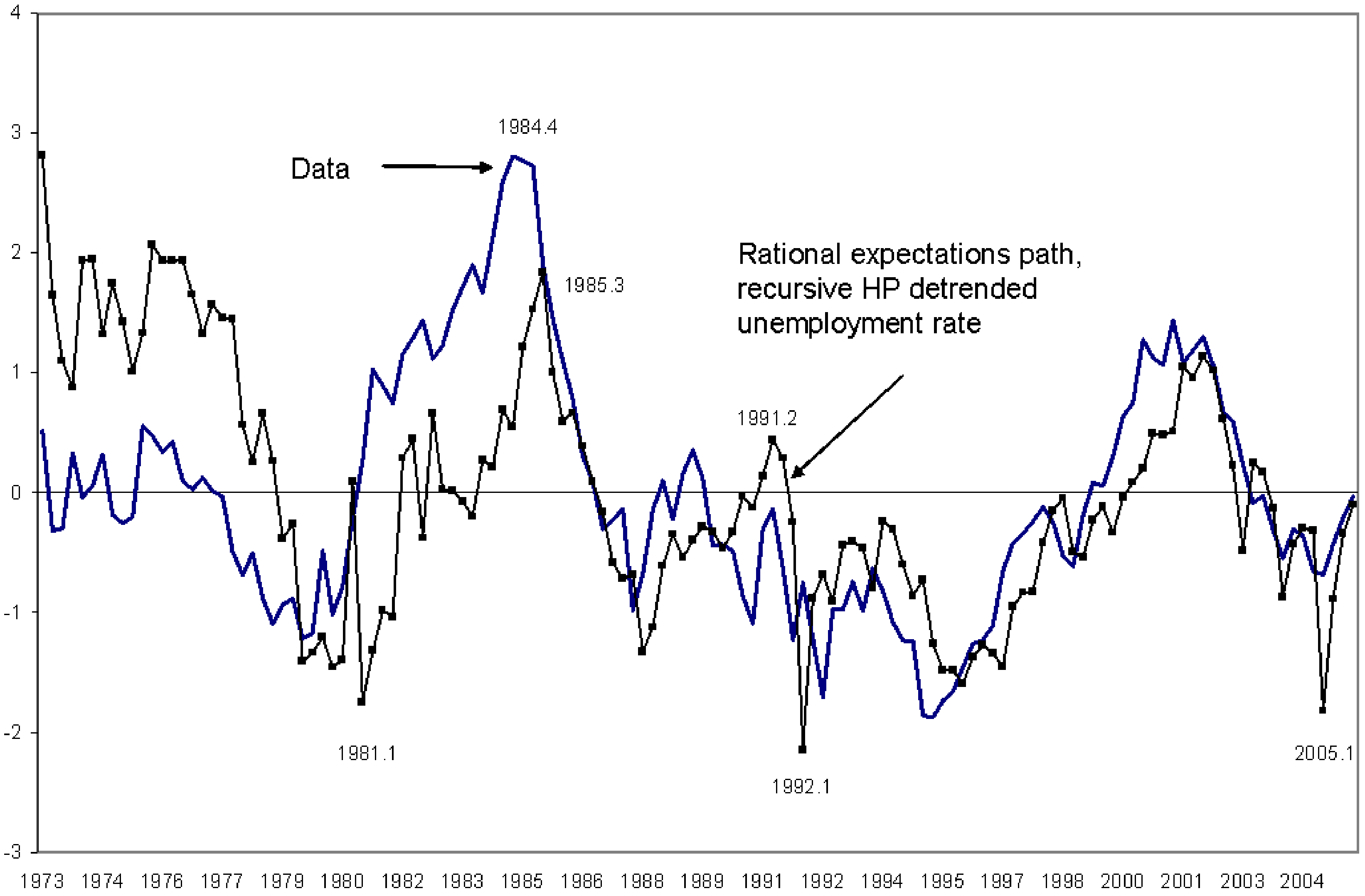


Figure 8: Rational path with HP unemployment gap.

Figure 8 plots the rational path using HP detrended unemployment. The comovements between the rational path and the data are generally quite close.

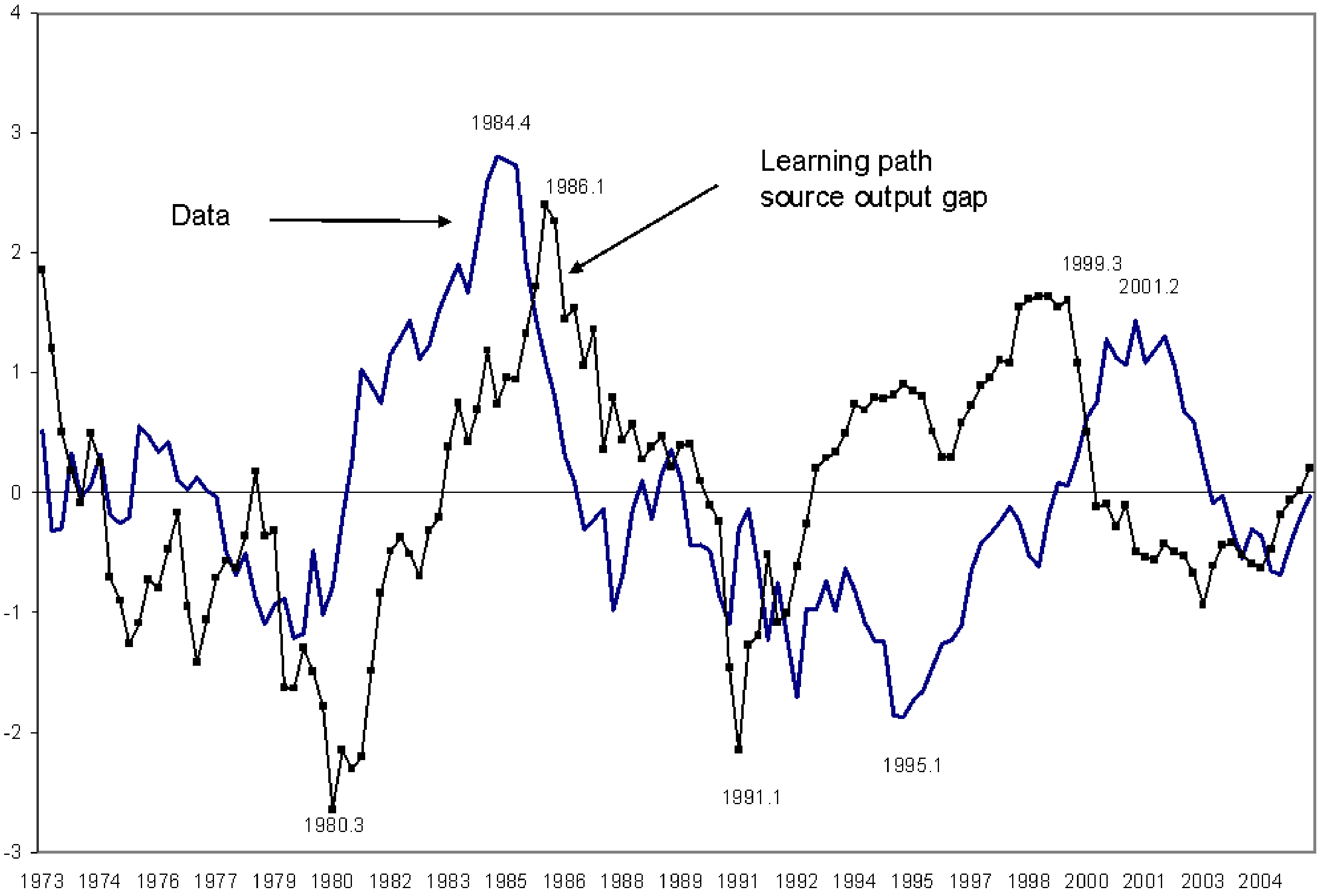


Figure 11. Learning path with contemporaneous exchange rate in Taylor rule, source output gap.

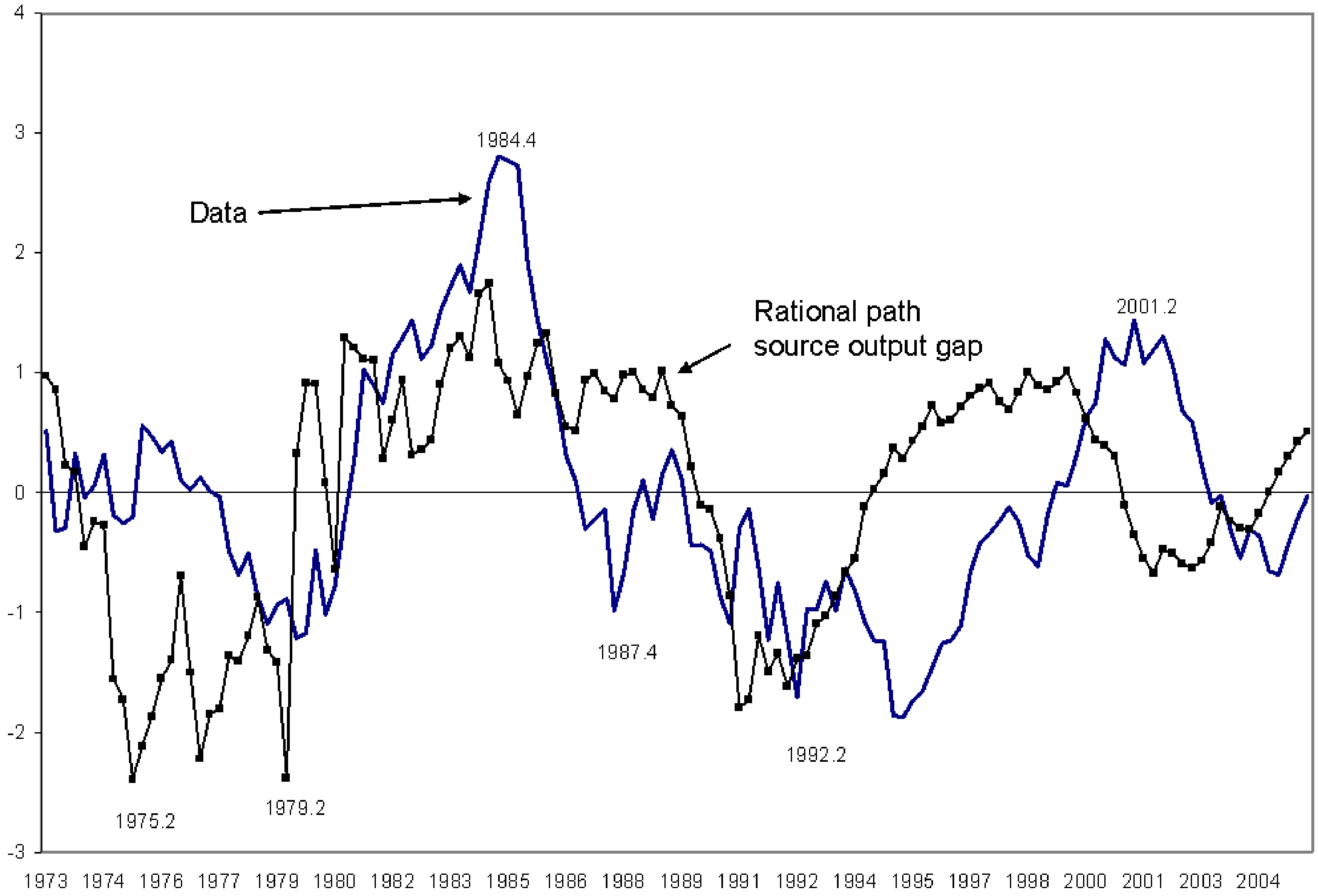


Figure 10 Rational path with contemporaneous exchange rate in Taylor rule, source output gap.

Table 3: Correlations and Relative Volatility

Form	Activity variable	Rational Corr	Rational T-ratio	Rational Volatility	Learning Corr	Learning T-ratio	Learning Volatility
	Source gap	0.308	2.461	0.965	0.346	2.170	1.054
Level	HP output	-0.029	-0.201	3.678	0.298	2.094	0.336
	HP unemployment	0.484	3.426	0.644	0.340	1.834	1.130
	Source gap	0.030	0.700	2.085	-0.039	-0.596	1.254
1-qtr return	HP output	0.019	0.536	7.918	0.033	0.601	0.458
	HP unemployment	0.031	0.499	1.561	-0.008	-0.135	1.870
	Source gap	0.235	3.142	1.525	0.044	0.393	1.164
4-qtr return	HP output	0.026	0.323	5.662	0.054	0.593	0.402
	HP unemployment	0.459	5.575	1.050	-0.026	-0.265	1.486
	Source gap	0.308	2.699	1.264	0.157	1.023	1.014
8-qtr return	HP output	0.066	0.639	4.374	0.037	0.338	0.357
	HP unemployment	0.624	5.418	0.848	0.109	0.706	1.142
	Source gap	0.424	4.106	1.055	0.335	2.393	1.086
16-qtr return	HP output	0.019	0.113	3.753	0.093	0.799	0.345
	HP unemployment	0.691	4.381	0.746	0.093	0.799	1.033

To sum up, there were six major swings in the real DM-dollar rate in the sample. The real dollar depreciation from 1973.1 to 1979.4, the sharp appreciation (1980.1–1984.4) and subsequent depreciation (1985.1–1987.4), a more tempered dollar depreciation (1988.1 to 1995.1), a dollar appreciation (1995.2–2001.2), and the dollar decline (2001.3–2005.4). Each of the rational paths falsely predicted a strong real dollar appreciation in 1991 and none of them adequately matched the volatility in the data. The learning model provides a plausible description for the data. Regardless of the definition used for the activity gap, each of the learning paths captured the major swings in the real exchange rate.

Two obvious extensions. (1) relax homogeneity restrictions on coefficients across countries. (2) Have an economic model for the output gap and inflation.

7 Taylor-rule fundamentals and exchange rate forecasts

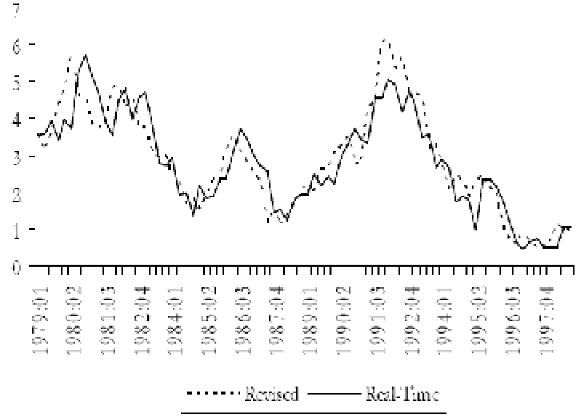
Molodtsova, Nikolsko-Rzhevskyy, and Papell, ‘Taylor Rules with Real-Time Data: A Tale of Two Countries and One Exchange Rate.’

- Real-time versus (revised) historical data.
  - Use the information (data) available to monetary authorities and other economic agents when they made their decisions.
  - Revision in output and output gap is more substantial than revision to price indices.
  - Alternative source of data: Greenbook forecasts of inflation and output gap observed by Fed but not the public until five years later.

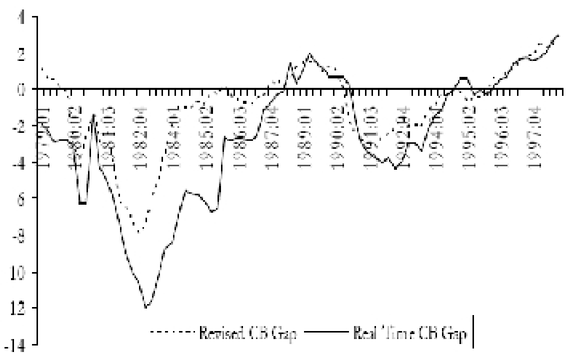
  - German real time data: Gerberding, Worms, and Seitz.
  - US real time data. Crouschore and Stark, Philadelphia Fed.

- Estimate Taylor rules with real-time data for US and Germany from 1979 to 1998.
  - U.S. potential GDP constructed by Orphanides.
  - German output gap is deviation of GDP to quadratic time trend.

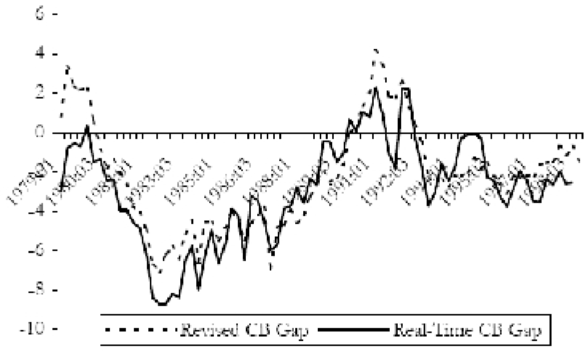
A. U.S. Inflation



B. German Inflation



A. U.S. Output Gap



B. German Output Gap

– Find that Taylor rule coefficients are robust to choice of real-time or historical data, and choice of forward looking forecasts, current and lagged inflation/output gap data.

Table 1. Estimated Taylor Rules for the United States

	Revised Data		*Real-Time Data*	
Inflation Coefficient, λ	1.39	1.50	1.32	1.36
	(0.17)	(0.21)	(0.16)	(0.21)
Output Gap Coefficient, γ	0.11	0.28	-0.01	0.01
	(0.15)	(0.20)	(0.08)	(0.12)
Smoothing Coefficient, ρ	-	0.55	-	0.54
		(0.18)		(0.18)
R-squared	0.73	0.82	0.73	0.82

Notes: The table reports nonlinear least squares estimates of the following equation, $i_t = (1-\rho)(\mu + \lambda\pi_t + \gamma y_t) + \rho i_{t-1} + v_t$, where i is the Federal Funds Rate, π is inflation, and y is the output gap. The sample size is 1979:Q1 – 1998:Q4. Standard errors are in parentheses.

Table 2. Estimated Taylor Rules for Germany

	Revised Data					*Real Time Data*				
Inflation Coefficient, λ	1.36	0.84	0.84	0.88	0.86	1.37	1.19	1.19	1.19	1.20
	(0.15)	(0.51)	(0.36)	(0.43)	(0.34)	(0.14)	(0.48)	(0.27)	(0.48)	(0.27)
Output Gap Coefficient, γ	0.09	1.13	1.27	1.02	1.20	0.23	0.78	0.99	0.78	1.00
	(0.09)	(0.77)	(0.64)	(0.67)	(0.59)	(0.07)	(0.35)	(0.28)	(0.38)	(0.29)
Smoothing Coefficient, ϱ	-	0.89	0.85	0.88	0.85	-	0.87	0.80	0.87	0.80
		(0.07)	(0.07)	(0.08)	(0.07)		(0.06)	(0.06)	(0.06)	(0.06)
Exchange Rate Coefficient, δ_1	-	-	0.09	-	0.08	-	-	0.08	-	0.08
			(0.05)		(0.05)			(0.03)		(0.03)
Money Growth Coefficient, δ_2				0.23	0.12				0.01	0.01
				(0.30)	(0.14)				(0.37)	(0.17)
R-squared	0.71	0.95	0.96	0.95	0.96	0.68	0.95	0.95	0.95	0.95

Notes: The table reports nonlinear least squares estimates of the following equation, $i_t = (1-\rho)(\mu + \lambda\pi_t + \gamma y_t + \delta z_t) + \rho i_{t-1} + v_t$, where i is the Federal Funds Rate, π is inflation, y is the output gap, and z is a vector containing the real DM/Dollar exchange rate and the money growth deviation.. The sample size is 1979:Q1 – 1998:Q4. Standard errors are in parentheses.

- Reduced form exchange rate forecasting equation. U.S. is home country

  $$\Delta s_{t+1} = \alpha_0 + \alpha_1 \pi_t - \alpha_2 \pi_t^* + \alpha_3 y_t - \alpha_4 y_t^* + \rho_1 i_{t-1} + \rho_2 i_{t-1}^* + \alpha_4 q_t + \eta_t$$

  Symmetric means homogeneity restrictions imposed on coefficients.

7.0.1 Clark-West test of equal forecast accuracy (in mean square sense).

1. Out-of-sample forecasting. Let the exchange rate return be

$$y_t \equiv s_t - s_{t-1}$$

   (a) RMSE, Theil's U-statistic.

$$\begin{aligned} H0 &: y_t = e_t, \qquad E_{t-1}e_t = 0 \\ Ha &: y_t = Z_t'\beta + e_t, \qquad E_{t-1}e_t = 0 \end{aligned}$$

   Under Ha,

$$y_t = \hat{f}_t + \hat{e}_t, \quad \hat{f}_t = Z_t'\hat{\beta}$$

Make P one-step ahead predictions.

$$\begin{aligned}
\text{MSPE}(1) &= \hat{\sigma}_1^2 = \frac{1}{P} \sum_{t=T-P+1}^{T} e_{t+1}^2 \\
\text{MSPE}(2) &= \hat{\sigma}_2^2 = \frac{1}{P} \sum_{t=T-P+1}^{T} \hat{e}_{t+1}^2 = \frac{1}{P} \sum_{t=T-P+1}^{T} \left(y_{t+1} - \hat{f}_{t+1}\right)^2 \\
U &= \frac{\hat{\sigma}_2}{\hat{\sigma}_1}
\end{aligned}$$

Can bootstrap Theil's U.

(b) Diebold-Mariano (1995)–West (1996) statistic. For non-nested hypotheses,

$$\begin{aligned}
F_{t+1} &= e_{t+1}^2 - \hat{e}_{t+1}^2 \\
\bar{F} &= \frac{1}{P} \sum_{t=T-P+1}^{T} F_{t+1} \\
\hat{V} &= \frac{1}{P} \sum_{t=T-P+1}^{T} \left(F_{t+1} - \bar{F} \right)^2 \\
DMW &= \frac{\bar{F}}{\hat{V}} \sim N(0,1)
\end{aligned}$$

To compute DMW: Regress F_{t+1} on a constant. DMW is the t-statistic for the constant. Asymptotics hold for non nested hypotheses.

i. DMW turns out not to be asymptotically normal for nested hypotheses. The random walk model is nested, causing DMR to be badly undersized.

ii. Under $H0$, sampling variability in estimated coefficients under alternative cause

$$E\left(\hat{\sigma}_1^2\right) < E\left(\hat{\sigma}_2^2\right)$$

Therefore, one accepts the random walk too often.

(c) Clark-West (2005) test. A simple adjustment.Works reasonably well for rolling or recursive regression. Both are slightly undersized but offer large improvements over DMW. Recall $\hat{f}_{t+1} = x'_{t+1}\hat{\beta}$.

$$\begin{aligned} F^a_{t+1} &= e^2_{t+1} - \hat{e}^2_{t+1} + \hat{f}^2_{t+1} \\ CW &= \frac{\bar{F}^a}{\hat{V}} \sim N(0,1) \end{aligned} \tag{3}$$

where a superscript stands for 'adjusted.'

i. CW statistic: Regress F^a_{t+1} on a constant. CW is the t-ratio from regression output.

ii. For long-horizon forecasts.

$$\begin{aligned} y_{t,k} &= y_{t+1} + y_{2+2} + \cdots + y_{t+k} \\ H0 &: y_{t,k} = e_{t+1,k} \\ Ha &: y_{t,k} = x'_{t+1}\beta + e_{t+1,k} = \hat{f}_{t,k} + \hat{e}_{t+1,k} \end{aligned}$$

Compute $F_t^a, \bar{F}^a$ as above.

$$F_{t+1,k}^a = e_{t+1,k} - \hat{e}_{t+1,k}^2 - \hat{f}_{t,k}^2$$

Due to serial correlation induced by overlapping forecasts, need to adjust $\hat{V}$. Ken West suggests the Hodrick (1992) or West (1997) covariance matrix

$$\begin{aligned}\hat{g}_t &= 2y_t\left(\hat{f}_{t,k} + \cdots + \hat{f}_{t-k+1}\right)\\ \hat{V} &= \frac{1}{P-2k+2}\sum_{t=t-P+k}^{T-k+1}\left(\hat{g}_{t+k} - \bar{g}\right)^2\end{aligned}$$

7.0.2 Evaluate choice of real-time or historical data for exchange rate forecasts.

Typical results

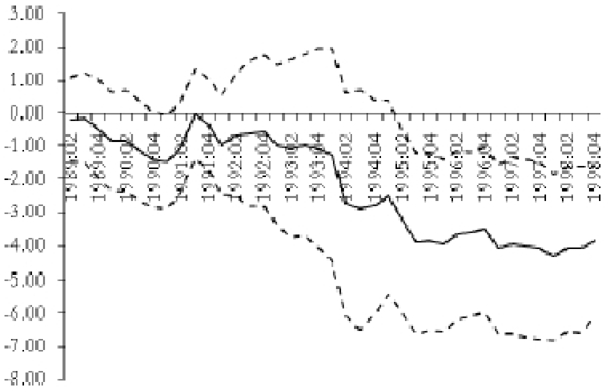
Table 5. CW Statistics: One-Quarter-Ahead USD/DM Exchange Rate Forecasts using Taylor Rules with Central Bank Output Gaps

	w/o Smoothing		*w/ Smoothing*	
	Symmetric	*Asymmetric*	*Symmetric*	*Asymmetric*
A. Revised and Real-Time Data				
Revised Data	0.358	0.548	0.258	-0.107
	(0.31)	(0.25)	(0.35)	(0.50)
Real-Time Data	-0.936	1.979**	-0.689	1.924**
	(0.77)	(0.02)	(0.70)	(0.03)

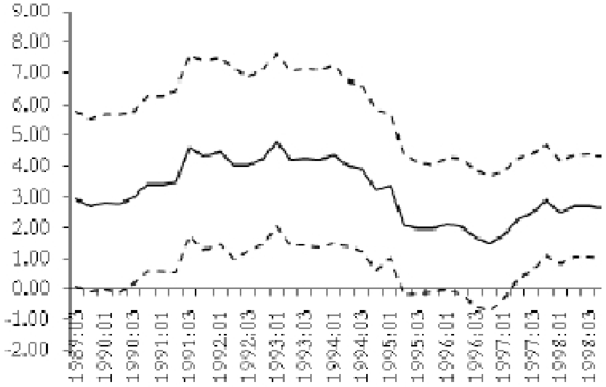
Table 6. CW Statistics: One-Quarter-Ahead USD/DM Exchange Rate Forecasts using Taylor Rules with Quadratic Detrended Output Gaps

	w/o Smoothing		*w/ Smoothing*	
	Symmetric	*Asymmetric*	*Symmetric*	*Asymmetric*
A. Revised and Real-Time Data				
Revised Data	0.201	0.526	0.210	-0.178
	(0.36)	(0.37)	(0.36)	(0.47)
Real-Time Data	0.009	1.775**	0.314	1.754**
	(0.44)	(0.03)	(0.34)	(0.04)

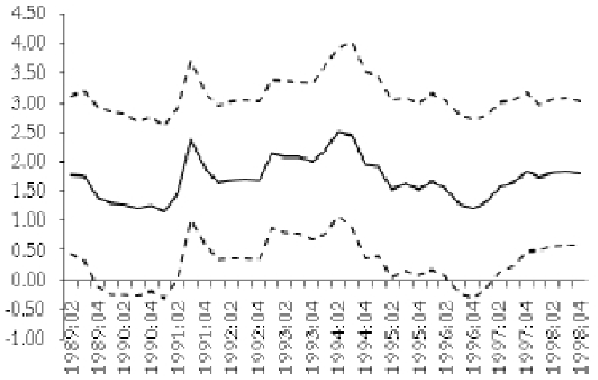
Significant predictability observed in asymmetric model using Greenbook inflation forecasts and Survey of Professional Forecaster's inflation forecasts.



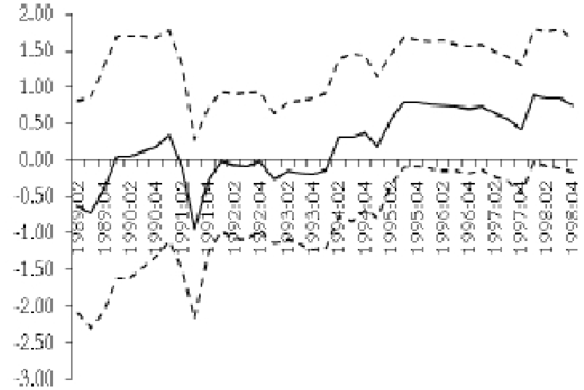
A. U.S. Inflation Coefficient



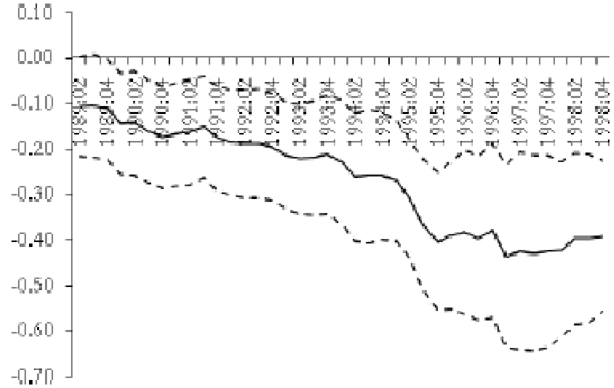
B. German Inflation Coefficient



C.U.S. Output Gap Coefficient



D.German Output Gap Coefficient



Inflation coefficients say bad news about inflation is good news for the exchange rate.

Increase in U.S. output gap predicts depreciation. German output gap coefficients never significant.

8 Conclusion

1. Explicit treatment of endogenous monetary policy and interest rate rules in exchange rate economics is relatively new.

2. The Taylor-rule approach might provide a resolution to the PPP puzzle.

3. The Taylor-rule approach identifies a very different set of macroeconomic fundamentals in discussions about exchange rate determination.

4. Calibrated partial equilibrium Taylor-rule models of the exchange rate seem to fit the data reasonably well, and evidently much better than PPP or traditional monetary approaches.

5. Taylor-rule fundamentals have statistically significant predictive power for the future exchange rate.

6. There is scope for more research in the area.