Third-Country Effects on the Exchange Rate

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Motivation

- Exchange Rate Disconnect Puzzle (Obstfeld-Rogoff, 2000)

1. Empirical puzzle
2. Meese & Rogoff (1983), Baxter & Stockman (1989), Diebold & Nason (1990), Obstfeld & Rogoff (1996), Devereux & Engel (2002), Cheung, Chinn, & Pascual (2005), Molodtsova & Papell (2009), Evans (2012), ...
3. Low R^2 in exchange rate (and predictive) regressions.

Research Question

Do third country effects explain the exchange rate disconnect puzzle?

Data

1. Sources:
   - Datastream and IMF IFS
2. Time Period:
   - 1999 - 2011 (Quarterly)
3. Countries:
   - Brazil, Canada, Denmark, Indonesia, Japan, Norway, Philippines, Singapore, Switzerland, Thailand, United Kingdom, and United States
4. Variables:
   - CPI, Exchange Rates, GDP, Industrial Production, Monetary Policy Interest Rate

Empirical Motivation

- Engel, Mark, and West (2012)
- Greenaway-McGrevy, Mark, Sul, and Wu (2012)

Motivation

- Regression of exchange rate between '1' and '2' using only macro variables from '1' and '2.'

$$e_{1,2,t+k} - e_{1,2,t} = \alpha + \beta(e_{1,2,t} + p_{2,t} - p_{1,t}) + \epsilon_{1,2,t+k}$$

- The world we live in is a multi-country environment.

  1. Low explanatory power is due to omitted third-country variables.

Motivation

- Regression of exchange rate between '1' and '2' using only macro variables from '1' and '2.' (One-period horizon)

$$e_{1,2,t+1} - e_{1,2,t} = \alpha + \beta(e_{1,2,t} + p_{2,t} - p_{1,t}) + \epsilon_{1,2,t+1}$$

Table: $\bar{R}^2$ from DPPP Exchange Rate Regressions

Country	DPPP relative to USD $\bar{R}^2$	Country	DPPP relative to USD $\bar{R}^2$
Brazil	-0.016	Norway	-0.004
Canada	-0.015	Philippines	-0.021
Denmark	-0.003	Singapore	-0.021
Great Britain	0.042	Switzerland	-0.018
Indonesia	-0.010	Thailand	-0.016
Japan	0.045		

Table: $\bar{R}^2$ from DPPP Exchange Rate Regressions

	DPPP relative to			
	USD	USD & euro	USD & yen	USD & SF
Country	$\bar{R}^2$	$\bar{R}^2$	$\bar{R}^2$	$\bar{R}^2$
	One-period horizon			
Brazil	-0.016	**0.097**	**0.088**	-0.036
Canada	-0.015	**-0.006**	-0.030	**0.013**
Denmark	-0.003	**0.000**	**0.124**	-0.024
Great Britain	0.042	0.024	**0.061**	0.024
Indonesia	-0.010	**-0.007**	-0.010	**0.027**
Japan	0.045	0.035		0.033
Norway	-0.004	**0.033**	**0.008**	**0.054**
Philippines	-0.021	**0.038**	**0.070**	**0.029**
Singapore	-0.021	-0.029	**0.045**	-0.025
Switzerland	-0.018	-0.033	**0.041**	
Thailand	-0.016	**0.000**	**0.065**	**-0.008**

Note: Bold face entries indicate that the addition of third-country variables increases $\bar{R}^2$.

Table: $\bar{R}^2$ from DPPP Exchange Rate Regressions

	DPPP relative to			
	USD	USD & euro	USD & yen	USD & SF
Country	$\bar{R}^2$	$\bar{R}^2$	$\bar{R}^2$	$\bar{R}^2$
	Four-period horizon			
Brazil	0.038	**0.488**	**0.429**	0.032
Canada	0.009	**0.042**	**0.013**	**0.100**
Denmark	0.070	**0.126**	**0.485**	0.062
Great Britain	0.250	**0.254**	**0.346**	0.243
Indonesia	0.216	**0.273**	**0.336**	**0.324**
Japan	0.199	**0.219**		**0.258**
Norway	0.070	0.061	**0.239**	0.064
Philippines	-0.008	**0.146**	**0.226**	**0.163**
Singapore	0.003	**0.095**	**0.147**	**0.124**
Switzerland	0.017	-0.001	**0.256**	
Thailand	-0.013	**0.041**	**0.297**	**0.069**

Note: Bold face entries indicate that the addition of third-country variables increases $\bar{R}^2$.

Exchange Rates and Taylor-Rule Fundamentals

Compare $\bar{R}^2$ from the predictive regressions:

1. $$e_{1,2,t+k} - e_{1,2,t} = b_0 + b_1 \pi_{1,t} + b_2 \pi_{2,t} + ... \\ ... + b_3 \tilde{y}_{1,t} + b_4 \tilde{y}_{2,t} + \epsilon_{1,2,t+k}$$

2. $$e_{1,2,t+k} - e_{1,2,t} = b_0 + b_1 \pi_{1,t} + b_2 \pi_{2,t} + b_3 \tilde{y}_{1,t} + b_4 \tilde{y}_{2,t} + ... \\ ... + b_5 \pi_{3,t} + b_6 \tilde{y}_{3,t} + \epsilon_{1,2,t+k}$$

Table: $\bar{R}^2$ from Taylor-Rule Exchange Rate Regressions

	Taylor-Rule Fundamentals of Home Country and			
	U.S.	U.S. & Euro	U.S. & Japan	U.S. & Switz.
Country	$\bar{R}^2$	$\bar{R}^2$	$\bar{R}^2$	$\bar{R}^2$
		One-period horizon		
Brazil	0.068	0.060	0.030	**0.126**
Canada	0.079	**0.084**	**0.144**	**0.115**
Denmark	-0.041	-0.086	**0.177**	-0.045
Great Britain	0.099	**0.155**	0.078	**0.111**
Indonesia	0.023	-0.012	**0.078**	**0.041**
Japan	0.011	**0.038**		-0.030
Norway	0.066	**0.079**	0.058	0.027
Philippines	-0.047	**0.072**	-0.077	-0.051
Singapore	-0.043	**0.037**	-0.050	**0.046**
Switzerland	-0.073	**0.169**	-0.114	
Thailand	0.054	0.045	0.034	0.021

Note: Bold face entries indicate that the addition of third-country variables increases $\bar{R}^2$.

Table: $\bar{R}^2$ from Taylor-Rule Exchange Rate Regressions

	Taylor-Rule Fundamentals of Home Country and			
	U.S.	U.S. & Euro	U.S. & Japan	U.S. & Switz.
Country	$\bar{R}^2$	$\bar{R}^2$	$\bar{R}^2$	$\bar{R}^2$
	Four-period horizon			
Brazil	-0.017	**0.053**	**0.001**	**0.253**
Canada	0.491	**0.494**	0.468	**0.738**
Denmark	0.208	**0.278**	**0.265**	**0.249**
GreatBritain	0.270	**0.333**	**0.336**	**0.275**
Indonesia	0.396	0.382	0.386	0.380
Japan	-0.013	**0.045**		-0.063
Norway	0.323	**0.426**	0.309	**0.338**
Philippines	-0.003	**0.112**	**0.061**	**0.135**
Singapore	0.095	**0.157**	0.073	**0.164**
Switzerland	0.085	**0.295**	**0.152**	
Thailand	0.182	0.167	0.160	**0.192**

Note: Bold face entries indicate that the addition of third-country variables increases $\bar{R}^2$.

Outline

- Partial Equilibrium Model
- General Equilibrium Model
- Model Results
- Empirical Results
- Conclusion

Partial Equilibrium Model

- Three Country Model:
  1. United States
  2. Home Country
  3. Rest of the World
- Monetary Policy
- Uncovered Interest Rate Parity (UIP)

Partial Equilibrium Model

- United States Monetary Policy:

$$i_{1,t} = \delta + \lambda E_t\left(\pi_{1,t+1}\right) + \mu \tilde{y}_{1,t} + \epsilon_{1,t}$$

- Home Country Monetary Policy:

$$i_{2,t} = \delta + \lambda E_t\left(\pi_{2,t+1}\right) + \mu \tilde{y}_{2,t} + \gamma\left(q_{1,3,t} - q_{1,2,t}\right) + \epsilon_{2,t}$$

Partial Equilibrium Model

- Uncovered Interest Rate Parity:

$$E_t\left(e_{1,2,t+1}\right) - e_{1,2,t} = i_{1,t} - i_{2,t}$$

- Real Uncovered Interest Rate Parity:

$$E_t\left(q_{1,2,t+1}\right) - q_{1,2,t} = \left[i_{1,t} - E_t\left(\pi_{1,t+1}\right)\right] - \left[i_{2,t} - E_t\left(\pi_{2,t+1}\right)\right]$$

Partial Equilibrium Model

Rearrange and Iterate Forward:

$$
\begin{aligned}
q_{1,2,t} = & \left(\frac{\lambda - 1}{1+\gamma}\right) E_t \sum_{k=0}^{\infty} \left(\frac{1}{1+\gamma}\right)^k (\pi_{2,t+k} - \pi_{1,t+k}) + \ldots \\
& \ldots + \left(\frac{\mu}{1+\gamma}\right) E_t \sum_{k=0}^{\infty} \left(\frac{1}{1+\gamma}\right)^k (\tilde{y}_{2,t+k} - \tilde{y}_{1,t+k}) + \ldots \\
& \ldots + \left(\frac{\gamma}{1+\gamma}\right) E_t \sum_{k=0}^{\infty} \left(\frac{1}{1+\gamma}\right)^k q_{1,3,t+k} + \frac{\epsilon_t}{1+\gamma}
\end{aligned}
$$

$$i_t = \delta + \rho i_{t-1} + (1-\rho)\left(\lambda E_t \pi_{t+1} + \phi \tilde{y}_t + \sigma q_{x,t}\right) + \epsilon_t$$

Table: Cross-Rate Management (Newey-West t-ratios in parentheses)

Country	Cross rate	σ	Country	Cross rate	σ
Brazil	euro	0.301**	Japan	euro	0.016**
		(3.664)			(3.274)
	SF	0.359**	Norway	yen	0.013*
		(4.573)			(1.885)
	yen	0.267**	Philippines	euro	0.103**
		(4.780)			(3.260)
Canada	euro	0.084*		yen	0.069**
		(1.849)			(2.399)
Denmark	euro	0.769**	Switzerland	euro	0.065**
		(4.616)			(3.141)
UK	yen	0.047*	Thailand	euro	0.124
		(1.844)			(1.531)
Indonesia	yen	0.072**	Singapore	yen	0.160
		(2.320)			(1.234)

Notes: * (**) indicates significance at the 10 (5) percent level.

Model

- Three Country Calvo Staggered Price-Setting Model
  1. The *United States* produces goods on $[0,a_1]$.
  2. The *Home country* produces goods on $[a_1,a_2]$.
  3. The *Rest of the World (ROW)* produces goods on $[a_2,1]$.
- Complete Markets Environment
- Monetary Policy
  1. Standard Taylor Rules (Independent Policy)
  2. Real Exchange Rate Targeting (Managed Float)

Household Problem

The household problem for country j is to maximize the expected discounted sum of future period utilities:

$$\max_{c_j(s^t), n_j(s^t), B_j(s_{t+1}), \frac{M_j(s^t)}{P_j(s^t)}} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) u\left(c_j(s^t), 1 - n_j(s^t), \frac{M_j(s^t)}{P_j(s^t)}\right)$$

subject to:

$$c_j(s^t) + \frac{M_j(s^t)}{P_j(s^t)} + \sum_{s_{t+1}} \frac{Q(s_{t+1}|s^t) B_j(s_{t+1})}{e_{1,j}(s^t) P_j(s^t)} =$$
$$\frac{W_j(s^t) n_j(s^t)}{P_j(s^t)} + \frac{\Pi_j(s^t)}{P_j(s^t)} + \frac{M_j(s^{t-1})}{P_j(s^t)} + \frac{B_j(s_t)}{e_{1,j}(s^t) P_j(s^t)}$$

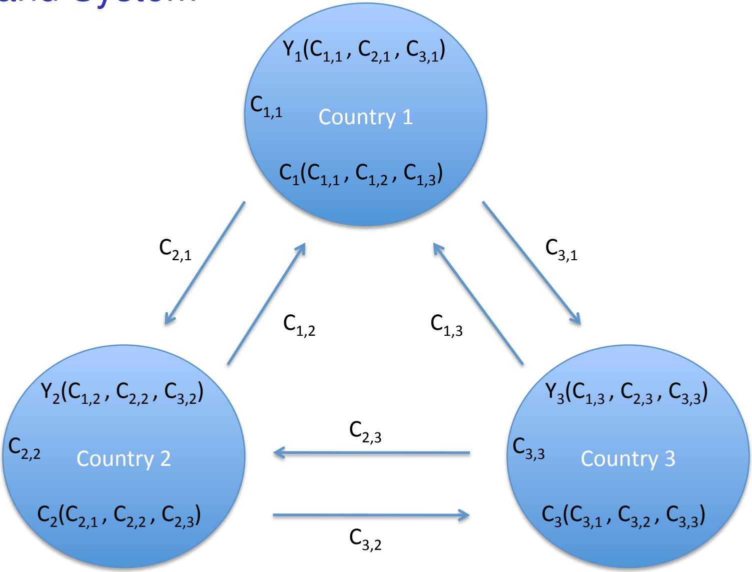
Functional Form

FONCs

Risk Sharing

$$
\begin{aligned}
q_{k,j}(s^t) &= \frac{e_{k,j}(s^t) P_j(s^t)}{P_k(s^t)} \\
&= h_{k,j,0} \left(\frac{c_j(s^t)}{c_k(s^t)} \right)^{-\gamma_1}
\end{aligned}
$$

Demand System



Consumption and Price Indices

$$c_{j,t} = \left[d_{j,1}^{\frac{1}{\mu}} c_{j,1,t}^{\frac{\mu-1}{\mu}} + d_{j,2}^{\frac{1}{\mu}} c_{j,2,t}^{\frac{\mu-1}{\mu}} + d_{j,3}^{\frac{1}{\mu}} c_{j,3,t}^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}$$

$$P_{j,t} = \left[d_{j,1} P_{j,1,t}^{1-\mu} + d_{j,2} P_{j,2,t}^{1-\mu} + d_{j,3} P_{j,3,t}^{1-\mu} \right]^{\frac{1}{1-\mu}}$$

where

$$d_{j,1} + d_{j,2} + d_{j,3} = 1$$

Consumption Sub-Indices

Price Sub-Indices

Firm Problem

Employment

$$\max_{p_{k,j,t}(\omega)} E_0 \sum_{s=0}^{\infty} (\alpha\beta)^s \frac{\Lambda_{1,t,t+s}}{P_{j,t+s}} \left[\frac{p_{1,j,t}(\omega)}{e_{1,j,t+s}} c_{1,j,t+s}(\omega) + \frac{p_{2,j,t}(\omega)}{e_{2,j,t+s}} c_{2,j,t+s}(\omega) \right.$$
$$\left. \ldots + \frac{p_{3,j,t}(\omega)}{e_{3,j,t+s}} c_{3,j,t+s}(\omega) - W_{j,t+s} n_{j,t+s}(\omega) \right]$$

subject to:

$$c_{1,j,t}(\omega) = \Phi_{1,j,t} c_{1,t} p_{1,j,t}(\omega)^{-\sigma}$$
$$c_{2,j,t}(\omega) = \Phi_{2,j,t} c_{2,t} p_{2,j,t}(\omega)^{-\sigma}$$
$$c_{3,j,t}(\omega) = \Phi_{3,j,t} c_{3,t} p_{3,j,t}(\omega)^{-\sigma}$$
$$c_{1,j,t}(\omega) + c_{2,j,t}(\omega) + c_{3,j,t}(\omega) = A_{j,t} n_{j,t}(\omega)$$
$$\Lambda_{1,t,t+s} = \left(\frac{c_{1,t+s}}{c_{1,t}} \right)^{-\gamma_1} \left(\frac{P_{1,t}}{P_{1,t+s}} \right)$$

Price Setting Equations

Log Linearization

$$p_{k,j,t}(\omega) = \left(\frac{\sigma}{\sigma-1}\right) \frac{E_t \sum_{s=0}^{\infty} (\alpha\beta)^s c_{j,t+s}^{-\gamma_1} c_{k,t+s} \Phi_{k,j,t+s} \left(\frac{W_{j,t+s}}{A_{j,t+s} P_{j,t+s}}\right)}{E_t \sum_{s=0}^{\infty} (\alpha\beta)^s c_{j,t+s}^{-\gamma_1} c_{k,t+s} \Phi_{k,j,t+s} \left(\frac{1}{e_{k,j,t+s} P_{j,t+s}}\right)}$$

where:

$$\Phi_{k,j,t} = d_{k,j}\phi_{k,j} \left(\frac{1}{P_{k,j,t}}\right)^{-\sigma} \left(\frac{P_{k,j,t}}{P_{k,t}}\right)^{-\mu}$$

$$\phi_{k,j} = \begin{cases} \frac{1}{a_1} & j = 1 \\ \frac{1}{a_2 - a_1} & j = 2 \\ \frac{1}{1 - a_2} & j = 3 \end{cases}$$

Goods Market Clearing Conditions

$$y_{1,t} = \int_0^{a_1} y_{1,t}(\omega)\, d\omega = c_{1,1,t} + c_{2,1,t} + c_{3,1,t}$$

$$y_{2,t} = \int_{a_1}^{a_2} y_{2,t}(\omega)\, d\omega = c_{1,2,t} + c_{2,2,t} + c_{3,2,t}$$

$$y_{1,t} + y_{2,t} + y_{3,t} = c_{1,t} + c_{2,t} + c_{3,t}$$

Technology Processes

$$ln(A_{1,t}) = \rho_1 ln(A_{1,t-1}) + \epsilon_{A_{1,t}}$$

$$ln(A_{2,t}) = \rho_2 ln(A_{2,t-1}) + \epsilon_{A_{2,t}}$$

$$ln(A_{3,t}) = \rho_3 ln(A_{3,t-1}) + \epsilon_{A_{3,t}}$$

$$\epsilon_{A_{j,t}} \overset{iid}{\sim} (0, \sigma^2_{\epsilon_A})$$

Monetary Policy Rules

1. Standard Taylor Rules (Independent Policy):

$$i_{1,t} = \psi_1 i_{1,t-1} + \xi_1 E_t \pi_{1,t+1} + \nu_1 \tilde{y}_{1,t} + \epsilon_{i_{1,t}}$$
$$i_{2,t} = \psi_2 i_{2,t-1} + \xi_2 E_t \pi_{2,t+1} + \nu_2 \tilde{y}_{2,t} + \epsilon_{i_{2,t}}$$
$$i_{3,t} = \psi_3 i_{3,t-1} + \xi_3 E_t \pi_{3,t+1} + \nu_3 \tilde{y}_{3,t} + \epsilon_{i_{3,t}}$$
$$\epsilon_{i_{j,t}} \overset{iid}{\sim} (0, \sigma^2_{\epsilon_i})$$

2. Real Exchange Rate Targeting (Managed Float):

$$i_{1,t} = \psi_1 i_{1,t-1} + \xi_1 E_t \pi_{1,t+1} + \nu_1 \tilde{y}_{1,t} + \epsilon_{i_{1,t}}$$
$$i_{2,t} = \psi_2 i_{2,t-1} + \xi_2 E_t \pi_{2,t+1} + \nu_2 \tilde{y}_{2,t} + \kappa_2 q_{2,3,t} + \epsilon_{i_{2,t}}$$
$$i_{3,t} = \psi_3 i_{3,t-1} + \xi_3 E_t \pi_{3,t+1} + \nu_3 \tilde{y}_{3,t} + \epsilon_{i_{3,t}}$$
$$\epsilon_{i_{j,t}} \overset{iid}{\sim} (0, \sigma^2_{\epsilon_i})$$

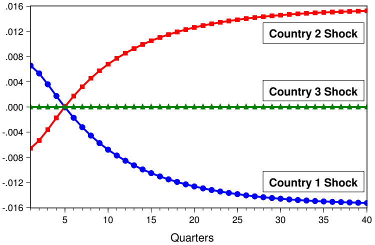
Equilibrium

An *equilibrium* for this economy is a collection of allocations for households of $c_{j,t}$, $n_{j,t}$, $M_{j,t}$, $B_j(s_t)$, allocations and prices for producers, $y_{j,t}$ and $p_{j,t}$, final goods prices, $P_{j,t}$, wages, $W_{j,t}$, and bond prices, $Q\left(s^{t+1}|s_t\right)$, such that the household allocations solve the household's problem, goods prices solve the producer's problem, market clearing conditions hold, and monetary policies are conducted as described above.

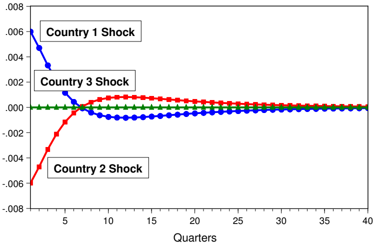
Parameterization

Preferences/Technology	
Discount Factor	$\beta = 0.99$
Coefficient of Relative Risk Aversion	$\gamma_1 = 2$
Leisure	$\gamma_2 = 2$
Real Balances	$\gamma_3 = 2$
Elasticity of Substitution	$\mu = 1.5$
Calvo Parameter	$\alpha = 0.75$
Home Bias	$\mathrm{d}_{k,j} = \frac{1}{3}$
Technology Process	
Persistence	$\rho_1 = \rho_2 = \rho_3 = 0.9$
Monetary Policy	
Policy Rate	$\psi_1 = \psi_2 = \psi_3 = 0.95$
Inflation	$\xi_1 = \xi_2 = \xi_3 = 1.5$
Output Gap	$\nu_1 = \nu_2 = \nu_3 = 0.5$
Cross Rate Real Exchange Rate	$\kappa_2 = 0.5$

Environment I: Technology Shock (Std. Taylor Rules & Symmetric Calvo Parameters)

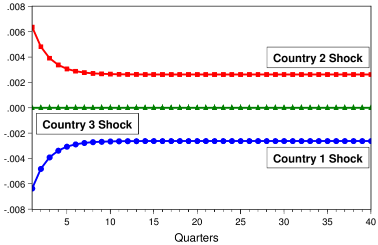


(a) Nominal Exchange Rate: $e_{1,2}$

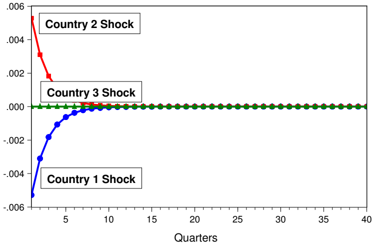


(b) Real Exchange Rate: $q_{1,2}$

Environment I: Monetary Policy Shock (Std. Taylor Rules & Symmetric Calvo Parameters)

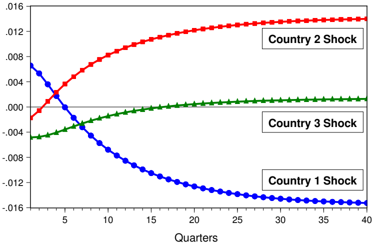


(c) Nominal Exchange Rate: $e_{1,2}$

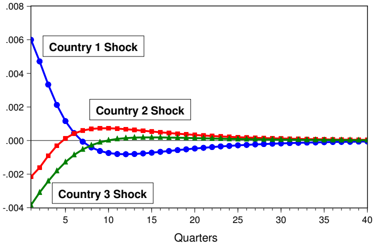


(d) Real Exchange Rate: $q_{1,2}$

Environment II: Technology Shock (Managed Float & Symmetric Calvo Parameters)

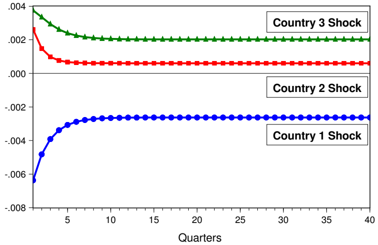


(e) Nominal Exchange Rate: $e_{1,2}$

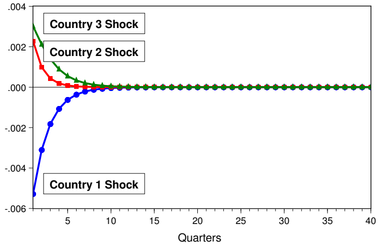


(f) Real Exchange Rate: $q_{1,2}$

Environment II: Monetary Policy Shock (Managed Float & Symmetric Calvo Parameters)



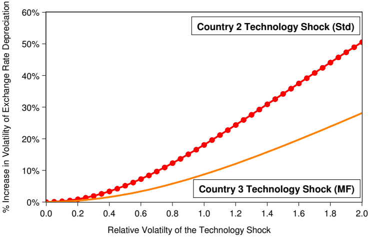
(g) Nominal Exchange Rate: $e_{1,2}$



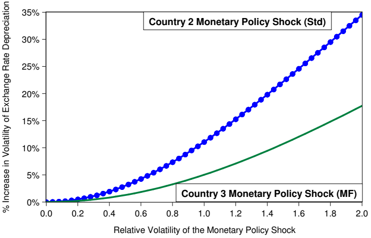
(h) Real Exchange Rate: $q_{1,2}$

% Increase in Volatility of Exchange Rate Depreciation

1. **Technology Shock**: Doubling the importance of Country 3 is more than half as important as doubling the importance of Country 2 in a two-country model.
2. **Monetary Policy Shock**: Doubling the importance of Country 3 is about half as important as doubling the importance of Country 2 in a two-country model.

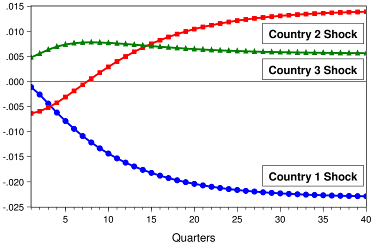


(i) Technology Shock

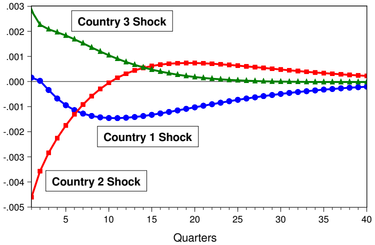


(j) Monetary Policy Shock

Environment III: Technology Shock (Std. Taylor Rules & Asymmetric Calvo Parameters, where $\alpha_1 < \alpha_3 < \alpha_2$)



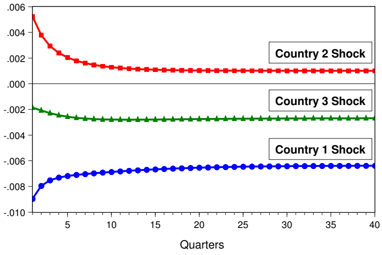
(k) Nominal Exchange Rate: $e_{1,2}$



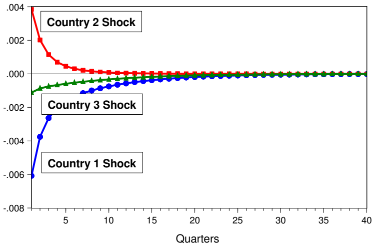
(l) Real Exchange Rate: $q_{1,2}$

r_j IRFs

Environment III: Monetary Policy Shock (Std. Taylor Rules & Asymmetric Calvo Parameters, where $\alpha_1 < \alpha_3 < \alpha_2$)



(m) Nominal Exchange Rate: $e_{1,2}$



(n) Real Exchange Rate: $q_{1,2}$

Exchange Rates and DPPP Fundamentals

Compare $\bar{R}^2$ from the predictive regressions:

1. $e_{1,2,t+k} - e_{1,2,t} = b_0 + b_1 q_{1,2,t} + \epsilon_{1,2,t+k}$
2. $e_{1,2,t+k} - e_{1,2,t} = b_0 + b_1 q_{1,2,t} + b_2 q_{2,3,t} + \epsilon_{1,2,t+k}$

Exchange Rates and DPPP Fundamentals

Table: Monte Carlo Mean $\bar{R}^2$ from DPPP Predictive Regressions

Horizon	Environment	2 Country Model $\bar{R}^2$	3 Country Model $\bar{R}^2$
1	II	0.018	0.059
1	III	0.014	0.057
1	IV	0.012	0.065
4	II	0.171	0.349
4	III	0.157	0.337
4	IV	0.140	0.345

Environment II: Managed Float & Symmetric Calvo Parameters
Environment III: Independent Policy & Asymmetric Calvo Parameters
Environment IV: Managed Float & Asymmetric Calvo Parameters

Exchange Rates and Taylor-Rule Fundamentals

Table: Monte Carlo Mean $\bar{R}^2$ from Taylor-Rule Fundamentals Regressions

Horizon	Environment	2 Country Model $\bar{R}^2$	3 Country Model $\bar{R}^2$
1	II	0.061	0.089
1	III	0.122	0.170
1	IV	0.155	0.206
4	II	0.249	0.337
4	III	0.335	0.520
4	IV	0.369	0.538

Environment II: Managed Float & Symmetric Calvo Parameters
Environment III: Independent Policy & Asymmetric Calvo Parameters
Environment IV: Managed Float & Asymmetric Calvo Parameters

Conclusion

- The exchange rate model can help explain the exchange rate disconnect puzzle.
- The three country model puts into context the factor structure of exchange rates and identification of factors with key USD exchange rates (with euro, SF and yen).

Functional Form

Household Problem

$$u\left(c_j\left(s^t\right), 1-n_j\left(s^t\right), \frac{M_j\left(s^t\right)}{P_j\left(s^t\right)}\right) = \frac{c_j\left(s^t\right)^{1-\gamma_1}-1}{1-\gamma_1} + \ldots$$

$$+ \theta_2\left(\frac{\left[1-n_j\left(s^t\right)\right]^{1-\gamma_2}-1}{1-\gamma_2}\right) + \ldots$$

$$+ \theta_3\left(\frac{\left[\frac{M_j\left(s^t\right)}{P_j\left(s^t\right)}\right]^{1-\gamma_3}-1}{1-\gamma_3}\right)$$

Household Problem Log Linearization

$$\theta_2 \left(1 - n_j\left(s^t\right)\right)^{-\gamma_2} = \left(\frac{W_j\left(s^t\right)}{P_j\left(s^t\right)}\right) c_j\left(s^t\right)^{-\gamma_1}$$

$$\theta_3 \left(\frac{M_j\left(s^t\right)}{P_j\left(s^t\right)}\right)^{-\gamma_3} = \left(\frac{i_j(s^t)}{1 + i_j(s^t)}\right) c_{j,t}(s^t)^{-\gamma_1}$$

$$Q\left(s_{t+1}|s^t\right) = \beta\pi\left(s_{t+1}|s^t\right)\left(\frac{c_j\left(s^{t+1}\right)}{c_j\left(s^t\right)}\right)^{-\gamma_1}\left(\frac{P_j\left(s^t\right)}{P_j\left(s^{t+1}\right)}\right)\frac{e_{1,j}\left(s^t\right)}{e_{1,j}\left(s^{t+1}\right)}$$

$$\frac{1}{1 + i_j(s^t)} = \beta E_t\left(\frac{c_j\left(s^{t+1}\right)}{c_j\left(s^t\right)}\right)^{-\gamma_1}\left(\frac{P_j\left(s^t\right)}{P_j\left(s^{t+1}\right)}\right)$$

$$= \sum_{s_{t+1}} Q\left(s_{t+1}|s^t\right)\left(\frac{e_{1,j}\left(s^{t+1}\right)}{e_{1,j}\left(s^t\right)}\right)$$

Log Linearization

FONCs

Labor Supply:

$$(\tilde{W}_{j,t} - \tilde{P}_{j,t}) = \gamma_1 \tilde{c}_{j,t} + \gamma_2 \left(\frac{n_j^*}{1 - n_j^*} \right) \tilde{n}_{j,t}$$

LM:

$$(\tilde{M}_{j,t} - \tilde{P}_{j,t}) = -\frac{1}{\gamma_3} \left(\frac{1}{i_j^*} - \frac{1}{1 + i_j^*} \right) \tilde{i}_{j,t} + \frac{\gamma_1}{\gamma_3} \tilde{c}_{j,t}$$

IS:

$$\tilde{c}_{j,t} = -\frac{1}{\gamma_1} (\tilde{i}_{j,t} - \tilde{\pi}_{j,t+1}) + \tilde{c}_{j,t+1}$$

Consumption Sub-Indices

Indices

$$
\begin{aligned}
c_{k,j,t} &= \left[\left(\phi_{k,j}\right)^{\frac{1}{\sigma}} \int_{\delta_1}^{\delta_2} c_{k,j,t}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} \\
&= d_{k,j} \left(\frac{P_{k,j,t}}{P_{k,t}} \right)^{-\mu} c_{k,t}
\end{aligned}
$$

where:

$$
c_{k,j,t}(\omega) = d_{k,j}\phi_{k,j} \left(\frac{p_{k,j,t}(\omega)}{P_{k,j,t}} \right)^{-\sigma} \left(\frac{P_{k,j,t}}{P_{k,t}} \right)^{-\mu} c_{k,t}
$$

$$
\phi_{k,j} = \begin{cases} \frac{1}{a_1} & j = 1 \\ \frac{1}{a_2 - a_1} & j = 2 \\ \frac{1}{1 - a_2} & j = 3 \end{cases}
$$

Price Sub-Indices

Indices

$$P_{k,j,t} = \begin{cases} \left[\left(\frac{1}{a_1}\right) \int_0^{a_1} p_{k,j,t}(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} & j = 1 \\ \left[\left(\frac{1}{a_2 - a_1}\right) \int_{a_1}^{a_2} p_{k,j,t}(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} & j = 2 \\ \left[\left(\frac{1}{1 - a_2}\right) \int_{a_2}^{1} p_{k,j,t}(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} & j = 3 \end{cases}$$

Alternatively:

$$P_{k,j,t}^{1-\sigma} = \alpha P_{k,j,t-1}^{1-\sigma} + (1-\alpha) p_{k,j,t}(\omega)^{1-\sigma}$$

Log Linearization of Price Setting Equations

FONCs

$$
\begin{aligned}
\tilde{\pi}_{k,j,t} = & \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left[(\tilde{W}_{j,t} - \tilde{P}_{j,t}) - \tilde{A}_{j,t} + d_{k,1}(\tilde{P}_{k,1,t} - \tilde{P}_{k,3,t})\right] + \\
& \ldots + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left[d_{k,2}(\tilde{P}_{k,2,t} - \tilde{P}_{k,3,t}) + (\tilde{P}_{k,3,t} - \tilde{P}_{k,j,t})\right] + \\
& \ldots + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left[\tilde{q}_{k,j,t}\right] + \beta E_t \tilde{\pi}_{k,j,t+1}
\end{aligned}
$$

Firm Problem

▸ Prices

The firm for country j optimizes along two dimensions: (i.) employment and (ii.) prices. This firm makes these decisions sequentially and will first choose employment.

$$\min_{n_{j,t}(\omega)} W_{j,t} n_{j,t}(\omega)$$

subject to:

$$c_{1,j,t}(\omega) = \Phi_{1,j,t} c_{1,t} p_{1,j,t}(\omega)^{-\sigma}$$
$$c_{2,j,t}(\omega) = \Phi_{2,j,t} c_{2,t} p_{2,j,t}(\omega)^{-\sigma}$$
$$c_{3,j,t}(\omega) = \Phi_{3,j,t} c_{3,t} p_{3,j,t}(\omega)^{-\sigma}$$
$$c_{1,j,t}(\omega) + c_{2,j,t}(\omega) + c_{3,j,t}(\omega) = A_{j,t} n_{j,t}(\omega)$$

Firm Problem

Prices

where:

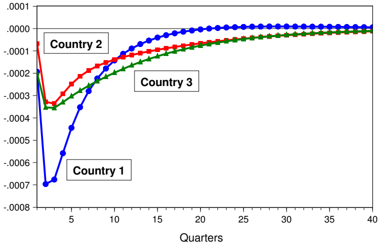
$$\Phi_{k,j,t} = d_{k,j}\phi_{k,j}\left(\frac{1}{P_{k,j,t}}\right)^{-\sigma}\left(\frac{P_{k,j,t}}{P_{k,t}}\right)^{-\mu}$$

$$\phi_{k,j} = \begin{cases} \frac{1}{a_1} & j = 1 \\ \frac{1}{a_2 - a_1} & j = 2 \\ \frac{1}{1 - a_2} & j = 3 \end{cases}$$

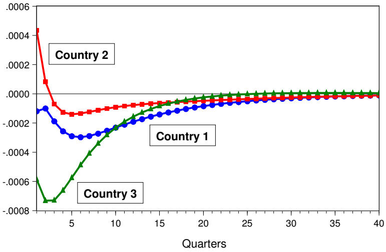
Log Linearized Flexible Price Output

$$\tilde{y}^f_{j,t} = \left[\frac{1 + \gamma_2 \left(\frac{n_j^*}{1-n_j^*} \right)}{\gamma_1 + \gamma_2 \left(\frac{n_j^*}{1-n_j^*} \right)} \right] \tilde{A}_{j,t}$$

Environment III: Technology Shock (Std. Taylor Rules & Asymmetric Calvo Parameters, where $\alpha_1 < \alpha_3 < \alpha_2$)



(o) Country 1 Shock to r_1, r_2, and r_3



(p) Country 3 Shock to r_1, r_2, and r_3

$e_{1,2}$ & $q_{1,2}$ IRFs