"Where's the Risk? The Forward Premium Bias, the Carry-Trade Premium, and Risk-Reversals in General Equilibrium"

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A Appendix (Not intended for publication)

This appendix fully develops the two-country model described in the text.

A.1 Recursive Utility Formulation

Recursive utility was introduced to macroeconomics and finance by Epstein and Zin (1989) and Weil (1989). We employ a functional form of recursive utility considered by Swanson (2016). Let c_t be the household's real consumption and ℓ_t be its labor supply. The utility function is,

$$V_t = (1-\beta)\left(\ln(c_t) - \eta\frac{\ell_t^{1+\chi}}{1+\chi}\right) - \frac{\beta}{\alpha}\ln\left[E_t\left(e^{-\alpha V_{t+1}}\right)\right], \tag{A.1}$$

where β is the subjective discount factor and η, χ, α are also parameters. The Epstein and Zin (1989) and Weil (1989) formulations separate the coefficient of relative risk aversion and the elasticity of intertemporal substitution. In (**??**), the elasticity of intertemporal substitution is 1 and risk aversion (RRA) is

$$RRA = \alpha + \left(\frac{1}{1+\frac{\eta}{\chi}}\right). \tag{A.2}$$

Also, the Frisch elasticity of labor supply is $1/\chi$.

A.2 Stationary Transformation of Utility

Productivity has a stochastic trend which causes the model to be nonstationary. Except for labor ℓ_t, the other variables in the model will inherit the stochastic trend from productivity, so the model must be transformed to induce stationarity. We do this by normalizing (dividing) the variables that grow over time, by the lagged level of total factor productivity, A_{t-1}. To transform (**??**), (i) multiply and divide c_t by A_{t-1}, where $\tilde{c}_t \equiv c_t/A_{t-1}$, (ii) subtract $\ln(A_{t-1})$ from both sides of (**??**), (iii) add and subtract $\ln(A_t)$ to V_{t+1}, and define $\tilde{V}_t \equiv$

$V_t - \ln(A_{t-1})$ and $G_t \equiv A_t/A_{t-1}$. These steps give the stationary form of utility,

$$\tilde{V}_t = \left\{(1-\beta)\left[\ln(\tilde{c}_t) - \eta\frac{\ell_t^{1+\chi}}{1+\chi}\right] - \frac{\beta}{\alpha}\ln\left[E_t e^{-\alpha\tilde{V}_{i,,t+1}}\right]\right\} + \beta\ln(G_t) \tag{A.3}$$

The **real stochastic discount factor** implied by (**??**) is $\beta M_{t,t+1}$,where

$$M_{t,t+1} = \left(\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\right)\left(\frac{e^{-\alpha\tilde{V}_{t+1}}}{E_t\left(e^{-\alpha\tilde{V}_{t+1}}\right)}\right)\left(\frac{1}{G_t}\right) \tag{A.4}$$

The nominal stochastic discount factor is $\beta N_{t,t+1} = \beta M_{t,t+1}e^{-\pi_{t+1}}$, where π_{t+1} is the inflation rate.

A.3 Households

Country $i = 1, 2$ households want to maximize

$$\tilde{V}_{i,t} = (1-\beta)\left[\ln(\tilde{c}_{i,t}) - \eta\frac{\ell_{i,t}^{1+\chi}}{1+\chi}\right] - \frac{\beta}{\alpha}\ln\left[E_t e^{-\alpha\tilde{V}_{i,t+1}}\right] + \beta\ln(G_{i,t}). \tag{A.5}$$

subject to its budget constraint which is describe below.

A.3.1 Complete markets

Denote the current state of the world by ω_t and the state history by $\omega^t = \{\omega_t, \omega_{t-1}, \ldots\}$. Households in Countries 1 and 2 have access to a full set of nominal state-contingent securities that pay one unit of Country 1's currency if the state occurs. Making the dependence on the state explicit, let $B_1(\omega^t)$ be the number of state ω^t contingent bonds held by the household. The price of a bond that pays off in state ω_{t+1} is $p_\omega(\omega_{t+1}|\omega^t)$. Country 1's household takes the payoffs from state-contingent bonds, its wages and firm profits to pay for consumption and its portfolio of state-contingent securities. Let $W_1(\omega^t)$ be the nominal wage, $P_1(\omega^t)$ be the price level and $\Pi_1(\omega^\tau)$ firm profits. Shares of Country 1 firms are not traded and are entirely owned by by Country 1 households. The household takes the flow resources of labor income, firm profits, and state-contingent bond payoffs to purchase consumption and a portfolio of state-contingent bonds. There is no physical capital in the model. Country 1's household budget constraint is,

$$c_1\left(\omega^t\right)+\sum_{\omega^{t+1}}\frac{p_\omega\left(\omega_{t+1}|\omega^t\right)B_1\left(\omega^{t+1}\right)}{P_1\left(\omega^t\right)}=\frac{W_1\left(\omega^t\right)}{P_1\left(\omega^t\right)}\ell_1\left(\omega^t\right)+\frac{\Pi_1\left(\omega^t\right)}{P_1\left(\omega^t\right)}+\frac{B_1\left(\omega^t\right)}{P_1\left(\omega^t\right)}. \tag{A.6}$$

To obtain a stationary representation for (**??**), divide both sides by $A_1\left(\omega^{t-1}\right)$,

$$\frac{c_1\left(\omega^t\right)}{A_1\left(\omega^{t-1}\right)}+\sum_{\omega^{t+1}}\frac{Q\left(\omega_{t+1}|\omega^t\right)B_1\left(\omega^{t+1}\right)}{P_1\left(\omega^t\right)A_1\left(\omega^{t-1}\right)}\frac{A_1\left(\omega^t\right)}{A_1\left(\omega^t\right)}=\frac{W_1\left(\omega^t\right)}{A_1\left(\omega^{t-1}\right)P_1\left(\omega^t\right)}\ell_1\left(\omega^t\right)+\frac{\Pi_1\left(\omega^t\right)}{A_1\left(\omega^{t-1}\right)P_1\left(\omega^t\right)}+... \tag{A.7}$$

$$...+\frac{B_1\left(\omega^t\right)}{A_1\left(\omega^{t-1}\right)P_1\left(\omega^t\right)},$$

and rewrite (**??**) by indicating those variables divided by $A_1\left(\omega^{t-1}\right)$ with a tilde,

$$\tilde{c}_1\left(\omega^t\right)+\sum_{\omega^{t+1}}\frac{p_\omega\left(\omega_{t+1}|\omega^t\right)\tilde{B}_1\left(\omega^{t+1}\right)}{P_1\left(\omega^t\right)}G_1\left(\omega^t\right)=\frac{\tilde{W}_1\left(\omega^t\right)}{P_1\left(\omega^t\right)}\ell_1\left(\omega^t\right)+\frac{\tilde{\Pi}_1\left(\omega^t\right)}{P_1\left(\omega^t\right)}+\frac{\tilde{B}_1\left(\omega^t\right)}{P_1\left(\omega^t\right)}. \tag{A.8}$$

where $G_1\left(\omega^t\right)=A_1\left(\omega^t\right)/A_1\left(\omega^{t-1}\right)$.

If $\pi\left(\omega_{t+1}|\omega^t\right)$ is the conditional probability of state ω_{t+1}, the optimality conditions for the household give the Euler equation for the state-contingent bond and the labor supply equation,

$$p_\omega\left(\omega^{t+1}|\omega^t\right)=\beta\pi\left(\omega_{t+1}|\omega^t\right)M_1\left(\omega_{t+1}|\omega^t\right)e^{-\pi_1\left(\omega^{t+1}\right)}, \tag{A.9}$$

$$\eta\tilde{c}_1\left(\omega^t\right)\ell_1\left(\omega^t\right)^\chi=\frac{\tilde{W}_1\left(\omega^t\right)}{P_1\left(\omega^t\right)}. \tag{A.10}$$

Summing over the prices of all state-contingent bonds gives the price of the nominally risk-free bond,

$$\frac{1}{1+i_1\left(\omega^t\right)}=\beta E_t\left(M_1\left(\omega_{t+1}|\omega^t\right)e^{-\pi_1\left(\omega^{t+1}\right)}\right). \tag{A.11}$$

The transformation of Country 2's budget constraint follows analogously, and gives the stationary form,

$$\tilde{c}_2\left(\omega^t\right)+\sum_{\omega^{t+1}}\frac{p_\omega\left(\omega^{t+1}|\omega^t\right)\tilde{B}_2\left(\omega^{t+1}\right)}{S_{1,2}\left(\omega^{t+1}\right)P_2\left(\omega^t\right)}G_2\left(\omega^t\right)=\frac{\tilde{W}_2\left(\omega^t\right)}{P_2\left(\omega^t\right)}\ell_2\left(\omega^t\right)+\frac{\tilde{\Pi}_2\left(\omega^t\right)}{P_2\left(\omega^t\right)}+\frac{\tilde{B}_2\left(\omega^t\right)}{S_{1,2}\left(\omega^t\right)P_2\left(\omega^t\right)} \tag{A.12}$$

where $S_{1,2}\left(\omega^t\right)$ is the nominal exchange rate (price of Country 2's currency). The optimality conditions for the foreign household gives the Euler equations for the state-contingent bond and labor supply,

$$p_{\omega}\left(\omega^{t+1}|\omega^{t}\right)=\beta\pi\left(\omega_{t+1}|\omega^{t}\right)M_{2}\left(\omega_{t+1}|\omega^{t}\right)\frac{S_{1,2}\left(\omega^{t}\right)}{S_{1,2}\left(\omega^{t+1}\right)}e^{-\pi_{2}\left(\omega^{t+1}\right)} \tag{A.13}$$

$$\eta\tilde{c}_{2}\left(\omega^{t}\right)\ell_{2}\left(\omega^{t}\right)^{\chi}=\frac{\tilde{W}_{2}\left(\omega^{t}\right)}{P_{2}\left(\omega^{t}\right)} \tag{A.14}$$

Let $Q_{1,2}\left(\omega^{t}\right)=\frac{S_{1,2}\left(\omega^{t}\right)P_{2}\left(\omega^{t}\right)}{P_{1}\left(\omega^{t}\right)}$ be the real exchange rate. Equating (**??**) to (**??**) gives the real exchange rate depreciation,

$$\frac{Q_{1,2}\left(\omega^{t+1}\right)}{Q_{1,2}\left(\omega^{t}\right)}=\frac{M_{2}\left(\omega_{t+1}|\omega^{t}\right)}{M_{1}\left(\omega_{t+1}|\omega^{t}\right)}, \tag{A.15}$$

and the nominal depreciation,

$$\frac{S_{1,2}\left(\omega^{t+1}\right)}{S_{1,2}\left(\omega^{t}\right)}=\frac{M_{2}\left(\omega_{t+1}|\omega^{t}\right)e^{-\pi_{2}\left(\omega^{t+1}\right)}}{M_{1}\left(\omega_{t+1}|\omega^{t}\right)e^{-\pi_{1}\left(\omega^{t+1}\right)}}.$$

A.3.2 Incomplete markets

Under incomplete markets, we can suppress the functional dependence on the state notation. Each country issues a non-state contingent nominal bond that is internationally traded. The country 1 bond is issued at a price of 1 and pays a gross return $1+i_{1,t}$ units of country 1 currency next period. Similarly, country 2 issues a bond, priced at 1 unit of currency 2 which pays $1+i_{2,t}$ units of currency 2 next period. Let $B_{i,j,t}$ be currency j bonds issued by j and held by i. This way of formulating incomplete markets renders bond holdings to be non-stationary even when productivity shocks are stationary. Following Schmidtt-Grohe and Uribe (2003), we impose a small fee on net foreign bond positions to induce stationarity in these positions. Let τ be the fee paid by country 1 households for holding country 2 bonds. The real cost of holding 'foreign' bonds valued at $S_{1,2,t}B_{1,2,t}$ is $\frac{\tau}{2A_{1,t-1}}\left(\frac{S_{1,2,t}B_{1,2,t}}{P_{1,t}}\right)^{2}$. As long as $\tau>0$, the Country 1 household will want $B_{1,2}=0$ in the steady state. The budget constraint facing the country 1 household is,

$$c_{1,t}+\frac{B_{1,1,t}}{P_{1,t}}+\frac{S_{1,2,t}B_{1,2,t}}{P_{1,t}}+\frac{\tau}{2A_{1,t-1}}\left(\frac{S_{1,2,t}B_{1,2,t}}{P_{1,t}}\right)^{2}=\frac{W_{1,t}\ell_{1,t}}{P_{1,t}}+\frac{\Pi_{1,t}}{P_{1,t}}+\frac{\left(1+i_{1,t-1}\right)B_{1,1,t-1}}{P_{1,t}}+\frac{\left(1+i_{2,t-1}\right)S_{1,2,t}B_{1,2,t-1}}{P_{1,t}} \tag{A.16}$$

The stationary transformation of (??) is obtained by dividing both sides by $A_{1,t-1}$, then re-arranging to get

$$\tilde{c}_{1,t}+\frac{\tilde{B}_{1,1,t}}{P_{1,t}}+\frac{Q_{1,2,t}\tilde{B}_{1,2,t}}{P_{2,t}}+\frac{\tau}{2}\left(\frac{Q_{1,2,t}\tilde{B}_{1,2,t}}{P_{2,t}}\right)^2=\frac{\tilde{W}_{1,t}\ell_{1,t}}{P_{1,t}}+\frac{\tilde{\Pi}_{1,t}}{P_{1,t}}+\frac{(1+i_{1,t-1})\,\tilde{B}_{1,1,t-1}}{P_{1,t}G_{1,t-1}}+\frac{(1+i_{2,t-1})\,Q_{1,2,t}\tilde{B}_{1,2,t-1}}{P_{2,t}G_{1,t-1}}, \tag{A.17}$$

where the tilde variables are divided by the lagged country 1 productivity (e.g., $\tilde{B}_{1,2,t-1}\equiv B_{1,2,t-1}/A_{1,t-2}$). The Euler equations associated with optimal bond holdings for country 1 are,

$$\frac{1}{1+i_{1,t}}=\beta E_t\left(M_{1,t,t+1}e^{-\pi_{1,t+1}}\right), \tag{A.18}$$

$$\left(\frac{1+\tau\left(Q_{1,2,t}\tilde{B}_{1,2,t}/P_{2,t}\right)}{1+i_{2,t}}\right)=\beta E_t\left(M_{1,t,t+1}e^{\Delta\log(Q_{1,2,t+1})}e^{-\pi_{2,t+1}}\right). \tag{A.19}$$

Similarly, country 2 begins with the budget constraint

$$c_{2,t}+\frac{B_{2,2,t}}{P_{2,t}}+\frac{B_{2,1,t}}{S_{1,2,t}P_{2,t}}+\frac{\tau}{2A_{2,t-1}}\left(\frac{B_{2,1,t}}{S_{1,2,t}P_{2,t}}\right)^2=\frac{W_{2,t}\ell_{2,t}}{P_{2,t}}+\frac{\Pi_{2,t}}{P_{2,t}}+\frac{(1+i_{2,t-1})\,B_{2,2,t-1}}{P_{2,t}}+\frac{(1+i_{1,t-1})\,B_{2,1,t-1}}{S_{1,2,t}P_{2,t}}. \tag{A.20}$$

Divide both sides of (??) by $A_{2,t-1}$ and re-arrange to get,

$$\tilde{c}_{2,t}+\frac{\tilde{B}_{2,2,t}}{P_{2,t}}+\frac{\tilde{B}_{2,1,t}}{Q_{1,2,t}P_{1,t}}+\frac{\tau}{2}\left(\frac{\tilde{B}_{2,1,t}}{Q_{1,2,t}P_{1,t}}\right)^2=\frac{\tilde{W}_{2,t}\ell_{2,t}}{P_{2,t}}+\frac{\tilde{\Pi}_{2,t}}{P_{2,t}}+\frac{(1+i_{2,t-1})\,\tilde{B}_{2,2,t-1}}{P_{2,t}G_{2,t-1}}+\frac{(1+i_{1,t-1})\,\tilde{B}_{2,1,t-1}}{Q_{1,2,t}P_{1,t}G_{2,t-1}}. \tag{A.21}$$

The Euler equations associated with optimal bond holdings for country 2 are,

$$\frac{1}{1+i_{2,t}}=\beta E_t\left(M_{2,t,t+1}e^{-\pi_{2,t+1}}\right), \tag{A.22}$$

$$\frac{1+\tau\left(\tilde{B}_{2,1,t}/\left(Q_{1,2,t}P_{1,t}\right)\right)}{1+i_{1,t}}=\beta E_t\left(M_{2,t,t+1}e^{\Delta\log(Q_{1,2,t+1})}e^{-\pi_{1,t+1}}\right). \tag{A.23}$$

The optimality conditions for the labor-leisure choice is unaffected by the change to incomplete markets and continue to be described by (??) and (??).

A.4 Demand functions

In each country, there are a continuum of firms, indexed by $f \in [0,1]$ each producing a differentiated product. Our convention on subscripts is $c_{i,j,t}$ is made in country j and consumed in country i. Let σ be the elasticity of substitution between varieties f. In country 1, consumption of 'home' produced goods and of imports are,

$$\tilde{c}_{1,1,t} = \left[\int_0^1 \tilde{c}_{1,1,t}(f)^{\frac{\sigma-1}{\sigma}} df\right]^{\frac{\sigma}{\sigma-1}}, \tag{A.24}$$

$$\tilde{c}_{1,2,t} = \left[\int_0^1 \tilde{c}_{1,2,t}(f)^{\frac{\sigma-1}{\sigma}} df\right]^{\frac{\sigma}{\sigma-1}}, \tag{A.25}$$

and the price indices for the $\tilde{c}_{1,1,t}$ and $\tilde{c}_{1,2,t}$ bundles associated with (**??**) and (**??**) are,

$$P_{1,1,t} = \left[\int_0^1 p_{1,1,t}(f)^{1-\sigma} df\right]^{\frac{1}{1-\sigma}}, \tag{A.26a}$$

$$P_{1,2,t} = \left[\int_0^1 p_{1,2,t}(f)^{1-\sigma} df\right]^{\frac{1}{1-\sigma}}. \tag{A.26b}$$

Aggregate consumption in country 1 is the constant elasticity of substitution (CES) index,

$$\tilde{c}_{1,t} = \left[d^{\frac{1}{\mu}} \tilde{c}_{1,1,t}^{\frac{\mu-1}{\mu}} + (1-d)^{\frac{1}{\mu}} \tilde{c}_{1,2,t}^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}}, \tag{A.27}$$

of goods produced in country 1, $\tilde{c}_{1,1,t} \equiv c_{1,1,t}/A_{1,t-1}$, and imports from country 2, $\tilde{c}_{1,2,t} \equiv c_{1,2,t}/A_{1,t-1}$. The elasticity of substitution between 'home' and 'foreign' goods is μ, and home-bias in consumption is represented by $d > 1/2$. The aggregate price level associated with (**??**) is

$$P_{1,t} = \left[dP_{1,1,t}^{1-\mu} + (1-d) P_{1,2,t}^{1-\mu}\right]^{\frac{1}{1-\mu}}. \tag{A.28}$$

The countries are symmetrical. Bundles of country 2 'domestic' consumption and its imports are,

$$\tilde{c}_{2,2,t} = \left[\int_0^1 \tilde{c}_{2,2,t}(f)^{\frac{\sigma-1}{\sigma}} df\right]^{\frac{\sigma}{\sigma-1}}, \tag{A.29}$$

$$\tilde{c}_{2,1,t} = \left[\int_0^1 \tilde{c}_{2,1,t}(f)^{\frac{\sigma-1}{\sigma}} df\right]^{\frac{\sigma}{\sigma-1}}, \tag{A.30}$$

where $\tilde{c}_{2,2,t} \equiv c_{2,2,t}/A_{2,t-1}$ and $\tilde{c}_{2,1,t} \equiv c_{2,1,t}/A_{2,t-1}$. The associated price indices are

$$P_{2,2,t} = \left[\int_0^1 p_{2,2,t}(f)^{1-\sigma} df\right]^{\frac{1}{1-\sigma}}, \tag{A.31}$$

$$P_{2,1,t} = \left[\int_0^1 p_{2,1,t}(f)^{1-\sigma} df\right]^{\frac{1}{1-\sigma}}. \tag{A.32}$$

Aggregate country 2 consumption and price level are,

$$\tilde{c}_{2,t} = \left[d^{\frac{1}{\mu}} \tilde{c}_{2,2,t}^{\frac{\mu-1}{\mu}} + (1-d)^{\frac{1}{\mu}} \tilde{c}_{2,1,t}^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}}, \tag{A.33}$$

$$P_{2,t} = \left[dP_{2,2,t}^{1-\mu} + (1-d) P_{2,1,t}^{1-\mu}\right]^{\frac{1}{1-\mu}}. \tag{A.34}$$

A.5 Intermediate Goods Firm Problem

In country 1, output of firm $f \in [0,1]$ is demand determined. Firm f can distinguish between domestic and foreign shoppers and is able to charge them different prices. Country 1 export prices are set in country 2's currency (local currency pricing, or LCP).

Labor is the only input of production. The production function for firm f is

$$y_{1,t}(f) = A_{1,t}\ell_{1,t}(f),$$

where $A_{1,t}$ is the level of productivity. Normalize by dividing both sides by $A_{1,t-1}$ to get

$$\tilde{y}_{1,t}(f) = G_{1,t}\ell_{1,t}(f). \tag{A.35}$$

Total costs are

$$\frac{\tilde{W}_{1,t}}{P_{1,t}}\ell_{1,t}(f),$$

where $W_{1,t}$ is the nominal wage and $\tilde{W}_{1,t} = W_{1,t}/A_{1,t-1}$. Output is demand determined, $\tilde{y}_{1,t}(f) = \tilde{c}_{1,1,t}(f) + \tilde{c}_{2,1,t}(f)$. The firm can always adjust its labor input. A Lagrangian for the firm is,

$$L = -\frac{\tilde{W}_{1,t}}{P_{1,t}}\ell_{1,t}(f) + \varphi_{1,t}\left(G_{1,t}\ell_{1,t}(f) - \tilde{c}_{1,1,t}(f) - \tilde{c}_{2,1,t}(f)\right). \tag{A.36}$$

The equation implied by choosing labor is,

$$\frac{\tilde{W}_{1,t}}{P_{1,t}} = \varphi_{1,t} G_{1,t}. \tag{A.37}$$

To determine optimal price setting, note that firm f faces these (normalized) domestic and foreign demands for its good

$$\tilde{c}_{1,1,t}(f) = d \left(\frac{p_{1,1,t}(f)}{P_{1,1,t}} \right)^{-\sigma} \left(\frac{P_{1,1,t}}{P_{1,t}} \right)^{-\mu} \tilde{c}_{1,t}, \tag{A.38}$$

$$\tilde{c}_{2,1,t}(f) = (1-d) \left(\frac{p_{2,1,t}(f)}{P_{2,1,t}} \right)^{-\sigma} \left(\frac{P_{2,1,t}}{P_{2,t}} \right)^{-\mu} \tilde{c}_{2,t} \Omega_{t-1}, \tag{A.39}$$

where $\Omega_t \equiv A_{2,t}/A_{1,t}$ is the ratio of country 2 to country 1 productivity. Since output is demand determined, we have

$$\tilde{y}_{1,t}(f) = G_{1,t} \ell_{1,t}(f) = \tilde{c}_{1,1,t}(f) + \tilde{c}_{2,1,t}(f), \tag{A.40}$$

from which it follows that firm f employment is,

$$\ell_{1,t}(f) = \frac{\tilde{c}_{1,1,t}(f) + \tilde{c}_{2,1,t}}{G_{1,t}}. \tag{A.41}$$

Let $\tilde{w}_{1,t} = \tilde{W}_{1,t}/P_{1,t}$ be the real wage. Current profit is,

$$\tilde{\Pi}_{1,t}(f) = \frac{p_{1,1,t}(f)}{P_{1,t}} \tilde{c}_{1,1,t}(f) + \frac{Q_{1,2,t} p_{2,1,t}(f)}{P_{2,t}} \tilde{c}_{2,1,t}(f) - \tilde{w}_{1,t} \ell_{1,t}(f). \tag{A.42}$$

Prices are sticky in the sense of Calvo (1983). Each period, the probability that the firm is allowed to change prices is $1 - \alpha_c$. As long as the contract is in effect, price automatically adjusts by (the continuously compounded) steady state inflation, $\bar{\pi}$. In periods when the firm does reset prices, it adjusts both the price for domestic and foreign markets, $p_{1,1,t}(f)$ and $p_{2,1,t}(f)$, to maximize

$$E_t \sum_{s=0}^{\infty} (\alpha_c \beta)^s M_{1,t,t+s} \left[\frac{p_{1,1,t}(f) e^{s\bar{\pi}_1}}{P_{1,t+s}} \tilde{c}_{1,1,t+s}(f) + \frac{Q_{1,2,t+s} p_{2,1,t}(f) e^{s\bar{\pi}_2}}{P_{2,t+s}} \tilde{c}_{2,1,t+s}(f) - \tilde{w}_{1,t+s} \ell_{1,t+s}(f) \right], \tag{A.43}$$

subject to the demand functions (**??**) and (**??**) and the labor demand function (**??**). When the firm chooses $p_{1,1,t}(f)$, the first-order condition can be re-arranged as,

$$p^*_{1,1,t} = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{a_{1,1,t}}{b_{1,1,t}}\right), \tag{A.44}$$

where

$$a_{1,1t} = E_t \sum_{s=0}^{\infty} (\alpha_c \beta)^s e^{-s\bar{\pi}_1\sigma} M_{1,t,t+s}\left(\frac{\tilde{w}_{1,t+s}}{G_{1,t+s}}\right) dP^{\sigma}_{1,1,t+s}\left(\frac{P_{1,1,t+s}}{P_{1,t+s}}\right)^{-\mu} \tilde{c}_{1,t+s}, \tag{A.45}$$

$$b_{1,1,t} = E_t \sum_{s=0}^{\infty} (\alpha_c \beta)^s e^{s\bar{\pi}_1(1-\sigma)} M_{1,t,t+s}\left(\frac{1}{P_{1,t+s}}\right) dP^{\sigma}_{1,1,t+s}\left(\frac{P_{1,1,t+s}}{P_{1,t+s}}\right)^{-\mu} \tilde{c}_{1,t+s}. \tag{A.46}$$

$a_{1,1,t}$ and $b_{1,1,t}$ can be represented recursively as,

$$a_{1,1,t} = \left(\frac{\tilde{w}_{1,t}}{G_{1,t}}\right) dP^{\sigma}_{1,1,t}\left(\frac{P_{1,1,t}}{P_{1,t}}\right)^{-\mu} \tilde{c}_{1,t} + (\alpha_c\beta)\, e^{-\bar{\pi}_1\sigma} E_t\left(M_{1,t,t+1} a_{1,1,t+1}\right), \tag{A.47}$$

$$b_{1,1,t} = \left(\frac{1}{P_{1,t}}\right) dP^{\sigma}_{1,1,t}\left(\frac{P_{1,1,t}}{P_{1,t}}\right)^{-\mu} \tilde{c}_{1,t} + (\alpha_c\beta)\, e^{\bar{\pi}_1(1-\sigma)} E_t\left(M_{1,t,t+1} b_{1,1,t+1}\right). \tag{A.48}$$

Now multiply both sides of (**??**) by $P^{\mu-\sigma}_{1,1,t} P^{-\mu}_{1,t}$ and multiply both sides of (**??**) by $P^{\mu-\sigma}_{1,1,t} P^{1-\mu}_{1,t}$. This gives

$$a_{1,1,t} P^{\mu-\sigma}_{1,1,t} P^{-\mu}_{1,t} = \left(\frac{\tilde{w}_{1,t}}{G_{1,t}}\right) \tilde{c}_{1,t} + \alpha_c \beta e^{-\bar{\pi}_1\sigma} E_t\left(M_{1,t,t+1}\left(\frac{P^{\mu-\sigma}_{1,1,t} P^{-\mu}_{1,t}}{P^{\mu-\sigma}_{1,1,t+1} P^{-\mu}_{1,t+1}}\right) a_{1,1,t+1} P^{\mu-\sigma}_{1,1,t+1} P^{-\mu}_{1,t+1}\right), \tag{A.49}$$

$$b_{1,1,t} P^{\mu-\sigma}_{1,1,t} P^{1-\mu}_{1,t} = \tilde{c}_{1,t} + \alpha_c \beta e^{\bar{\pi}_1(1-\sigma)} E_t\left(M_{1,t+1}\left(\frac{P^{\mu-\sigma}_{1,1,t} P^{1-\mu}_{1,t}}{P^{\mu-\sigma}_{1,1,t+1} P^{1-\mu}_{1,t+1}}\right) b_{1,1,t+1} P^{\mu-\sigma}_{1,1,t+1} P^{1-\mu}_{1,t+1}\right). \tag{A.50}$$

The d has canceled out between numerator and denominator. Because of the separable form of the recursive utility function, we can simplify further by dividing both sides of (**??**) and (**??**) by $\tilde{c}_{1,t}$. The price-reset can be rewritten as

$$\left(\frac{p^*_{1,1,t}}{P_{1,1,t}}\right)\left(\frac{P_{1,1,t}}{P_{1,t}}\right) = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{z^n_{1,1,t}}{z^d_{1,1,t}}\right), \tag{A.51}$$

where

$$z^n_{1,1,t} = \frac{\tilde{w}_{1,t}}{G_{1,t}} + \alpha_c \beta e^{-\bar{\pi}_1 \sigma} E_t \left(\Lambda_{1,t,t+1} \left(\frac{P_{1,1,t}}{P_{1,1,t+1}} \right)^{\mu-\sigma} \left(\frac{P_{1,t}}{P_{1,t+1}} \right)^{-\mu} z^n_{1,1,t+1} \right), \tag{A.52}$$

$$z^d_{1,1,t} = 1 + \alpha_c \beta e^{\bar{\pi}_1 (1-\sigma)} E_t \left(\Lambda_{1,t,t+1} \left(\frac{P_{1,1,t}}{P_{1,1,t+1}} \right)^{\mu-\sigma} \left(\frac{P_{1,t}}{P_{1,t+1}} \right)^{1-\mu} z^d_{1,1,t+1} \right), \tag{A.53}$$

and

$$\Lambda_{1,t,t+1} \equiv \left(\frac{e^{-\alpha \tilde{V}_{1,t+1}}}{E_t \left(e^{-\alpha \tilde{V}_{1,t+1}} \right)} \right) \left(\frac{1}{G_{1,t}} \right). \tag{A.54}$$

Re-arrangement of the first-order condition for choosing $p_{2,1,t}(f)$ gives

$$p^*_{2,1,t} = \left(\frac{\sigma}{\sigma - 1} \right) \frac{E_t \sum_{s=0}^{\infty} (\alpha_c \beta)^s e^{-s\bar{\pi}_1 \sigma} M_{1,t,t+s} \left(\frac{\tilde{w}_{1,t+s}}{G_{1,t+s}} \right) (1-d) P^{\sigma}_{2,1,t+s} \left(\frac{P_{2,1,t+s}}{P_{2,t+s}} \right)^{-\mu} \tilde{c}_{2,t+s} \Omega_{t-1+s}}{E_t \sum_{s=0}^{\infty} (\alpha_c \beta)^s e^{s\bar{\pi}_1 (1-\sigma)} M_{1,t,t+s} \frac{Q_{1,2,t+s}}{P_{2,t+s}} (1-d) P^{\sigma}_{2,1,t+s} \left(\frac{P_{2,1,t+s}}{P_{2,t+s}} \right)^{-\mu} \tilde{c}_{2,t+s} \Omega_{t-1+s}} \tag{A.55}$$

$$= \left(\frac{\sigma}{\sigma - 1} \right) \left(\frac{a_{2,1,t}}{b_{2,1,t}} \right) \tag{A.56}$$

where

$$a_{2,1,t} = E_t \sum_{s=0}^{\infty} (\alpha_c \beta)^s e^{-s\bar{\pi}_1 \sigma} M_{1,t,t+s} \left(\frac{\tilde{w}_{1,t+s}}{G_{1,t+s}} \right) (1-d) P^{\sigma}_{2,1,t+s} \left(\frac{P_{2,1,t+s}}{P_{2,t+s}} \right)^{-\mu} \tilde{c}_{2,t+s} \Omega_{t-1+s}, \tag{A.57}$$

$$b_{2,1,t} = E_t \sum_{s=0}^{\infty} (\alpha_c \beta)^s e^{s\bar{\pi}_1 (1-\sigma)} M_{1,t,t+s} \frac{Q_{1,2,t+s}}{P_{2,t+s}} (1-d) P^{\sigma}_{2,1,t+s} \left(\frac{P_{2,1,t+s}}{P_{2,t+s}} \right)^{-\mu} \tilde{c}_{2,t+s} \Omega_{t-1+s}. \tag{A.58}$$

Noting that $(1-d)$ will cancel, multiply both sides of (**??**) by $P^{\mu-\sigma}_{2,1,t} P^{-\mu}_{2,t}$ and multiply both sides of (**??**) by $P^{\mu-\sigma}_{2,1,t} P^{1-\mu}_{2,t}$,where

$$a_{2,1,t} P^{\mu-\sigma}_{2,1,t} P^{-\mu}_{2,t} = \left(\frac{\tilde{w}_{1,t}}{G_{1,t}} \right) \tilde{c}_{2,t} \Omega_{t-1} + \alpha_c \beta e^{-\bar{\pi}_1 \sigma} E_t \left(M_{1,t,t+1} \left(\frac{P^{\mu-\sigma}_{2,1,t} P^{-\mu}_{2,t}}{P^{\mu-\sigma}_{2,1,t+1} P^{-\mu}_{2,t+1}} \right) a_{2,1,t+1} P^{\mu-\sigma}_{2,1,t+1} P^{-\mu}_{2,t+1} \right),$$

$$b_{2,1,t} P^{\mu-\sigma}_{2,1,t} P^{1-\mu}_{2,t} = Q_{1,2,t} \tilde{c}_{2,t} \Omega_{t-1} + \alpha_c \beta e^{\bar{\pi}_1 (1-\sigma)} E_t \left(M_{1,t+1} \left(\frac{P^{\mu-\sigma}_{2,1,t} P^{1-\mu}_{2,t}}{P^{\mu-\sigma}_{2,1,t+1} P^{1-\mu}_{2,t+1}} \right) b_{2,1,t+1} P^{\mu-\sigma}_{2,1,t+1} P^{1-\mu}_{2,t+1} \right).$$

Now divide both equations by $\tilde{c}_{2,t}$. This gives,

$$\left(\frac{p^*_{2,1,t}}{P_{2,1,t}}\right)\left(\frac{P_{2,1,t}}{P_{2,t}}\right)=\left(\frac{\sigma}{\sigma-1}\right)\left(\frac{z^n_{2,1,t}}{z^d_{2,1,t}}\right), \tag{A.59}$$

where

$$z^n_{2,1,t}=\left(\frac{\tilde{w}_{1,t}}{G_{1,t}}\right)\Omega_{t-1}+\alpha_c\beta e^{-\bar{\pi}_1\sigma}E_t\left(\Lambda_{1,t,t+1}\left(\frac{P_{2,1,t}}{P_{2,1,t+1}}\right)^{\mu-\sigma}\left(\frac{P_{2,t}}{P_{2,t+1}}\right)^{-\mu}z^n_{2,1,t+1}\right), \tag{A.60}$$

$$z^d_{2,1,t}=Q_{1,2,t}\Omega_{t-1}+\alpha_{2,1}\beta e^{\bar{\pi}_1(1-\sigma)}E_t\left(\Lambda_{1,t,t+1}\left(\frac{P_{2,1,t}}{P_{2,1,t+1}}\right)^{\mu-\sigma}\left(\frac{P_{2,t}}{P_{2,t+1}}\right)^{1-\mu}z^d_{2,1,t+1}\right). \tag{A.61}$$

Firm $f\in[0,1]$ in country 2 faces an analogous and symmetrical environment. It has the same Calvo probability for price-reset opportunities, $1-\alpha_c$. The production functions for its domestic and foreign markets are,

$$y_{2,t}(f)=A_{2,t}\ell_{2,t}(f).$$

Normalize by dividing both sides by $A_{2,t-1}$ to get

$$\tilde{y}_{2,t}(f)=G_{2,t}\ell_{2,t}(f). \tag{A.62}$$

Total costs are

$$\frac{\tilde{W}_{2,t}}{P_{2,t}}\ell_{2,t}(f).$$

Let $\tilde{w}_{2,t}=\tilde{W}_{2,t}/P_{2,t}$ be the real wage. The firm can always adjust its labor input. A Lagrangian for the firm is,

$$L=-\tilde{w}_{2,t}\ell_{2,t}(f)+\varphi_{2,t}\left(G_{2,t}\ell_{2,t}(f)-\tilde{c}_{2,2,t}(f)-\tilde{c}_{1,2,,t}(f)\right) \tag{A.63}$$

where $\tilde{c}_{1,2,t}(f)=c_{1,2,t}(f)/A_{2,t-1}$. The equation implied by choosing labor input yield the Euler equation,

$$\tilde{w}_{2,t}=\varphi_{2,t}G_{2,t}. \tag{A.64}$$

To determine optimal price setting, note that firm f faces these (normalized) domestic and foreign demands for its good

$$\tilde{c}_{2,2,t}(f) = d\left(\frac{p_{2,2,t}\,(f)}{P_{2,2,t}}\right)^{-\sigma}\left(\frac{P_{2,2,t}}{P_{2,t}}\right)^{-\mu}\tilde{c}_{2,t}, \tag{A.65}$$

$$\tilde{c}_{1,2,,t}(f) = (1-d)\left(\frac{p_{1,2,t}(f)}{P_{1,2,t}}\right)^{-\sigma}\left(\frac{P_{1,2,t}}{P_{1,t}}\right)^{-\mu}\tilde{c}_{1,t}\left(\frac{1}{\Omega_{t-1}}\right). \tag{A.66}$$

Since output is demand determined, it follows that

$$\tilde{y}_{2,t}\,(f) = G_{2,t}\ell_{2,t}\,(f) = \tilde{c}_{2,2,t}\,(f) + \tilde{c}_{1,2,t}\,(f). \tag{A.67}$$

Firm $f \in [0,1]$ in country 2 firm wants to maximize

$$E_t\sum_{s=0}^{\infty}(\alpha_c\beta)^s M_{2,t,t+s}\left[\frac{p^*_{2,2,t}(f)e^{-s\bar{\pi}_2}}{P_{2,t+s}}\tilde{c}_{2,2,t+s}\,(f) + \frac{p_{1,2,t}(f)e^{-s\bar{\pi}_2}}{Q_{1,2,t+s}P_{1,t+s}}\tilde{c}_{1,2,t+s}(f) - \tilde{w}_{2,t+s}\left(\frac{\tilde{c}_{2,2,t+s}\,(f)+\tilde{c}_{1,2,t}\,(f)}{G_{2,t+s}}\right)\right]. \tag{A.68}$$

By inspection, the choice for $p_{2,2,t}\,(f)$ is analogous and symmetrical to the choice for $p_{1,1,t}\,(f)$. The first-order condition can be re-arranged to give

$$\left(\frac{p^*_{2,2,t}}{P_{2,2,t}}\right)\left(\frac{P_{2,2,t}}{P_{2,t}}\right) = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{z^n_{2,2t}}{z^d_{2,2t}}\right), \tag{A.69}$$

where

$$z^n_{2,2,t} = \frac{\tilde{w}_{2,t}}{G_{2,t}} + \alpha_c\beta e^{-\bar{\pi}_2\sigma}E_t\left(\Lambda_{2,t,t+1}\left(\frac{P_{2,2,t}}{P_{2,2,t+1}}\right)^{\mu-\sigma}\left(\frac{P_{2,t}}{P_{2,t+1}}\right)^{-\mu}z^n_{2,2,t+1}\right), \tag{A.70}$$

$$z^d_{2,2,t} = 1 + \alpha_c\beta e^{\bar{\pi}_2(1-\sigma)}E_t\left(\Lambda_{2,t,t+1}\left(\frac{P_{2,2,t}}{P_{2,2,t+1}}\right)^{\mu-\sigma}\left(\frac{P_{2,t}}{P_{2,t+1}}\right)^{1-\mu}z^d_{2,2,t+1}\right). \tag{A.71}$$

Re-arrangement of the first-order conditions for choosing $p_{1,2,t}\,(f)$ gives,

$$p^*_{1,2,t} = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{a_{1,2,t}}{b_{1,2,t}}\right), \tag{A.72}$$

where

$$a_{1,2,t} = E_t \sum_{s=0}^{\infty} (\alpha_c \beta)^s e^{-s\bar{\pi}_2\sigma} M_{2,t,t+s} \left(\frac{\tilde{w}_{2,t+s}}{G_{2,t+s}} \right) (1-d)\, P_{1,2,t+s}^{\sigma} \left(\frac{P_{1,2,t+s}}{P_{1,t+s}} \right)^{-\mu} \tilde{c}_{1,t+s} \left(\frac{1}{\Omega_{t-1+s}} \right),$$

$$b_{1,2,t} = E_t \sum_{s=0}^{\infty} (\alpha_c \beta)^s e^{s\bar{\pi}_2(1-\sigma)} M_{2,t,t+s} \frac{1}{Q_{1,2,t+s} P_{1,t+s}} (1-d)\, P_{1,2,t+s}^{\sigma} \left(\frac{P_{1,2,t+s}}{P_{1,t+s}} \right)^{-\mu} \tilde{c}_{1,t+s} \left(\frac{1}{\Omega_{t-1+s}} \right),$$

or in terms of the recursive form,

$$a_{1,2,t} P_{1,2,t}^{\mu-\sigma} P_{1,t}^{-\mu} = \left(\frac{\tilde{w}_{2,t}}{G_{2,t}} \right) \tilde{c}_{1,t} \left(\frac{1}{\Omega_{t-1}} \right) + \alpha_c \beta E_t e^{-\bar{\pi}_2\sigma} \left(M_{2,t,t+1} \left(\frac{P_{1,2,t}^{\mu-\sigma} P_{1,t}^{-\mu}}{P_{1,2,t+1}^{\mu-\sigma} P_{1,t+1}^{-\mu}} \right) a_{1,2,t+1} P_{1,2,t+1}^{\mu-\sigma} P_{1,t+1}^{-\mu} \right), \tag{A.73}$$

$$b_{1,2,t} P_{1,2,t}^{\mu-\sigma} P_{1,t}^{1-\mu} = \left(\frac{1}{Q_{1,2,t}} \right) \tilde{c}_{1,t} \left(\frac{1}{\Omega_{t-1}} \right) + \alpha_c \beta e^{\bar{\pi}_2(1-\sigma)} E_t \left(M_{2,t,t+1} \left(\frac{P_{1,2,t}^{\mu-\sigma} P_{1,t}^{1-\mu}}{P_{1,2,t+1}^{\mu-\sigma} P_{1,t+1}^{1-\mu}} \right) b_{1,2,t+1} P_{1,2,t+1}^{\mu-\sigma} P_{1,t+1}^{1-\mu} \right). \tag{A.74}$$

Dividing both sides of (**??**) and (**??**) by $\tilde{c}_{1,t}$, the optimal reset price is

$$\left(\frac{p_{1,2,t}^*}{P_{1,2,t}} \right) \left(\frac{P_{1,2,t}}{P_{1,t}} \right) = \left(\frac{\sigma}{\sigma-1} \right) \left(\frac{z_{1,2,t}^n}{z_{1,2,t}^d} \right), \tag{A.75}$$

$$z_{1,2,t}^n = \left(\frac{\tilde{w}_{2,t}}{G_{2,t}} \right) \left(\frac{1}{\Omega_{t-1}} \right) + \alpha_c \beta E_t e^{-\bar{\pi}_2\sigma} \left(\Lambda_{2,t,t+1} \left(\frac{P_{1,2,t}}{P_{1,2,t+1}} \right)^{\mu-\sigma} \left(\frac{P_{1,t}}{P_{1,t+1}} \right)^{-\mu} z_{1,2,t+1}^n \right), \tag{A.76}$$

$$z_{1,2,t}^d = \left(\frac{1}{Q_{1,2,t}} \right) \left(\frac{1}{\Omega_{t-1}} \right) + \alpha_c \beta e^{\bar{\pi}_2(1-\sigma)} E_t \left(\Lambda_{2,t,t+1} \left(\frac{P_{1,2,t}}{P_{1,2,t+1}} \right)^{\mu-\sigma} \left(\frac{P_{1,t}}{P_{1,t+1}} \right)^{1-\mu} z_{1,2,t+1}^d \right). \tag{A.77}$$

A.6 Equilibrium conditions

Equating country 1's firm $f's$ supply to its demand gives,

$$G_{1,t} \ell_{1,t}(f) = d \left(\frac{p_{1,1,t}(f)}{P_{1,1.t}} \right)^{-\sigma} \left(\frac{P_{1,1,t}}{P_{1,t}} \right)^{-\mu} \tilde{c}_{1,t} + (1-d) \left(\frac{p_{2,1,t}(f)}{P_{2,1,t}} \right)^{-\sigma} \left(\frac{P_{2,1,t}}{P_{2,t}} \right)^{-\mu} \tilde{c}_{2,t} \Omega_{t-1}, \tag{A.78}$$

and integrating (**??**) gives,

$$G_{1,t}\ell_{1,t} = d\left(\frac{P_{1,1,t}}{P_{1,t}}\right)^{-\mu}\tilde{c}_{1,t}v^p_{1,1,t} + (1-d)\left(\frac{P_{2,1,t}}{P_{2,t}}\right)^{-\mu}\tilde{c}_{2,t}\Omega_{t-1}v^p_{2,1,t}, \tag{A.79}$$

where $\ell_{1,t} = \int_0^1 \ell_{1,t}(f)\,df$ is total employment at firm f, $v^p_{1,1,t} \equiv \int_0^1 \left(\frac{p_{1,1,t}(f)}{P_{1,1,t}}\right)^{-\sigma} df$ is a measure of price dispersion for domestic goods in the domestic market and $v^p_{2,1,t} = \int_0^1 \left(\frac{p_{2,1,t}(f)}{P_{2,1,t}}\right) df$ is import price dispersion in country 2. A fraction α_c of firms are stuck with last period's price $p_{1,1,t-1}(f)$ and $p_{2,1,t-1}(f)$. Since there are a large number of firms charging what they charged last period, it will also be the case that $\int_0^{\alpha_c} p_{1,1,t-1}(f)^{-\sigma} df = \alpha_c P^{-\sigma}_{1,1,t-1}$.[1] Similarly, $\int_0^{\alpha_c} p_{2,1,t-1}(f)^{-\sigma} df = \alpha_c P^{-\sigma}_{2,1,t-1}$. The complementary fraction $(1-\alpha_c)$ are able to reset price, and they all reset to the same price, $p^*_{1,1,t}$ and $p^*_{2,1,t}$. This gives the recursive representation for price dispersion,

$$v^p_{1,1,t} = (1-\alpha_c)\left(\frac{p^*_{1,1,t}}{P_{1,1,t}}\right)^{-\sigma} + \alpha_c e^{(1-\sigma)\bar{\pi}_1}\left(\frac{P_{1,1,t-1}}{P_{1,1,t}}\right)^{-\sigma} v^p_{1,1,t-1}, \tag{A.80}$$

$$v^p_{2,1,t} = (1-\alpha_c)\left(\frac{p^*_{2,1,t}}{P_{2,1,t}}\right)^{-\sigma} + \alpha_c e^{(1-\sigma)\bar{\pi}_2}\left(\frac{P_{2,1,t-1}}{P_{2,1,t}}\right)^{-\sigma} v^p_{2,1,t-1}. \tag{A.81}$$

[1] We have, as definition of the price index, $P_{1,1,t} = \left[\int_0^1 p_{1,1,t}(f)^{1-\sigma} df\right]^{\frac{1}{1-\sigma}}$, which we know can be represented as

$$P^{1-\sigma}_{1,1,t} = (1-\alpha_c)\,p^{*(1-\sigma)}_{1,1,t} + \alpha_c P^{1-\sigma}_{1,1,t-1}.$$

Now the price dispersion term is defined to be

$$\begin{aligned}
v^p_{1,1,t} &= \int_0^1 \left(\frac{p_{1,1,t}(f)}{P_{1,1,t}}\right)^{-\sigma} df \\
&= \int_0^{1-\alpha_c} \left(\frac{p^*_{1,1,t}}{P_{1,1,t}}\right)^{-\sigma} df + \int_{1-\alpha_c}^1 \left(\frac{p_{1,1,t-1}(f)}{P_{1,1,t}}\right)^{-\sigma} df \\
&= (1-\alpha_c)\left(\frac{p^*_{1,1,t}}{P_{1,1,t-1}}\right)^{-\sigma} + \int_{1-\alpha_c}^1 \left(\frac{p_{1,1,t-1}(f)}{P_{1,1,t-1}}\right)^{-\sigma}\left(\frac{P_{1,1,t-1}}{P_{1,1,t}}\right)^{-\sigma} df \\
&= (1-\alpha_c)\left(\frac{p^*_{1,1,t}}{P_{1,1,t-1}}\right)^{-\sigma} + \alpha_c\left(\frac{P_{1,1,t-1}}{P_{1,1,t}}\right)^{-\sigma} v^p_{1,1,t-1}
\end{aligned}$$

Price dynamics also have a recursive formulation. To obtain, we illustrate with $P_{1,1,t}$.

$$\begin{aligned}
P_{1,1,t}^{1-\sigma} &= \int_0^1 p_{1,1,t}\left(f\right)^{1-\sigma} df \\
&= \int_0^{1-\alpha_c} p_{1,1,t}^{*(1-\sigma)} df + \int_{1-\alpha_c}^1 p_{1,1,t-1}\left(f\right)^{1-\sigma} df \\
&= \left(1-\alpha_c\right) p_{1,1,t}^{*(1-\sigma)} + \underbrace{\alpha_c \int_0^1 p_{1,1,t-1}\left(f\right)^{1-\sigma} df}_{\alpha_c P_{1,1,t-1}^{1-\sigma}} \\
&= \left(1-\alpha_c\right) p_{1,1,t}^{*(1-\sigma)} + \alpha_c e^{(1-\sigma)\bar{\pi}_1} P_{1,1,t-1}^{1-\sigma}.
\end{aligned}$$

Analogous calculations hold for country 2.

A.6.1 Collection of Equilibrium Conditions

Let us collect the equilibrium conditions here.

$$\tilde{c}_{1,1,t} = d\left(\frac{P_{1,1,t}}{P_{1,t}}\right)^{-\mu} \tilde{c}_{1,t} = \left(\int_0^1 \tilde{c}_{1,1,t}\left(f\right)^{\frac{\sigma-1}{\sigma}} df\right)^{\frac{\sigma}{\sigma-1}} \tag{A.82}$$

$$\tilde{c}_{1,2,t} = (1-d)\left(\frac{P_{1,2,t}}{P_{1,t}}\right)^{-\mu} \tilde{c}_{1,t}\left(\frac{1}{\Omega_{t-1}}\right) = \left(\int_0^1 \tilde{c}_{1,2,t}\left(f\right)^{\frac{\sigma-1}{\sigma}} df\right)^{\frac{\sigma}{\sigma-1}} \tag{A.83}$$

$$\tilde{c}_{2,2,t} = d\left(\frac{P_{2,2,t}}{P_{2,t}}\right)^{-\mu} \tilde{c}_{2,t} = \left(\int_0^1 \tilde{c}_{2,2,t}\left(f\right)^{\frac{\sigma-1}{\sigma}} df\right)^{\frac{\sigma}{\sigma-1}} \tag{A.84}$$

$$\tilde{c}_{2,1,t} = (1-d)\left(\frac{P_{2,1,t}}{P_{2,t}}\right)^{-\mu} \tilde{c}_{2,t}\Omega_{t-1} = \left(\int_0^1 \tilde{c}_{2,1,t}\left(f\right)^{\frac{\sigma-1}{\sigma}} df\right)^{\frac{\sigma}{\sigma-1}} \tag{A.85}$$

Output:

$$G_{1,t}\ell_{1,t} = \tilde{c}_{1,1,t} v_{1,1,t}^p + \tilde{c}_{2,1,t} v_{2,1,t}^p \tag{A.86}$$

$$G_{2,t}\ell_{2,t} = \tilde{c}_{2,2,t} v_{2,2,t}^p + \tilde{c}_{1,2,t} v_{1,2,t}^p \tag{A.87}$$

Price dispersion:

$$v^p_{1,1,t} = (1-\alpha_c)\left(\frac{p^*_{1,1,t}}{P_{1,1,t}}\right)^{-\sigma} + \alpha_c e^{-\sigma\bar{\pi}_{1,1}}\left(\frac{P_{1,1,t-1}}{P_{1,1,t}}\right)^{-\sigma} v^p_{1,1,t-1} = \int_0^1 \left(\frac{p_{1,1,t}(f)}{P_{1,1,t}}\right)^{-\sigma} df \tag{A.88}$$

$$v^p_{2,1,t} = (1-\alpha_c)\left(\frac{p^*_{2,1,t}}{P_{2,1,t}}\right)^{-\sigma} + \alpha_c e^{-\sigma\bar{\pi}_{2,1}}\left(\frac{P_{2,1,t-1}}{P_{2,1,t}}\right)^{-\sigma} v^p_{2,1,t-1} = \int_0^1 \left(\frac{p_{2,1,t}(f)}{P_{2,1,t}}\right)^{-\sigma} df \tag{A.89}$$

$$v^p_{2,2,t} = (1-\alpha_c)\left(\frac{p^*_{2,2,t}}{P_{2,2,t}}\right)^{-\sigma} + \alpha_c e^{-\sigma\bar{\pi}_{2,2}}\left(\frac{P_{2,2,t-1}}{P_{2,2,t}}\right)^{-\sigma} v^p_{2,2,t-1} = \int_0^1 \left(\frac{p_{2,2,t}(f)}{P_{2,2,t}}\right)^{-\sigma} df \tag{A.90}$$

$$v^p_{1,2,t} = (1-\alpha_c)\left(\frac{p^*_{1,2,t}}{P_{1,2,t}}\right)^{-\sigma} + \alpha_c e^{-\sigma\bar{\pi}_{1,2}}\left(\frac{P_{1,2,t-1}}{P_{1,2,t}}\right)^{-\sigma} v^p_{1,2,t-1} = \int_0^1 \left(\frac{p_{1,2,t}(f)}{P_{1,2,t}}\right)^{-\sigma} df \tag{A.91}$$

Profits:

$$\tilde{\Pi}_{1,t} = \left(\frac{P_{1,1,t}}{P_{1,t}}\right)\tilde{c}_{1,1,t} + Q_{1,2,t}\left(\frac{P_{2,1,t}}{P_{2,t}}\right)\tilde{c}_{2,1,t} - \tilde{w}_{1,t}\ell_{1,t} \tag{A.92}$$

$$\tilde{\Pi}_{2,t} = \left(\frac{P_{2,2,t}}{P_{2,t}}\right)\tilde{c}_{2,2,t} + \left(\frac{1}{Q_{1,2,t}}\right)\left(\frac{P_{1,2,t}}{P_{1,t}}\right)\tilde{c}_{1,2,t} - \tilde{w}_{2,t}\ell_{2,t} \tag{A.93}$$

Price dynamics:

$$1 = \alpha_c e^{(1-\sigma)\bar{\pi}_1}\left(\frac{P_{1,1,t-1}}{P_{1,1,t}}\right)^{1-\sigma} + (1-\alpha_c)\left(\frac{p^*_{1,1,t}}{P_{1,1,t}}\right)^{1-\sigma} \tag{A.94}$$

$$1 = \alpha_c e^{(1-\sigma)\bar{\pi}_1}\left(\frac{P_{1,2,t-1}}{P_{1,2,t}}\right)^{1-\sigma} + (1-\alpha_c)\left(\frac{p^*_{1,2,t}}{P_{1,2,t}}\right)^{1-\sigma} \tag{A.95}$$

$$1 = \alpha_c e^{(1-\sigma)\bar{\pi}_2}\left(\frac{P_{2,2,t-1}}{P_{2,2,t}}\right)^{1-\sigma} + (1-\alpha_c)\left(\frac{p^*_{2,2,t}}{P_{2,2,t}}\right)^{1-\sigma} \tag{A.96}$$

$$1 = \alpha_c e^{(1-\sigma)\bar{\pi}_2}\left(\frac{P_{2,1,t-1}}{P_{2,1,t}}\right)^{1-\sigma} + (1-\alpha_c)\left(\frac{p^*_{2,1,t}}{P_{2,1,t}}\right)^{1-\sigma} \tag{A.97}$$

A.7 Steady State

In the steady state, the reset price is equal to the sub-category price index, $p^*_{i,j} = P_{i,j}$, consumption is equal across countries, $\tilde{c}_1 = \tilde{c}_2 = \tilde{c}$, and labor allocations are equal across activities $\ell_1 = \ell_2 = \ell$. In addition, $\Omega = 1$, $G_1 = G_2 = 1$, $Q_{1,2} = 1$, $P_{1,1}/P_1 = P_{1,2}/P_1 = P_{2,2}/P_2 = P_{2,1}/P_2 = 1$. The household Euler equation in the steady state is identical for both countries,

$$w = \eta \tilde{c} \ell^{\chi}. \tag{A.98}$$

Steady state demand is,

$$\tilde{c}_{1,1} = d\tilde{c} \tag{A.99}$$

$$\tilde{c}_{1,2} = (1-d)\,\tilde{c} \tag{A.100}$$

$$\tilde{c}_{2,2} = d\tilde{c} \tag{A.101}$$

$$\tilde{c}_{2,1} = (1-d)\,\tilde{c} \tag{A.102}$$

Labor input,

$$\ell_1 = \tilde{c}_{1,1} + \tilde{c}_{2,1} \tag{A.103}$$

$$\ell_2 = \tilde{c}_{2,2} + \tilde{c}_{1,2} \tag{A.104}$$

Imposing $\ell_1 = \ell_2 = \ell$ in (**??**) and (**??**) tells us $\tilde{c}_{1,1} = \tilde{c}_{2,2} = d\ell, \tilde{c}_{2,1} = \tilde{c}_{1,2} = (1-d)\,\ell$. Take the steady state values of the price resets and substitute out the wage $w = \eta \tilde{c} \ell^{\chi}$,to get

$$1 = \left(\frac{\sigma}{\sigma - 1}\right) \eta \tilde{c} \ell^{\chi} \tag{A.105}$$

We also have,

$$\tilde{y}_1 = \tilde{c}_{1,1} + \tilde{c}_{2,1} = \tilde{c} = \tilde{y}_2 = \tilde{c}_{2,2} + \tilde{c}_{1,2.} \tag{A.106}$$

For price setting

$$1 = \left(\frac{\sigma}{\sigma - 1}\right) \left(\frac{z^n_{1,1}}{z^d_{1,1}}\right) \tag{A.107}$$

$$z^n_{1,1} = \frac{\tilde{w}_1}{1 - \alpha_c \beta} \tag{A.108}$$

$$z^d_{1,1} = \frac{1}{1 - \alpha_c \beta} \tag{A.109}$$

$$1 = \left(\frac{\sigma}{\sigma - 1}\right)\left(\frac{z^n_{2,1}}{z^d_{2,1}}\right) \tag{A.110}$$

$$z^n_{2,1} = \frac{\tilde{w}_1}{(1 - \alpha_c \beta)} \tag{A.111}$$

$$z^d_{2,1} = \frac{1}{1 - \alpha_c \beta} \tag{A.112}$$

$$1 = \left(\frac{\sigma}{\sigma - 1}\right)\left(\frac{z^n_{2,2}}{z^d_{2,2}}\right) \tag{A.113}$$

$$z^n_{2,2,t} = \frac{\tilde{w}_2}{(1 - \alpha_c \beta)} \tag{A.114}$$

$$z^d_{2,2} = \frac{1}{1 - \alpha_c \beta} \tag{A.115}$$

$$1 = \left(\frac{\sigma}{\sigma - 1}\right)\left(\frac{z^n_{1,2}}{z^d_{1,2}}\right) \tag{A.116}$$

$$z^n_{1,2,t} = \frac{\tilde{w}_{2,t}}{(1 - \alpha_c \beta)} \tag{A.117}$$

$$z^d_{1,2,t} = \frac{1}{1 - \alpha_c \beta} \tag{A.118}$$

B Equations of interest

Equations of interest are taken from the text above.

Value function

$$\tilde{V}_t = \left\{(1 - \beta)\left[\ln(\tilde{c}_t) - \eta \frac{\ell_t^{1+\chi}}{1 + \chi}\right] - \frac{\beta}{\alpha} \ln\left[E_t e^{-\alpha \tilde{V}_{i,t+1}}\right]\right\} + \beta \ln(G_t) \tag{B.1}$$

Auxiliary Variable for Value Function

$$U_t = E_t\left(e^{-\alpha \tilde{V}_{t+1}}\right) \tag{B.2}$$

Discount term in Price Setting Equations

$$\Lambda_{t-1,t} = \frac{e^{\left(-\alpha \tilde{V}_t\right)}}{U_{t-1}} \frac{1}{G_{t-1}} \tag{B.3}$$

Real SDF

$$\beta M_{t,t+1} = \beta \left(\frac{\tilde{c}_t}{\tilde{c}_{t+1}} \right) \Lambda_{t,t+1} \tag{B.4}$$

Nominal SDF

$$\beta N_{t,t+1} = \beta M_{t,t+1} e^{-\pi_{t+1}} \tag{B.5}$$

Labor supply

$$\eta \tilde{c}_t \ell_t^{\chi} = \tilde{w}_t \tag{B.6}$$

Incomplete Markets: Bonds Consumption Euler Equations

$$\frac{1}{1+i_{1,t}} = \beta E_t N_{1,t,t+1} \tag{B.7}$$

$$\frac{1}{1+i_{2,t}} = \beta E_t N_{2,t,t+1} \tag{B.8}$$

$$\left(\frac{1}{1+i_{2,t}} \right) \left(1 + \tau \left(\frac{Q_{1,2,t} \tilde{B}_{1,2,t}}{P_{2,t}} \right) \right) = \beta E_t \left(M_{1,t,t+1} \left(\frac{Q_{1,2,t+1}}{Q_{1,2,t}} \right) e^{-\pi_{2,t+1}} \right) \tag{B.9}$$

$$\left(\frac{1}{1+i_{1,t}} \right) \left(1 + \tau \left(\frac{\tilde{B}_{2,1,t}}{Q_{1,2,t} P_{1,t}} \right) \right) = \beta E_t \left(M_{2,t,t+1} \left(\frac{Q_{1,2,t}}{Q_{1,2,t+1}} \right) e^{-\pi_{1,t+1}} \right) \tag{B.10}$$

Real Interest Rate

$$r_t = i_t - E_t \pi_{t+1} \tag{B.11}$$

Budget Constraint (Incomplete Markets)

$$\tilde{c}_{1,t} + \frac{\tilde{B}_{1,1,t}}{P_{1,t}} + \frac{Q_{1,2,t} \tilde{B}_{1,2,t}}{P_{2,t}} + \frac{\tau}{2} \left(\frac{Q_{1,2,t} \tilde{B}_{1,2,t}}{P_{2,t}} \right)^2 = \frac{\tilde{W}_{1,t} \ell_{1,t}}{P_{1,t}} + \frac{\tilde{\Pi}_{1,t}}{P_{1,t}} + \frac{(1+i_{1,t-1}) \tilde{B}_{1,1,t-1}}{P_{1,t} G_{1,t-1}} + \frac{(1+i_{2,t-1}) Q_{1,2,t} \tilde{B}_{1,2,t-1}}{P_{2,t} G_{1,t-1}}$$

$$\tilde{c}_{2,t} + \frac{\tilde{B}_{2,2,t}}{P_{2,t}} + \frac{\tilde{B}_{2,1,t}}{Q_{1,2,t} P_{1,t}} + \frac{\tau}{2} \left(\frac{\tilde{B}_{2,1,t}}{Q_{1,2,t} P_{1,t}} \right)^2 = \frac{\tilde{W}_{2,t} \ell_{2,t}}{P_{2,t}} + \frac{\tilde{\Pi}_{2,t}}{P_{2,t}} + \frac{(1+i_{2,t-1}) \tilde{B}_{2,2,t-1}}{P_{2,t} G_{2,t-1}} + \frac{(1+i_{1,t-1}) \tilde{B}_{2,1,t-1}}{Q_{1,2,t} P_{1,t} G_{2,t-1}}$$

Net Zero International Bonds (incomplete markets)

$$0 = \tilde{B}_{1,1,t} + \tilde{B}_{2,1,t}\Omega_{t-1} \tag{B.12}$$

$$0 = \frac{\tilde{B}_{1,2,t}}{\Omega_{t-1}} + \tilde{B}_{2,2,t} \tag{B.13}$$

Complete Markets: Consumption Euler Equations and Real Exchange Rate

$$\frac{1}{1+r_t} = \beta E_t M_{t,t+1} \tag{B.14}$$

$$\frac{Q_{1,2,t+1}}{Q_{1,2,t}} = \frac{M_{2,t,t+1}}{M_{1,t,t+1}}$$

Nominal exchange rate depreciation

$$\frac{S_{1,2,t}}{S_{1,2,t-1}} = \frac{Q_{1,2,t}}{Q_{1,2,t-1}} \frac{e^{\pi_{1,t}}}{e^{\pi_{2,t}}} \tag{B.15}$$

Profits

$$\tilde{\Pi}_{1,t} = \left(\frac{P_{1,1,t}}{P_{1,t}}\right) \tilde{c}_{1,1,t} + Q_{1,2,t} \left(\frac{P_{2,1,t}}{P_{2,t}}\right) \tilde{c}_{2,1,t} - \tilde{w}_{1,t}\ell_{1,t} \tag{B.16}$$

$$\tilde{\Pi}_{2,t} = \left(\frac{P_{2,2,t}}{P_{2,t}}\right) \tilde{c}_{2,2,t} + \left(\frac{1}{Q_{1,2,t}}\right) \left(\frac{P_{1,2,t}}{P_{1,t}}\right) \tilde{c}_{1,2,t} - \tilde{w}_{2,t}\ell_{2,t} \tag{B.17}$$

Demand functions

$$\tilde{c}_{1,1,t} = d\left(\frac{P_{1,1,t}}{P_{1,t}}\right)^{-\mu} \tilde{c}_{1,t} \tag{B.18}$$

$$\tilde{c}_{1,2,t} = (1-d)\left(\frac{P_{1,2,t}}{P_{1,t}}\right)^{-\mu} \tilde{c}_{1,t} \tag{B.19}$$

$$\tilde{c}_{2,2,t} = d\left(\frac{P_{2,2,t}}{P_{2,t}}\right)^{-\mu} \tilde{c}_{2,t} \tag{B.20}$$

$$\tilde{c}_{2,1,t} = (1-d)\left(\frac{P_{2,1,t}}{P_{2,t}}\right)^{-\mu} \tilde{c}_{2,t} \tag{B.21}$$

Equilibrium conditions

$$\tilde{y}_{1,t} = \tilde{c}_{1,1,t} + \tilde{c}_{2,1,t} \tag{B.22}$$

$$\tilde{y}_{2,t} = \tilde{c}_{2,2,t} + \tilde{c}_{1,2,t} \tag{B.23}$$

$$G_{1,t}\ell_{1,t} = \tilde{c}_{1,1,t}v^p_{1,1,t} + \tilde{c}_{2,1,t}v^p_{2,1,t} \tag{B.24}$$

$$G_{2,t}\ell_{2,t} = \tilde{c}_{2,2,t}v^p_{2,2,t} + \tilde{c}_{1,2,t}v^p_{1,2,t} \tag{B.25}$$

$$v^p_{1,1,t} = (1-\alpha_c)\left(\frac{p^*_{1,1,t}}{P_{1,1,t}}\right)^{-\sigma} + \alpha_c e^{-\sigma\bar{\pi}_{1,1}}\left(\frac{P_{1,1,t-1}}{P_{1,1,t}}\right)^{-\sigma} v^p_{1,1,t-1} \tag{B.26}$$

$$v^p_{2,1,t} = (1-\alpha_c)\left(\frac{p^*_{2,1,t}}{P_{2,1,t}}\right)^{-\sigma} + \alpha_c e^{-\sigma\bar{\pi}_{2,1}}\left(\frac{P_{2,1,t-1}}{P_{2,1,t}}\right)^{-\sigma} v^p_{2,1,t-1} \tag{B.27}$$

$$v^p_{2,2,t} = (1-\alpha_c)\left(\frac{p^*_{2,2,t}}{P_{2,2,t}}\right)^{-\sigma} + \alpha_c e^{-\sigma\bar{\pi}_{2,2}}\left(\frac{P_{2,2,t-1}}{P_{2,2,t}}\right)^{-\sigma} v^p_{2,2,t-1} \tag{B.28}$$

$$v^p_{1,2,t} = (1-\alpha_c)\left(\frac{p^*_{1,2,t}}{P_{1,2,t}}\right)^{-\sigma} + \alpha_c e^{-\sigma\bar{\pi}_{1,2}}\left(\frac{P_{1,2,t-1}}{P_{1,2,t}}\right)^{-\sigma} v^p_{1,2,t-1} \tag{B.29}$$

Evolution of Prices

$$1 = \alpha_c e^{(1-\sigma)\bar{\pi}_1}\left(\frac{P_{1,1,t-1}}{P_{1,1,t}}\right)^{1-\sigma} + (1-\alpha_c)\left(\frac{p^*_{1,1,t}}{P_{1,1,t}}\right)^{1-\sigma} \tag{B.30}$$

$$1 = \alpha_c e^{(1-\sigma)\bar{\pi}_1}\left(\frac{P_{1,2,t-1}}{P_{1,2,t}}\right)^{1-\sigma} + (1-\alpha_c)\left(\frac{p^*_{1,2,t}}{P_{1,2,t}}\right)^{1-\sigma} \tag{B.31}$$

$$1 = \alpha_c e^{(1-\sigma)\bar{\pi}_2}\left(\frac{P_{2,2,t-1}}{P_{2,2,t}}\right)^{1-\sigma} + (1-\alpha_c)\left(\frac{p^*_{2,2,t}}{P_{2,2,t}}\right)^{1-\sigma} \tag{B.32}$$

$$1 = \alpha_c e^{(1-\sigma)\bar{\pi}_2}\left(\frac{P_{2,1,t-1}}{P_{2,1,t}}\right)^{1-\sigma} + (1-\alpha_c)\left(\frac{p^*_{2,1,t}}{P_{2,1,t}}\right)^{1-\sigma} \tag{B.33}$$

Price Indices

$$P_{1,t} = \left[dP^{1-\mu}_{1,1,t} + (1-d)\,P^{1-\mu}_{1,2,t}\right]^{\frac{1}{1-\mu}} \tag{B.34}$$

$$P_{2,t} = \left[dP^{1-\mu}_{2,2,t} + (1-d)\,P^{1-\mu}_{2,1,t}\right]^{\frac{1}{1-\mu}} \tag{B.35}$$

Price setting equations

$$\left(\frac{p^*_{1,1,t}}{P_{1,1,t}}\right)\left(\frac{P_{1,1,t}}{P_{1,t}}\right)=\left(\frac{\sigma}{\sigma-1}\right)\left(\frac{z^n_{1,1,t}}{z^d_{1,1,t}}\right) \tag{B.36}$$

$$z^n_{1,1,t}=\frac{\tilde{w}_{1,t}}{G_{1,t}}+\alpha_c\beta e^{-\bar{\pi}_1\sigma}E_t\left(\Lambda_{1,t,t+1}\left(\frac{P_{1,1,t}}{P_{1,1,t+1}}\right)^{\mu-\sigma}\left(\frac{P_{1,t}}{P_{1,t+1}}\right)^{-\mu}z^n_{1,1,t+1}\right) \tag{B.37}$$

$$z^d_{1,1,t}=1+\alpha_c\beta e^{\bar{\pi}_1(1-\sigma)}E_t\left(\Lambda_{1,t,t+1}\left(\frac{P_{1,1,t}}{P_{1,1,t+1}}\right)^{\mu-\sigma}\left(\frac{P_{1,t}}{P_{1,t+1}}\right)^{1-\mu}z^d_{1,1,t+1}\right) \tag{B.38}$$

$$\left(\frac{p^*_{2,1,t}}{P_{2,1,t}}\right)\left(\frac{P_{2,1,t}}{P_{2,t}}\right)=\left(\frac{\sigma}{\sigma-1}\right)\left(\frac{z^n_{2,1,t}}{z^d_{2,1,t}}\right) \tag{B.39}$$

$$z^n_{2,1,t}=\left(\frac{\tilde{w}_{1,t}}{G_{1,t}}\right)\Omega_{t-1}+\alpha_c\beta e^{-\bar{\pi}_2\sigma}E_t\left(\Lambda_{1,t,t+1}\left(\frac{P_{2,1,t}}{P_{2,1,t+1}}\right)^{\mu-\sigma}\left(\frac{P_{2,t}}{P_{2,t+1}}\right)^{-\mu}z^n_{2,1,t+1}\right) \tag{B.40}$$

$$z^d_{2,1,t}=Q_{1,2,t}\Omega_{t-1}+\alpha_{2,1}\beta e^{\bar{\pi}_2(1-\sigma)}E_t\left(\Lambda_{1,t,t+1}\left(\frac{P_{2,1,t}}{P_{2,1,t+1}}\right)^{\mu-\sigma}\left(\frac{P_{2,t}}{P_{2,t+1}}\right)^{1-\mu}z^d_{2,1,t+1}\right) \tag{B.41}$$

$$\left(\frac{p^*_{2,2,t}}{P_{2,2,t}}\right)\left(\frac{P_{2,2,t}}{P_{2,t}}\right)=\left(\frac{\sigma}{\sigma-1}\right)\left(\frac{z^n_{2,2t}}{z^d_{2,2t}}\right), \tag{B.42}$$

$$z^n_{2,2,t}=\frac{\tilde{w}_{2,t}}{G_{2,t}}+\alpha_c\beta e^{-\bar{\pi}_2\sigma}E_t\left(\Lambda_{2,t,t+1}\left(\frac{P_{2,2,t}}{P_{2,2,t+1}}\right)^{\mu-\sigma}\left(\frac{P_{2,t}}{P_{2,t+1}}\right)^{-\mu}z^n_{2,2,t+1}\right) \tag{B.43}$$

$$z^d_{2,2,t}=1+\alpha_c\beta e^{\bar{\pi}_2(1-\sigma)}E_t\left(\Lambda_{2,t,t+1}\left(\frac{P_{2,2,t}}{P_{2,2,t+1}}\right)^{\mu-\sigma}\left(\frac{P_{2,t}}{P_{2,t+1}}\right)^{1-\mu}z^d_{2,2,t+1}\right) \tag{B.44}$$

$$\left(\frac{p^*_{1,2,t}}{P_{1,2,t}}\right)\left(\frac{P_{1,2,t}}{P_{1,t}}\right)=\left(\frac{\sigma}{\sigma-1}\right)\left(\frac{z^n_{1,2,t}}{z^d_{1,2,t}}\right) \tag{B.45}$$

$$z^n_{1,2,t} = \left(\frac{\tilde{w}_{2,t}}{G_{2,t}}\right)\left(\frac{1}{\Omega_{t-1}}\right) + \alpha_c \beta e^{-\bar{\pi}_1 \sigma} E_t \left(\Lambda_{2,t,t+1}\left(\frac{P_{1,2,t}}{P_{1,2,t+1}}\right)^{\mu-\sigma}\left(\frac{P_{1,t}}{P_{1,t+1}}\right)^{-\mu} z^n_{1,2,t+1}\right) \tag{B.46}$$

$$z^d_{1,2,t} = \left(\frac{1}{Q_{1,2,t}}\right)\left(\frac{1}{\Omega_{t-1}}\right) + \alpha_c \beta e^{\bar{\pi}_1 (1-\sigma)} E_t \left(\Lambda_{2,t,t+1}\left(\frac{P_{1,2,t}}{P_{1,2,t+1}}\right)^{\mu-\sigma}\left(\frac{P_{1,t}}{P_{1,t+1}}\right)^{1-\mu} z^d_{1,2,t+1}\right) \tag{B.47}$$