Third-Country Effects on the Exchange Rate*

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Abstract

Predictive regressions for bilateral exchange rates are typically run using variables from the associated bilateral country pairs. These regressions characteristically show limited explanatory power. Considerable relative increases in adjusted R^2 are obtained by augmenting the regressions with third-country variables. A three-country exchange rate model is presented in which cross-country heterogeneity opens up channels for third-country effects to influence the bilateral rate.

Keywords: Exchange Rates, Disconnect Puzzle, Multi-Country Model

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Introduction

In this paper, we addresses the *exchange-rate disconnect puzzle,* which is the term coined by Obstfeld and Rogoff (2000) to describe the "exceedingly weak relationship (except, perhaps, in the longer run) between the exchange rate and virtually *any* macroeconomic aggregates." There are two facets to the puzzle. On one side, whether a country's exchange rate floats, is fixed, or takes some intermediate regime seems to be irrelevant for macroeconomic performance. Baxter and Stockman (1989), Flood and Rose (1995) find neither economic growth nor macro aggregate volatility to be sensitive to a country's exchange rate regime while Reinhart and Rogoff (2004), Levy-Yeyati and Sturzenegger (2003), Dubas et al. (2010), and Talvas et al. (2008) report conflicting conclusions.

This paper addresses the other side of the puzzle, which is under floating, macroeconomic variables have virtually no explanatory or predictive power for exchange rate movements. This seems especially puzzling since the the exchange rate is itself a macroeconomic variable. While there are many ways to characterize this aspect of the exchange-rate disconnect puzzle, we frame our discussion in terms of the *adjusted* R^2 (or $\bar{R}^2$) from predictive regressions of the future change in the log exchange rate on current macroeconomic fundamentals.

The literature is replete with reports of low explanatory power from such regressions. We illustrate here by regressing the one-quarter ahead Danish krone depreciation against the U.S. dollar on the deviation from purchasing power parity (PPP). This commonly used formulation is based on a restricted error-correction model that views the PPP as the long-run equilibrium value of the nominal exchange rate. Using quarterly observations from 1999 to 2011, the regression has no explanatory power ($\bar{R}^2 = -0.003$). Clearly, there is a serious omitted variables problem. Note that the bilateral depreciation is regressed only on U.S. and Danish variables–the two countries associated with the bilateral rate. However, augmenting the regression with the Danish-Japanese deviation from PPP raises the $\bar{R}^2$ to 0.124.

This example is a preview to the general approach taken in this paper; that improved explanatory power can be obtained by incorporating information from beyond the associated bilateral country pairs. We generically refer to the extended information as "third-country" effects. This terminology should not be taken to mean that the same 'third country' is involved in determination of all bilateral exchange rates as different exchange rates may be affected by different third countries, by many such third countries or even the "rest of the world." Our strategy for viewing third-country effects as omitted variables draws its motivation from research that employs factor analyses to study exchange rates, as in Engel et al. (2012) who find multiple common factors (sources of cross-sectional correlation) in panels of bilateral exchange rates. We view the *multiplicity* of the common factors to suggest that variables beyond the two associated countries explain some of the bilateral exchange rate variation. Two related papers are Verdelhan (2013), who uses country-level heterogeneity to identify contributions from global (third-country) sources to bilateral exchange rate variation and Greenaway-McGrevy et al. (2012) identify specific exchange rates as factors.

Building on this existing empirical work, we report evidence that third-country variables can improve explanatory power in exchange rate regressions. The empirical evidence is presented in the context of two common formulations. Our first specification is as in the example above where we regress the bilateral depreciation on the deviation from PPP. The second regression recognizes the endogeneity of monetary policy and regresses the future change in the exchange rate variables, such as the inflation rate and the output gap, that are thought to enter the monetary policy reaction function. We refer to these variables as Taylor rule fundamentals.[1] We measure the importance of third-country effects on the bilateral exchange rate as the contribution of third-country variables to the adjusted R^2. Our analysis is based on in-sample estimates. Although we work with the predictive regression framework, the paper is not about forecasting.

These regressions show correlations among endogenous variables, however, and are not explicit about the channels by which third-country effects matter. To put some structure on the regression evidence, we first develop intuition for third-country influences using a conventional partial-equilibrium asset-pricing framework where the exchange rate is 'priced' as the expected present value of a linear combination of macroeconomic fundamentals and unobservable shocks.

The key element that admits third-country effects on the bilateral rate are structural heterogeneities across countries. In the presence of these heterogeneities, countries 1 and 2 respond differently to shocks originating in country 3 which results in variation in the bilateral exchange rate between 1 and 2. We show that differences in exchange rate mangement create a role for the deviation from PPP or alternatively on third country fundamentals of inflation and the output gap to influence the bilateral rate in a way that conforms to the regressions from the empirical work. Country-level heterogeneity is also emphasized by Verdelhan (2013) as key in identifying bilateral exchange rate variation from global shocks and by Benigno (2004) as a necessary feature in generating real exchange rate persistence. We also report empirical estimates of policy rules that is consistent with the idea that cross-rate movements have influenced policy rates.

Next, we extend the analysis to a general equilibrium environment where inflation and output gaps are determined by policy and productivity shocks. The framework we use is largely borrowed from Benigno (2004) which we extend to incorporate a third country. We have two primary motivations in using this framework. First, it provides the minimum amount of structure needed to get a useful theory of nominal exchange rates and their dependence on third-country effects. Second, two-country versions of the model have been studied extensively and many of its properties are well understood (Chari et al., 2002, Steinsson, 2008, Bergin, 2006, and Kollman, 2001)). In particular, Benigno (2004) shows in the two-country context that the model can account for the persistence of the real exchange rate observed in the data when countries exhibit heterogeneity in the duration of nominal contracts and policy is characterized by interest rate smoothing. In our work, we show that these sorts of heterogeneity create an important pathway for third-country variables to affect the bilateral exchange rate. Impulse response analyses performed on the parameterized model show that country 3 shocks

[1] Empirical exchange rate research has intensively examined the explanatory / predictive power of monetary and purchasing-power parity fundamentals (Mark, 1995, Chinn and Meese, 1995, Cheung et al., 2005, Mark and Sul, 2001, Rapach and Wohar, 2004, Groen 2005 and Cerra and Saxena (2010). Other work has incorporated monetary policy endogeneity via interest-rate feedback rules (Engel and West (2006), Mark, 2009, Molodtsova and Papell, 2009, Molodtsova et al., 2008, 2011).

can be quantitatively important and have as large an impact on the bilateral rate as country 2 shocks. Using the model as the data-generating mechanism in Monte Carlo experiments, we show that median $\bar{R}^2$ values from 'two-country' predictive regressions (in which future short-horizon changes in the log exchange rate between 1 and 2 are regressed on macroeconomic fundamentals only of Countries 1 and 2) and 'third-country' regressions (that condition on the fundamentals of all three countries) are broadly consistent with the estimates obtained from the data.

We contrast our paper from Engel and West (2005) and Devereux and Engel (2002), whose work is driven by rationalizing the disconnect puzzle by deriving conditions under which the exchange rate is theoretically predicted to be disconnected from the macroeconomy. Our approach, which attempts to identify omitted variables from the standard exchange rate regressions, is foreshadowed by Hodrick and Vassalou (2002) who found that multi-country models are better able to explain the dynamics of exchange rates than two-country models. Our work is also related to Evans (2012) who also seeks to solve the exchange-rate disconnect puzzle. His explanation for low explanatory power in exchange rate regressions is that most exchange-rate variation is driven by unobserved, non-fundamental risk (taste) shocks whereas we emphasize observed third-country.

The remainder of the paper is organized as follows. The next section presents empirical evidence on the contribution of third-country effects to explain bilateral exchange rate movements. Section 2 presents an illustrative partial equilibrium model that lays out direct third country effects arising from heterogeneity in monetary policy reaction functions. Some evidence for policy heterogeneity is also reported. Section 3 presents the complete three-country general equilibrium model. The model's properties and predictions are discussed in Section 4 and Section 5 concludes.

1 Third-Country Effects on the adjusted R^2 in Exchange Rate Prediction Equations

In this section, we present in-sample estimates of two predictive regression formulations that have performed relatively well in the forecasting context (see Engel et al. (2007)). Althought the empirical work is set in a predictive regression framework, we emphasize that this paper is not about forecasting. We confine the analysis to in-sample estimates and avoid issues involving the use of revised versus real time data, whether out-of-sample information is exploited in estimation, criteria for evaluating forecast accuracy, economic versus statistical significance and so forth. While these are important issues in their own right, they are not central to the point of our paper.

This work is motivated by recent exchange-rate research using factor analysis. This work begins with Engel et al. (2012) who show that a three-factor model explains a large proportion of bilateral exchange rate variation. Verdelhan (2013) and Greenaway-McGrevy (2012) identify the factors with specific exchange rates. Verdelhan's two-factor model consists of a dollar factor and a carry factor. The dollar factor is the cross-sectional average of exchange rates which is approximately the first principal component (PC) and the carry factor are exchange rates between high and low interest rate countries. Interpreting his estimates in the context of the stochastic discount factor approach to exchange rates establishes that global risks are priced into bilateral exchange rates and represent an important source of bilateral variation. Greenaway-McGrevy et al. identification is even more specific. They identify the US dollar–euro exchange rate, which closely approximates the first PC, as the dominant factor and the dollar–yen and dollar-Swiss franc rates as subordinate factors. The dollar–euro is rationalized by its dominate role in the foreign exchange market. Their identification of the Swiss franc and the yen as factors accords with their role as 'safe haven' currencies and as their role as funding sources for the carry trade, which is consistent with Verdelhan's carry factor. Greenaway-Mcgrevy et al. also find that the post euro-creation period constitutes a separate regime. Because of the important role played by the euro in international finance, and to avoid complexities involved in estimation across different regimes, we limit our empirical analyses to the period since the launch of the euro. Hence, the data set consists of 15 bilateral exchange rates against the US dollar with the time-span of the observations ranging from 1999Q1 to 2011Q4. We follow Greenaway-McGrevy et al. identification as candidates for third-country effects.

1.1 Deviations from PPP regressions

The U.S. serves as the numeraire country. Notation is $\tilde{e}_{j,t}$ for the log U.S. dollar price of one unit of currency j. The log real exchange rate between the U.S. and country j is $\tilde{q}_{j,t} = \tilde{e}_{jt,} + \tilde{p}_{j,t} - \tilde{p}_{us,t}$ where $\tilde{p}_{j,t}$ is the (log) general price level of country j. We first consider the regression of the future percent change in the exchange rate on the deviation from PPP, and compare the adjusted R^2 between

$$\tilde{e}_{j,t+k} - \tilde{e}_{j,t} = b_0 + b_1\tilde{q}_{j,t} + \epsilon_{j,t+k}, \tag{1}$$

which regresses the $k-$period proportionate change in the exchange rate on the deviation from PPP between the U.S. and country j, and a regression augmented by the deviation from PPP between the U.S. and country $i \neq j$,

$$\tilde{e}_{j,t+k} - \tilde{e}_{j,t} = b_0 + b_1\tilde{q}_{j,t} + b_2\tilde{q}_{i,t} + \epsilon_{j,t+k.} \tag{2}$$

Eq.(1) is a formulation that attempts to exploit an error-correction mechanism where the PPP fundamentals serve as the long-run attractor for the nominal exchange rate.

The one-period horizon results are shown in Panel A of Table 1. $\bar{R}^2$ from estimates of the two-country regression (1) are shown under column (1). The bilateral deviation from PPP has almost no explanatory power at the one-period horizon. We note a preponderance of negative $\bar{R}^2$ from these regressions. Positive $\bar{R}^2$ are obtained only in four instances.

Augmenting the regression with the US-euro deviation from PPP raises the $\bar{R}^2$ in 8 cases. In 3 of those instances, the slope on the augmented variable is significant at the 5% level.[2] Augmentation with the U.S.-Japanese deviation from PPP raises the $\bar{R}^2$ in 9 cases, 8 of which have significant slopes. Similarly, including the U.S.-Swiss deviation from PPP increases $\bar{R}^2$ in 8 cases. Relatively large improvements in explanatory power are found when adding the U.S.-Japanese real exchange rate to Denmark ($\bar{R}^2$ increases from -0.03 to 0.12) and adding the U.S.-euro real exchange rate to New Zealand

[2]Standard errors calculated by method of Newey and West (1983).

($\bar{R}^2$ increase from -0.008 to 0.11) and to Brazil ($\bar{R}^2$ increases from -0.02 to 0.10). Taken together, augmentation fails to increase the $\bar{R}^2$ only for Japan and Korea, and with the exception of Indonesia, the slope on the augmented variable in those cases is significant. The average of the maximum $\bar{R}^2$ from the augmented regressions is 0.054 whereas the average $\bar{R}^2$ in the standard two-country regression is -0.002.

The four-period horizon results are shown in Panel B. $\bar{R}^2$ for eq.(1) are higher than the one-horizon $\bar{R}^2$ in every case and exceeds 0.2 in four cases (Gt. Britain, Indonesia, Korea and Sweden) but explanatory power generally remains low with negative $\bar{R}^2$ for Australia, Philippines and Thailand. Augmentation with the U.S.-euro real exchange rater raises $\bar{R}^2$ in 11 of 15 cases (significant at the 5% level in 4 cases). Augmentation with the U.S.-Japanese real exchange rate raises $\bar{R}^2$ in 13 of 14 cases (significant at the 5% level in 11 cases). Proportionately large improvements in explanatory power occur when augmenting Brazil with the euro and Swiss Franc real exchange rates ($\bar{R}^2$ increases from 0.038 to 0.49 and 0.43 respectively), augmenting Denmark with the yen real exchange rate ($\bar{R}^2$ increases from 0.07 to 0.49) and augmenting New Zealand with the yen real exchange rate ($\bar{R}^2$ increases from 0.06 to 0.48). The average of the maximum $\bar{R}^2$ increases from 0.09 in the two-country regression to 0.30 in the augmented regression.

1.2 Taylor rule fundamentals regressions

Our second regression employs Taylor-rule fundamentals (inflation and the output gap). This approach recognizes that since the mid 1990s, central banks have increasingly conducted policy through interest-rate reaction functions (Taylor rules) that depend on expected domestic inflation $E_t(\pi_{j,t+1})$ and the output gap $\tilde{y}_{j,t}$,

$$i_{j,t} = \delta + \lambda E_t(\pi_{j,t+1}) + \phi \tilde{y}_{j,t} + \epsilon_{j,t}. \tag{3}$$

The Taylor-rule fundamentals regression can be motivated beginning with the quasi-UIP relationship,

$$E_t(\tilde{e}_{j,t+1}) - \tilde{e}_{j,t} = i_t - i_{j,t} + \zeta_{j,t}. \tag{4}$$

where i_t is the interest rate of the numeraire country, the U.S. As is well known, UIP is routinely and decisively rejected by the data, although a consensus as to the source of these violations has not emerged. Here, we acknowledge the failure of UIP by including an exogenous UIP deviation term $\zeta_{j,t}$. For convenience, assume that inflation follows a first-order autoregressive process (AR(1)). Then $E_t(\pi_{j,t+1}) = \rho_\pi \pi_{j,t}$, where $|\rho_\pi| < 1$ is the autocorrelation coefficient. Employ this expectation formula in the policy rule (3) and substituting the policy rules into (4) implies a predictive regression relationship with bilateral Taylor rule fundamentals

$$\tilde{e}_{j,t+1} - \tilde{e}_{j,t} = b_0 + b_1 \pi_t + b_2 \pi_{j,t} + b_3 \tilde{y}_t + b_4 \tilde{y}_{j,t} + v_{j,t+1}, \tag{5}$$

where the deviation from UIP, $\zeta_{j,t}$, has been impounded into the regression error term, $v_{j,t+1}$. The predictive power of regressions of the form (5) has been studied in a series of papers by Molodtsova and Papell (2009) and Molodtsova et al. (2008, 2011). We measure the output gap with the cyclical component of the Hodrick-Prescott filter.

To assess the value of third-country effects, we compare the adjusted R^2 from (5) to the $\bar{R}^2$ from

$$\tilde{e}_{j,t+1} - \tilde{e}_{j,t} = b_0 + b_1\pi_t + b_2\pi_{j,t} + b_3\tilde{y}_t + b_4\tilde{y}_{j,t} + b_5\pi_{m,t} + b_6\tilde{y}_{m,t} + v_{j,t+1}, \tag{6}$$

$(m \neq j)$. Although the quasi-UIP relation implies a horizon that coincides with the maturity of the interest rates, we run the regressions at both the one- and four-quarter horizons.

The one-period horizon results are shown in Panel A of Table 2. Column (1) shows the $\bar{R}^2$ from estimating (5). Explanatory power varies across the exchange rates with $\bar{R}^2$ ranging from 0.16 for Australia to -0.07 for Switzerland.

Augmenting the regression with euro-area Taylor-rule fundamentals raises the $\bar{R}^2$ in 10 of 15 cases. The slope coefficient on at least one of the third-country variables are significant at the 10% level in 7 of those cases. Augmenting with Swiss fundamentals raises $\bar{R}^2$ in 9 of 14 cases where a third-country slope is significant in 8 of those cases. Somewhat less improvement is obtained from augmentation by Japanese Taylor-rule fundamentals. These regressions yield increases in $\bar{R}^2$ in only 4 cases. The average of the maximum $\bar{R}^2$ from the augmented regressions is 0.13, whereas the average from the unaugmented regressions is 0.04. A potential 11% improvement in explanatory power is attributable to third-country effects at the one-period horizon.

The four-period horizon results are shown in Panel B of Table 2. Again, explanatory power varies across exchange rates. We obtain a relatively large $\bar{R}^2$ of 0.49 for Canada and continue to see negative values in three cases (Brazil, Japan, Philippines).

Augmenting the regression with euro-area fundamentals raise the $\bar{R}^2$ in 11 cases. At lease one of the third-country variables are significant in 7 of those cases. Augmenting with Swiss fundamentals raise the $\bar{R}^2$ in 12 of 14 cases where a third-country slope is significant in 8 of those cases. Fairly dramatic improvement is achieved for Canada where the $\bar{R}^2$ increases from 0.49 to 0.74. Augmenting with Japanese fundamentals raise the $\bar{R}^2$ in 7 of 14 cases with third-country slope significance in 4 of those cases. The average of the maximum $\bar{R}^2$ from the augmented regressions is 0.34, whereas the average from the unaugmented regressions is 0.24.

2 A Channel of Third-Country Dependence through Monetary Policy

To provide structure on the regression results, this section shows how heterogeneity in monetary policy across countries creates a direct channel for third-country effects on the bilateral exchange rate. The model presented here, as in Engel and West (2006) and Mark (2009), follows the partial equilibrium asset-approach where the exchange rate can be represented as the present value of future observable and unobservable fundamentals and endogenous monetary policy is conducted through interest rate feedback rules, which we refer to as 'Taylor rules.'

Let country 1 be the U.S., country 2 be 'home' and country 3 be the third country. For country $j = 1, 2, 3$, let $\pi_{j,t}$ be its general price level inflation from $t-1$ to t, $i_{j,t}$ its policy interest rate and $\tilde{y}_{j,t}$ its output gap. Suppressing constant terms, the monetary authorities in country 1 respond to expected domestic inflation and the output gap

$$i_{1,t} = \lambda E_t \left(\pi_{1,t+1}\right) + \phi \tilde{y}_{1,t} + \epsilon_{1,t}, \tag{7}$$

where $\epsilon_{1,t}$ is a shock to monetary policy.

For simplicity, we assume a constant inflation target of zero. Empirical formulations typically include the lagged policy rate to capture the central bank's desire for interest rate smoothing. We omit the lagged rate here since including it complicates the algebra but does not change the qualitative results. In the general equilibrium model presented below, we include the lagged interest rate.

The monetary policy rule in country 2 is

$$i_{2,t} = \lambda E_t \left(\pi_{2,t+1}\right) + \phi \tilde{y}_{2,t} + \sigma q_{2,3,t} + \epsilon_{2,t}. \tag{8}$$

The assumption that the policy parameters λ and ϕ are identical across countries is for convenience. Note that country 2's monetary authority is assumed to engage in exchange rate management. In the typical formulation (e.g., Engel and West (2006) and Mark(2007)), country 2 manages the bilateral exchange rate between itself and country 1 and $q_{1,2,t}$ is the variable that appears in the rule. However, we posit that country 2 manages not the bilateral rate between itself and 1, but with 3 so that $q_{2,3,t}$ appears in eq. (8). Although the U.S. dollar is the dominant currency in foreign exchange markets, geographical, trade and other considerations might influence country 2 to pursue a management policy with respect to country 3. In this scheme, country 2 authorities view the PPP relative to country 3 as the 'natural' level of the bilateral rate between 2 and 3. Stabilizing exchange-rate management implies that $\sigma > 0$, indicating that country 2 should raise the interest rate when it experiences a real depreciation ($q_{2,3,t}$ increases).

Let the exchange rate–interest rate relation be given by the quasi-interest parity condition

$$E_t \left(\tilde{e}_{1,2,t+1}\right) - \tilde{e}_{1,2,t} = i_{1,t} - i_{2,t} + \zeta_{1,2,t} \tag{9}$$

where $\zeta_{1,2,t}$ is an exogenous deviation from UIP.

For notational efficiency, for any variable x, let $x_{i,j,t}$ denote the differential between countries i and j, so that $\pi_{i,j,t} = \pi_{i,t} - \pi_{j,t}$ is the inflation differential and $\tilde{y}_{i,j,t} = \tilde{y}_{i,t} - \tilde{y}_{j,t}$ is the output gap differential. Substitute the Taylor rules (7) and (8) into (9). From this result, subtract the expected inflation differential between 1 and 2, $E_t \left(\pi_{1,2,t+1}\right)$. This gives a stochastic difference equation in the real exchange rate, $q_{1,2,t}$, between 1 and 2, which depends on $q_{2,3,t}$. Noting that $q_{2,3,t} = q_{1,3,t} - q_{1,2,t}$, the difference equation for $q_{1,2,t}$ can be written showing its dependence on the bilateral real exchange rate between countries 1 and 3,[3]

$$\begin{aligned} q_{1,2,t} = {} & (1+\sigma)^{-1} \left[\sigma q_{1,3,t} + \zeta_{2,1,t} + (\lambda - 1) E_t \left(\pi_{2,1,t+1}\right) + \phi \tilde{y}_{2,1,t} + \epsilon_{2,1,t}\right] \\ & + (1+\sigma)^{-1} E_t \left(q_{1,2,t+1}\right). \end{aligned} \tag{10}$$

Following Engel and West (2005), let $f_t = (\lambda - 1) E_t \left(\pi_{2,1,t+1}\right) + \phi \tilde{y}_{2,1,t} + \epsilon_{2,1,t}$ denote the observable fundamentals and let $\sigma q_{1,3,t} + \zeta_{2,1,t}$ denote the 'unobservable' fundamentals. Here, $q_{1,3,t}$ is unobservable

[3] Note that deviation from UIP, inflation and output gaps are now expressed as differentials between countries 2 and 1.

in the sense that it is typically an omitted variable in these models. Forward iteration of (10) then yields the present value formula

$$q_{1,2,t} = \frac{1}{1+\sigma} E_t \sum_{j=0}^{\infty} \left(\frac{1}{1+\sigma} \right)^j f_{t+j} + \frac{1}{1+\sigma} E_t \sum_{j=0}^{\infty} \left(\frac{1}{1+\sigma} \right)^j \left[\sigma q_{1,3,t+j} + \zeta_{2,1,t+j} \right]. \tag{11}$$

In addition to the observable fundamentals and the UIP deviation, $q_{1,2,t}$ depends on expectations of future values of the real exchange rate $q_{1,3,t}$. For simplicity, assume that the driving processes for inflation, the output gap, the deviation from UIP and the real exchange rate between 1 and 3 as independent AR(1) processes,

$$\begin{aligned} \pi_{j,t} &= \rho_\pi \pi_{j,t-1} + u_{j,t}, \\ \tilde{y}_{j,t} &= \rho_y \tilde{y}_{j,t-1} + v_{j,t}, \\ \zeta_{2,1,t} &= \rho_p \zeta_{2,1,t-1} + w_{2,1,t}, \\ q_{1,3,t} &= \rho_q q_{1,3,t-1} + x_{1,3,t}, \end{aligned}$$

where the innovations u, v, w, x are *i.i.d.* with zero mean and finite variances. Using these processes to compute expectations in conjunction with (10) gives a solution for the real exchange rate, $q_{1,2,t}$, and the expected real depreciation $E_t(q_{1,2,t+1}) - q_{1,2,t}$. Adding the expected inflation differential $E_t(\pi_{1,2,t+1})$ to both sides of the expected real depreciation solution gives the expected *nominal* depreciation,

$$\begin{aligned} E_t e_{1,2,t+1} - e_{1,2,t} = {} & \left(\frac{\lambda(\rho_\pi - 1) - \sigma}{1+\sigma-\rho_\pi} \right) \rho_\pi \pi_{2,1,t} + \frac{\phi(\rho_y - 1)}{1+\sigma-\rho_y} \tilde{y}_{2,1,t} \\ & + \left(\frac{\sigma(\rho_q - 1)}{1+\sigma-\rho_q} q_{1,3,t} + \frac{(\rho_p - 1)}{1+\sigma-\rho_p} \zeta_{2,1,t} - \frac{1}{1+\sigma} \epsilon_{2,1,t} \right). \end{aligned} \tag{12}$$

The feature of the solution that we highlight is the dependence of the expected future nominal depreciation on the cross deviation from PPP between 1 and 3. Country 2's management of its exchange rate against country 3 creates a direct pathway for cross-real exchange rates to enter into the predictive regressions as third-country effects. In the Taylor-rule regression (5), the composite error term included the deviation from UIP, $\zeta_{2,1,t}$, and the cross-real exchange rate, $q_{1,3,t}$. The presence of the UIP deviation, $\zeta_{2,1,t}$, does not change the qualitative prediction that third-country effects, in the form of the real exchange rate, account for some of the variation in nominal exchange rates.

The empirical specification implied by (12) only partially conforms to the deviation from PPP regressions in that it suggests including the cross-real exchange rate. To establish a closer connection to the Taylor-rule regressions, assume that country 3's policy rule is symmetric to country 1's

$$i_{3,t} = \lambda E_t(\pi_{3,t+1}) + \phi y_{3,t} + \epsilon_{3,t}, \tag{13}$$

and that the quasi-UIP relationship between 1 and 3 is,

$$E_t(e_{1,3,t+1}) - e_{1,3,t} = i_{1,t} - i_{3,t} + \zeta_{1,3,t}. \tag{14}$$

From equations (7), (13) (14) and the AR(1) processes for inflation and output gaps gives a solution for the real cross rate

$$q_{1,3,t} = \frac{\rho_\pi(\lambda - 1)}{1-\rho_\pi} \pi_{3,1,t} + \frac{\phi}{1-\rho_y} \tilde{y}_{3,1,t} + \frac{1}{1-\rho_p} \zeta_{3,1,t} + \epsilon_{3,1,t}. \tag{15}$$

Substituting (15) into (10), solving and performing variable manipulation as above gives the expected nominal depreciation as a linear function of the Taylor-rule fundamentals for countries 1,2, and 3,

$$E_t e_{1,2,t+1} - e_{1,2,t} = -\frac{\sigma \rho_\pi (\lambda - 1)}{1 + \sigma - \rho_\pi} \pi_{3,t} - \frac{\sigma \phi}{(1 + \sigma - \rho_y)} \tilde{y}_{3,t} + F(\pi_{1,t}, \pi_{2,t}, \tilde{y}_{1,t}, \tilde{y}_{2,t}) + v_{1,2,t+1}, \tag{16}$$

where $F(\pi_{1,t}, \pi_{2,t}, \tilde{y}_{1,t}, \tilde{y}_{2,t})$ is a linear function and $v_{1,2,t+1}$ is a composite regression error that contains the UIP deviations $\zeta_{2,1,t}$ and $\zeta_{3,1,t}$.[4] This formulation identifies an additional third-country impact through the deviation from UIP between countries 1 and 3, $\zeta_{3,1,t}$. However, in estimating this relationship, these terms are unobservable and become impounded into the regression error. Hence, eq.(16) provides an explanation for why one might run the regression (6) with third-country Taylor rule fundamentals.

We note that the specific form of the managed float assumed here is not critical for this conclusion. While we assume that the authorities react to variations in the real exchange rate, cross-rate influence will also appear if Country 2's policy rate reacts to the *change* in its exchange rate with Country 3.[5] Furthermore, the general result is not sensitive to the assumption that the authorities are managing relative to a single cross-rate. Exchange rate management against a basket of currencies (as done since 2005 by the central banks of Russia, China and Malaysia, see Sokolov (2012)) will produce similar qualitative results.

The essential point of this section is that differences between countries 1 and 2 create potential pathways for third-country effects to matter for the bilateral exchange rate. The country heterogeneity assumed in this section is in the conduct of monetary policy. In the next subsection, we present some evidence consistent with the cross-rate exchange rate management assumed here, we do not believe that this is the only sort of heterogeneity or even necessarily the most important type of cross-country heterogeneity for third-country effects. Exploring the consequences of other forms of cross-country heterogeneity requires a richer economic environment, which we address in section 3 below.

2.1 Some Evidence Consistent with Cross-Rate Management

This subsection presents evidence consistent with the idea that cross-rate movements have influenced interest rates. Our policy rates are from *Datastream*.[6] We construct the output gap as the deviation of log of industrial production from the Hodrick-Prescott filtered trend. Inflation is measured by the rate of increase in the CPI.

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[4] $F(\pi_{1,t}, \pi_{2,t}, \tilde{y}_{1,t}, \tilde{y}_{2,t}) = \left(1 + \frac{\sigma(\lambda-1)}{1+\sigma-\rho_\pi} - \frac{(\lambda-1)(\rho_\pi-1)}{(1+\sigma-\rho_\pi)}\right) \rho_\pi \pi_{1,t} + \left(\frac{(\lambda-1)(\rho_\pi-1)}{(1+\sigma-\rho_\pi)} - 1\right) \rho_\pi \pi_{2,t} + \frac{\phi(\rho_y-1)}{(1+\sigma-\rho_\pi)} \tilde{y}_{2,t} - \left(\frac{\phi(\rho_y-1)}{(1+\sigma-\rho_\pi)} - \frac{\sigma\phi}{(1+\sigma-\rho_y)}\right) y_{1,t}$, and $v_{1,2,t+1} = \left(\frac{(\rho_p-1)}{(1+\sigma-\rho_p)} p_{2,1,t} - (1+\sigma)^{-1} \tilde{\epsilon}_{2,1,t} - \frac{\sigma}{(1+\sigma-\rho_p)} p_{3,1,t} - \frac{\sigma}{\sigma+1} \tilde{\epsilon}_{3,1,t}\right)$

[5] In this case, UIP implies a second-order stochastic difference equation. The solution has the same qualitative implication that the exchange rate between 1 and 2 depends in part on the expected present value of the cross rate.

[6] The interest rates used for the countries in our analysis are as follows. Brazil, Selic target rate. Canada, Bank of Canada rate. Denmark, interbank one-month offered rate. Great Britain, UK clearing banks base rate. Indonesia, BI rate and interbank rate. Japan, uncollateralized overnight rate. Philippines, reverse repo and interbank call loan rate. Switzerland, 3 month LIBOR. Thailand, interbank overnight rate. Singapore, interbank one-month rate. The euro, ECB interest rate. The U.S., Federal Funds rate.

As in Clarida et al. (1998, 2000), we allow for interest rate smoothing by the authorities in the empirical work. If the target rate i_t^T is given by eq. (8), interest rate smoothing of the policy rate is represented as $i_{j,t} = \rho i_{j,t-1} + (1-\rho) i_{j,t}^T$. Hence we estimate monetary policy feedback rules of the form

$$i_{j,t} = \delta + \rho i_{j,t-1} + (1-\rho)\left(\lambda E_t \pi_{j,t+1} + \phi \tilde{y}_{j,t} + \sigma q_{j,m,t}\right) + \epsilon_{j,t} \tag{17}$$

where $q_{j,m}$ is the log real exchange rate between the country in question j, and either the euro, the Swiss franc, or the yen. Estimation is performed by substituting $E_t \pi_{j,t+1} = \pi_{j,t+1} + v_{j,t+1}$ in eq. (17) where $v_{j,t+1}$ is the rational expectations forecast error, and estimating the resulting equation by generalized method of moments. We use the lagged policy rate and current and three lagged values of inflation, the output gap, and real exchange rates as instruments.

Table 3 reports the estimation results. Panel A reports estimates of and asymptotic t-ratios (in parentheses) for σ, which is the coefficient of primary interest (Estimation results for the best case scenarios are provided in the Appendix). For each country, the estimates are obtained from separate regressions. The cross-rate management term always shows up with the predicted sign and is almost always statistically significant.[7] In some cases (e.g., Brazil), the evidence is consistent with exchange rate management against more than one currency. In many cases, estimated coefficients on expected inflation and/or the output gap are either insignificant and/or have signs inconsistent with conventional notions of inflation targeting or of the Taylor rule.

Estimates of σ in (17) for Australia were insignificant. However, in panel B, we replace the deviation from PPP with the real *depreciation,* from which we see that the policy rate appears to have responded to the real depreciation between the Australian dollar and the euro.

To summarize, our estimates provide evidence of monetary policy induced heterogeneity across countries that may generate third-country effects on bi-lateral exchange rates.

3 A Three Country General Equilibrium Exchange-Rate Model

This section develops and analyzes a general equilibrium three-country exchange rate model to investigate how *exogenous* third-country shocks impact the bilateral exchange rate. We work within the familiar structure of the New Keynesian model that has been a popular framework for studying a variety of exchange rate and international business cycle issues (Chari et al., 2002, Bergin 2006, Kollman, 2001, Steinsson, 2008). Many properties of the model are well understood and it has served as a successful device for understanding aspects of exchange rate behavior. The sticky-price aspect delivers a theory for the nominal exchange rate and our extension to three countries allows an explicit examination of third-country effects. We consider this to be the simplest model available for the purposes of our investigation. As such, it is not a theory of everything. Specifically, we do not provide a theory for deviations from UIP. For some authors, deviations from UIP play a prominent role in their analyses (Kollman (2001), Devereux and Engel (2002), Bergin (2006), Evans (2013)) and they incorporate features in their

[7]Quarterly industrial production data were not available for Indonesia, Norway, the Philippines, Thailand and Singapore.

models to generate such deviations. Kollman (2001) and Bergin (2006) introduce exogenous UIP deviations, Devereux and Engel (2002) feature a noise-trader component and Evans (2013) introduces taste shocks that affects agent's risk-tolerance. Unlike these papers, it is not necessary to create deviations from UIP to make our point. To avoid unnecessary complications, the model is presented in a complete markets environment in which UIP holds. A similar set of comments apply to the Backus-Smith (1993) condition, which is also implied by the model but rejected by the data. We acknowledge that the model has some counterfactual implications but they do not invalidate its predictions about third-country effects on bilateral exchange rates.

3.1 The Model

As in the previous section, we loosely think of the U.S. as Country 1 and a representative home country as Country 2. Country 3 is the third country (obviously). There is no capital in the model and the production technology requires only labor. Each country is populated by a continuum of economic agents with population size proportional to the range of produced goods. The model is quite standard and our presentation of its formal structure is brief. Complete derivations of the model's equations are given in the appendix.

The Household's Problem

In period t, any one of N possible states of nature can occur. Let s_t denote the state at t and $s^t = (s_t, s_{t-1}, ..., s_0)$ denote the history. Financial markets are complete. A full set of state contingent bonds with payoffs in Country 1 money are traded internationally. Output is supplied by a continuum of monopolistically competitive firms each producing a differentiated product using only labor. Ownership of the firms is not internationally traded. Hence, households of country $j = 1, 2, 3$ own their country's firms and claims to their profits. Household resources accrue from firm profits, $\Pi_j(s^t)$, sales of labor, $n_j(s^t)$, previously unspent money balances, $M_j(s^t)$, and payoffs from the state-contingent bonds.

Let $C_j(s^t)$ be the household's consumption index (elaboration of the composition of the index follows below), $P_j(s^t)$ be the general price level, $Q(s_{t+1}|s^t)$ be the Country 1 currency price of a state-contingent security, $W_j(s^t)$ be the nominal wage, $B_j(s_t)$ be the number of state s_t securities held, and $e_{i,j}(s^t)$ be the nominal exchange rate expressed as the country i currency price of a unit of country j money. Households face the sequential budget constraints

$$C_j(s^t) + \frac{M_j(s^t)}{P_j(s^t)} + \sum_{s_{t+1}} \frac{Q(s_{t+1}|s^t) B_j(s_{t+1})}{e_{1,j}(s^t) P_j(s^t)} = \frac{W_j(s^t) n_j(s^t)}{P_j(s^t)} + \frac{\Pi_j(s^t)}{P_j(s^t)} + \frac{M_j(s^{t-1})}{P_j(s^t)} + \frac{B_j(s_t)}{e_{1,j}(s^t) P_j(s^t)}, \tag{18}$$

where current period resources are on the right side and uses of those resources on the left side of (18).

Preferences are defined over consumption, C_j, leisure, $(1 - n_j)$, and real money balances, M_j/P_j, where the functional form for flow utility is

$$u\left(C_j,(1-n_j),\frac{M_j}{P_j}\right)=\left(\frac{{C_j}^{1-\gamma_1}-1}{1-\gamma_1}\right)+\theta_2\left(\frac{(1-n_j)^{1-\gamma_2}-1}{1-\gamma_2}\right)+\theta_3\left(\frac{(M_j/P_j)^{1-\gamma_3}-1}{1-\gamma_3}\right). \tag{19}$$

A household in country $j=1,2,3$ maximizes lifetime expected utility

$$\sum_{t=0}^{\infty}\beta^t\sum_{s^t}\pi\left(s^t\right)u\left(C_j\left(s^t\right),\left(1-n_j\left(s^t\right)\right),\frac{M_j\left(s^t\right)}{P_j\left(s^t\right)}\right), \tag{20}$$

subject to eq.(18) and the functional form eq.(19). Due to the complete-markets environment, the real exchange rate in this model (as in Chari et al. (2002), Benigno (2004) and Steinsson (2008)) has the Backus-Smith (1993) form.

The Demand System

The household's consumption problem is broken into two parts. The first part is the intertemporal decision of expenditures and savings discussed above. The second part is a cost-minimizing problem for allocating consumption expenditures across the different choices of goods, which we now describe.

At this point, we lighten the notation by suppressing the functional dependence on the state. The underlying goods are differentiated on a unit interval continuum with country 1 producing goods on $\omega\in[a_0,a_1)$, country 2 on $\omega\in[a_1,a_2)$ and country 3 on $\omega\in[a_2,a_3]$ where $(0=a_0<a_1<a_2<a_3=1)$. Our notational convention is that the first subscript indicates where the good is consumed and the second subscript indicates where the good is produced ($C_{i,j,t}$ is produced in j and exported to i at time t). The consumption index for the household of country j is formed by the CES (constant elasticity of substitution) index

$$C_{j,t}=\left((d_{j,1})^{\frac{1}{\mu}}\left(C_{j,1,t}\right)^{\frac{\mu-1}{\mu}}+(d_{j,2})^{\frac{1}{\mu}}\left(C_{j,2,t}\right)^{\frac{\mu-1}{\mu}}+(d_{j,3})^{\frac{1}{\mu}}\left(C_{j,3,t}\right)^{\frac{\mu-1}{\mu}}\right)^{\frac{\mu}{\mu-1}},$$

of consumption subindices and $0\le\mu\le\infty$ is the elasticity of substitution and $d_{j,1}+d_{j,2}+d_{j,3}=1$. The general price level associated with this consumption index is

$$P_{j,t}=\left(d_{j,1}\left(P_{j,1,t}\right)^{1-\mu}+d_{j,2}\left(P_{j,2,t}\right)^{1-\mu}+d_{j,3}\left(P_{j,3,t}\right)^{1-\mu}\right)^{\frac{1}{1-\mu}}.$$

Each of the underlying country baskets $C_{i,j,t}$ are themselves CES indices of the individual goods purchased, $c_{i,j,t}(\omega)$, from country j and consumed by residents of country i,

$$C_{i,j,t}=\left(\left(\frac{1}{a_j-a_{j-1}}\right)^{\frac{1}{\sigma}}\int_{a_{j-1}}^{a_j}c_{i,j,t}(\omega)^{\frac{\sigma-1}{\sigma}}d\omega\right)^{\frac{\sigma}{\sigma-1}}\qquad j=1,2,3\ ,$$

and have corresponding CES price indices $P_{i,j,t}$ of the individual good prices, $p_{i,j,t}(\omega)$, produced in country j and consumed by residents of country i,

$$P_{i,j,t}=\left(\left(\frac{1}{a_j-a_{j-1}}\right)\int_{a_{j-1}}^{a_j}p_{i,j,t}(\omega)^{1-\sigma}d\omega\right)^{\frac{1}{1-\sigma}}\qquad j=1,2,3\ .$$

The solution to the cost-minimization problem gives the demand functions for the underlying goods

$$c_{i,j,t}(\omega) = \phi_{i,j,t}\left(\frac{p_{i,j,t}(\omega)}{P_{i,j},t}\right)^{-\sigma} C_{i,t}, \tag{21}$$

where

$$\phi_{i,j,t} = \left\{\left(\frac{d_{i,j}}{a_j - a_{j-1}}\right)\left(\frac{P_{i,j,t}}{P_{i,t}}\right)^{-\mu}\right. .$$

The Firm's Problem

Firms in country j have access to a linear (in labor input) technology

$$y_{j,t}(\omega) = A_{j,t} n_{j,t}(\omega), \tag{22}$$

where $A_{j,t}$ is an economy-wide technology shock and $n_{j,t}(\omega)$ is the labor input into producing commodity ω.[8] The firm's output is demand determined so that

$$y_{j,t}(\omega) = c_{1,j,t}(\omega) + c_{2,j,t}(\omega) + c_{3,j,t}(\omega). \tag{23}$$

Real current period profits for a firm in country j is

$$\Pi_{j,t}(\omega) = \sum_{i=1}^{3} \frac{e_{j,i,t}}{P_{j,t}} p_{i,j,t}(\omega) c_{i,j,t}(\omega) - \frac{W_{j,t}}{P_{j,t}} n_{j,t}(\omega). \tag{24}$$

Substituting $n_{j,t}(\omega)$ from eq. (22), $y_{j,t}(\omega)$ from eq. (23) and the individual goods demands from eq. (21) into eq. (24) gives current period profits as

$$\Pi_{j,t}(\omega) = \sum_{i=1}^{3}\left(\frac{e_{j,i,t}}{P_{j,t}} p_{i,j,t}(\omega) - \frac{W_{j,t}}{A_{j,t}P_{j,t}}\right)\phi_{i,j,t}\left(\frac{p_{i,j,t}(\omega)}{P_{i,j,t}}\right)^{-\sigma} C_{i,t}. \tag{25}$$

Firms engage in local-currency pricing and prices are sticky in the sense of Calvo (1983). As in Benigno (2004), we allow heterogeneity in price stickiness by country of origin. That is, firms of Country j can reset prices of all its sales (whether they be domestic sales or exports) with probability $(1 - \alpha_j)$. A firm in country j who is chosen to reset price this period does so to maximize

$$E_t \sum_{k=0}^{\infty} (\alpha_j \beta)^k C_{j,t+k}^{-\gamma_1} \Pi_{j,t+k}(\omega),$$

subject to (25). Rearrangement of the first-order condition gives the optimal price for the country j firm who sells its product in country i as

$$p_{i,j,t}^{*}(\omega) = \frac{\sigma}{(\sigma - 1)} \frac{E_t \sum_{k=0}^{\infty} (\alpha_j \beta)^k C_{i,t+k} C_{j,t+k}^{-\gamma_1} \phi_{i,j,t+k} P_{i,j,t+k}^{\sigma} \frac{W_{j,t+k}}{A_{j,t+k}P_{j,t+k}}}{E_t \sum_{k=0}^{\infty} (\alpha_j \beta)^k C_{i,t+k} C_{j,t+k}^{-\gamma_1} \phi_{i,j,t+k} P_{i,j,t+k}^{\sigma} \frac{e_{j,i,t+k}}{P_{j,t+k}}}. \tag{26}$$

With the fraction $(1 - \alpha_j)$ of the firms resetting price to the value of $p_{i,j,t}^{*}(\omega)$ and the fraction α_j that maintain price at the previous level, the price index of goods produced in j and sold in i evolves according to

$$P_{i,j,t}^{(1-\sigma)} = (1 - \alpha_j)\left(p_{i,j,t}^{*}\right)^{(1-\sigma)} + \alpha_j P_{i,j,t-1}^{(1-\sigma)}. \tag{27}$$

[8] National output is $Y_{1,t} = \int_0^{a_1} y_{1,t}(\omega)\,d\omega$, $Y_{2,t} = \int_{a_1}^{a_2} y_{2,t}(\omega)\,d\omega$, and $Y_{3,t} = \int_{a_2}^{1} y_{3,t}(\omega)\,d\omega$.

Monetary Policy

The monetary authorities conduct policy through interest rate reaction functions. We include the lagged interest rate to model interest rate smoothing by the authorities. The form of the rule (28) follows Benigno (2004) and Steinsson (2008). We consider two variants of policy, conduct–independent and managed float. Under *independence*, countries pursue only domestic objectives so that authorities in country $j = 1, 2, 3$ set their interest rates according to

$$i_{j,t} = \delta + \rho i_{j,t-1} + \lambda E_t \pi_{j,t+1} + \phi \tilde{y}_{j,t} + \epsilon_{j,t}. \tag{28}$$

In a second variant, which we'll refer to as "managed float," Countries 1 and 3 run policy according to eq.(28) but Country 2's policy function also includes the real exchange rate between 2 and 3,

$$i_{2,t} = \delta + \rho i_{2,t-1} + \lambda E_t \pi_{2,t+1} + \phi \tilde{y}_{2,t} + \sigma q_{2,3,t} + \epsilon_{2,t}, \tag{29}$$

which is the form studied in Section 1.1. Here, purchasing power parity is viewed as the equilibrium for the nominal exchange rate and the authorities intervene to stabilize the exchange rate against Country 3 around the PPP value.

Equilibrium

Equilibrium requires that national outputs, $Y_{j,t}$, be consumed,

$$Y_{j,t} = \int_{a_{j-1}}^{a_j} y_{j,t}(\omega)\, d\omega = \sum_{i=1}^{3} C_{i,j,t} \qquad j = 1, 2, 3 \ , \tag{30}$$

and labor supply be allocated,

$$n_{j,t} = \int_{a_{j-1}}^{a_j} n_{j,t}(\omega)\, d\omega \qquad j = 1, 2, 3 \ . \tag{31}$$

An *equilibrium* for this economy is a collection of allocations for households of $C_{j,t}, n_{j,t}, M_{j,t}, B_j(s_t)$, allocations and prices for producers, $y_{j,t}$ and $p_{i,j,t}$, final goods prices, $P_{j,t}$, wages, $W_{j,t}$, and bond prices, $Q\left(s^{t+1}|s_t\right)$, such that the household allocations solve the household's problem, goods prices solve the producer's problem, market clearing conditions hold, and monetary policies are conducted as described above.

We specify the exogenous monetary shocks to be $\epsilon_{j,t} \overset{iid}{\sim} \left(0, \sigma_\epsilon^2\right)$ processes and the technology shocks to be univariate first-order autoregressive processes with no spill-overs,

$$ln\left(A_{j,t}\right) = \psi ln\left(A_{j,t-1}\right) + \nu_{j,t},$$

where $\nu_{j,t} \overset{iid}{\sim} (0, \sigma_\nu^2)$.

3.2 Solution Method and Parameterization

We take a first-order approximation around the zero-inflation steady state and then solve the model numerically. The derivation of the approximated model is given in the appendix. Our model parameterization, which is summarized in Table 4, draws on values used in the literature and assumes that a period is one quarter. The subjective discount factor is set at $\beta = 0.99$ which implies an annualized steady state interest rate of four percent. Preference parameters for consumption (γ_1), leisure (γ_2), and real money balances (γ_3) are all set at 2. There is no home bias in consumption ($d_{j,k} = \frac{1}{3}$ for $j, k = 1, 2, 3$). Also, following Benigno (2004), we set the elasticity of substitution between domestic and foreign goods (μ) to 1.5.

We begin with a benchmark specification of symmetry in price setting and policy rules. Here, we set $\alpha_j = 0.75$ for $j = 1, 2, 3$, which corresponds to firms updating prices, on average, once per year. To examine the impact of heterogeneity in price stickiness across countries, we assume the same degree of price-setting heterogeneity as Benigno (2004). His parameterizations are informed by estimates of U.S nominal price rigidity that range between 0.407 and 0.66 (Gali et al. (2001)) and estimates for the EMU area that range between 0.78 and 0.89 (Benigno and Lopez-Salido (2002). Using these studies as guidance, we set country 2 firms to experience the most price stickiness, followed by country 3, and lastly country 1 ($\alpha_2 = 0.89 > \alpha_3 = 0.65 > \alpha_1 = 0.407$).

The persistence of the exogenous productivity process (ψ) is set to 0.9. For the monetary policy rules we assume that the monetary policy interest rate is persistent, with a coefficient (ρ) equal to 0.95. We assume that the central bank adjusts the policy rate more than one-for-one with changes in inflation ($\lambda = 1.5$), but less than one-for-one with movements in the output gap ($\phi = 0.5$), as is standard in the literature. We set $\sigma = 0.5$, which is at the higher end of our empirical estimates.

4 Model Implied Third-Country Effects on the Bilateral Exchange Rate between Countries 1 and 2

In this section, we investigate the model's predictions regarding third-country effects on the exchange rate. The analysis proceeds in two parts. First, we conduct an impulse response analysis for the bilateral exchange rate between countries 1 and 2. We do this for both the nominal and real exchange rate. The question is whether or not third-country effects in the model can exert a quantitatively important effect on the exchange rate. Our primary interest is in observing the relative magnitude of the bilateral exchange rate response to shocks originating in country 3 compared to country 1 and country 2 shocks.

The second part of our analysis examines the extent to which the model can explain the empirical evidence on third-country contributions to the prediction regression $\bar{R}^2$ presented in section 2. To test our hypothesis that an important part of the *exchange rate disconnect puzzle* is due to omission of third country effects, we use the model as the data generating process in a Monte Carlo experiment. The equilibrium observations generated by the model are then used to generate $\bar{R}^2$ in the standard two-country and third-country augmented regressions. These experiments show that improvement in explanatory power by including third-country macroeconomic fundamentals is consistent with the improvements we found in the data.

Cross-country heterogeneity is necessary for third-country effects to be present. We consider two types of country heterogeneity. The first is, as in section 2, differences in the monetary policy rules. The second source of heterogeneity is in cross-country differences in the duration of nominal contracts or price stickiness.

4.1 Impulse Response Analysis

Environment I: Independent monetary policy; symmetric price setting. We present impulse responses in an environment of symmetric monetary policies and price stickiness to establish a benchmark set of results against which responses under asymmetries can be compared.[9] Since there are no differences across countries, these benchmark results for the exchange rate between 1 and 2 are identical to predictions that would be obtained from a two-country model under symmetry. Countries 1 and 2 have identical responses to any shocks originating in country 3. Since the country 3 shock does not induce any *relative* changes between 1 and 2, there is no effect on their bilateral exchange rate

Figure 1 shows impulse responses of the nominal exchange rate, $e_{1,2}$, and the real exchange rate, $q_{1,2}$, from a positive technology shock originating from each country. The favorable country 1 technology shock generates country 1 deflation as improved efficiency leads 1's firms to cut prices on home sales as well as on exports. Country 2 (and 3) experiences inflation as its firms raise prices in response to increasing demand. The divergent inflation responses lead the real interest rate to fall in 1 and to rise in 2. Relative consumption in 1 increases resulting in both real and nominal depreciations from Country 1's perspective (increases in $q_{1,2}$ and $e_{1,2}$).

Figure 2 shows impulse responses to monetary policy (tightening) shocks. Monetary tightening in 1 causes its currency to appreciate relative to 2. $e_{1,2,t}$ and $q_{1,2,t}$ fall upon impact and there is instantaneous overshooting of the nominal exchange rate. The initial policy shock is persistent on account of interest rate smoothing which keeps 1's real interest rate above 2's for several periods. This pushes consumption in 1 below consumption in 2 which results in a country 1 real and nominal appreciation relative to country 2. We note that these monetary policy shocks generate exchange rate overshooting.

Environment II. Managed float with symmetric price setting. We now introduce asymmetries in monetary policy rules by assuming that country 2 pursues a managed float against country 3's currency. Price stickiness across countries remains identical. We begin with exchange rate responses to technology shocks, which are shown in Figure 3.

A favorable technology shock in Country 1 produces the same responses from $e_{1,2}$ and $q_{1,2}$ as under environment I (independent policy, symmetric price setting). This is because the country 1 shock affects 2 and 3 identically and therefore has no effect on $q_{2,3}$. Hence, the fact that 2 manages its exchange rate against 3 is of no consequence for the exchange rate between 1 and 2.

A favorable technology shock in country 2 produces initial responses of $e_{1,2}$ and $q_{1,2}$ that are qualitatively the same as under environment I, but of smaller magnitude. Country 2's technology improvement lowers its marginal cost. Country 2 firms respond by lowering prices which leads to a period of deflation

[9] As mentioned earlier, shocks from Country 3 have no effect on the exchange rate between 1 and 2. Because the exchange rate depends on Country 2 variables *relative* to Country 1 variables, under symmetry, these variables respond in exactly the same way to Country 3 shocks which render the exchange rate unaffected.

in 2. Countries 1 and 3 experience inflation as their firms raise prices of domestic sales and of exports to each other in response to rising demand. The relatively low country 2 real interest rate and relatively high consumption in 2 relative to 3 generates a real country 2 depreciation (increase in $q_{2,3}$), to which the monetary authorities respond. This depreciation causes 2's interest rate to be higher than it would be if it were not managing the exchange rate. The managed float policy response attenuates the increase in country 2 consumption and therefore 2's currency depreciation against 1.

Under Environment II, third-country technology shocks can have measurable effects on the exchange rate. A favorable technology shock in country 3 produces an initial real and nominal appreciation of currency 1 relative to 2. The initial impact on the exchange rate between 1 and 2 is of the same order of magnitude as the impact effect of a country 2 technology shock. The country 3 technology shock generates country 3 deflation and increases its consumption. It also generates inflation in countries 1 and 2 (firms in 1 and 2 raise prices of domestic sales and exports to each other). This raises country 3 consumption above country 2 consumption and generates a real country 2 appreciation relative to 3 ($q_{2,3}$ falls). Part of the managed float policy response in 2 is to lower the interest rate whereas the real interest rate in 1 increases as monetary policy in 1 reacts primarily to increased inflation. As a result, consumption in 2 rises above consumption in 1 which generates a country 1 real and nominal appreciation relative to 2 (decrease in $q_{1,2}$ and $e_{1,2}$).

Figure 4 shows the exchange rate responses to monetary policy shocks. The responses of $e_{1,2}$ and $q_{1,2}$ following a country 1 monetary policy (tightening) shock are similar to the responses under Environment I. The tightening affects countries 2 and 3 symmetrically, which has no effect on $q_{2,3}$ and hence no difference in 2's policy response.

A country 2 monetary policy shock results in an initial responses of $e_{1,2}$ and $q_{1,2}$ that are dampened relative to the response under Environment I. Country 2's tightening initially generates a real appreciation in 2 relative to 1 and 3, but the decrease in $q_{2,3}$ causes 2's central bank to loosen. This subsequent loosening attenuates the effects of the initial shock on 2's exchange rate with 1.

A surprise country 3 monetary tightening generates the same qualitative response in $e_{1,2}$ and $q_{1,2}$ as that from a country 2 monetary tightening but the magnitude is larger. The tightening in 3 raises $q_{2,3}$ on impact. This causes 2 to raise its interest rate which leads to an appreciation of 2 relative to 1 (an increase in $e_{1,2}$ and $q_{1,2}$).

The third-country effects obtained thus far assume that shock volatility is the same across countries. Figure 5 shows how varying relative shock volatility of country 3 affects the volatility of the *depreciation* ($\Delta e_{1,2}$). To form a basis of comparison, we also show the effect of varying the volatility of country 2 shocks.[10] The effect of varying the size of technology shocks is shown on the figure on the left, the effect of varying policy shocks shown on the right. The line marked with symbols shows the relative increase in exchange rate depreciation volatility (measured as the standard deviation) for a relative increase in the volatility of country 2 technology shocks obtained from a symmetric two-country model. The solid line shows the analogous information when varying the volatility of Country 3's technology shock under

[10] This is comparing the contribution to exchange rate (depreciation) volatility by the third country in a three-country model to the contribution from a second country in a two-country model.

a country 2 managed float. At 0.0 on the horizontal axis, the volatility of 1 and 2's technology shocks are equal. At 1.0, 2's technology shock is twice as volatile as 1's. Doubling the importance (volatility) of country 3 is more than half as important as doubling the importance of country 2 in a two-country model.

For monetary policy, doubling the size of the policy shock volatility contributes less to exchange rate volatility than a doubling of technology shock volatility. However, we see a similar contribution to exchange rate volatility (between 1 and 2) generated by increasing country 3 policy volatility relative to increasing country 2 volatility in a two-country environment; it is about half the size.

Environment III. Independent monetary policies with asymmetric price stickiness. Here we assume different reset price probabilities but symmetric independent monetary policies (no exchange rate management). Prices are stickiest in country 2 and most flexible in country 1 ($\alpha_2 = 0.89 > \alpha_3 = 0.65 > \alpha_1 = 0.407$). Figure 6 shows the exchange rate responses to technology shocks. They are considerably different from those obtained under the symmetric price stickiness case of Environment I. The indirect channel created by price-stickiness asymmetries results in country 3 shocks generating $e_{1,2}$ and $q_{1,2}$ responses of the same order of magnitude as shocks originating in countries 1 and 2.

A country 1 technology shock creates a small impact effect on the real and nominal exchange rates but the delayed *appreciation* of 1's currency relative to 2 is relatively large. This contrasts with the response under Environment I where 1's technology shock initially led to a depreciation of 1 relative to 2. Under Environment I, country 2's real interest rate increases while country 1's real interest rate declines on impact. Higher country 1 consumption and lower country 2 consumption result in a country 1 depreciation (increase $q_{1,2}$) on impact. As seen on the left panel of Figure 7, under Environment III, the real interest rate in 2 declines initially and the resulting effect on country 2 consumption mitigates the initial effect on the exchange rate. r_2 declines because of the initial impact on inflation. Country 1 firms lower prices while country 2 and 3 firms initially raise prices. Initially, due to longer contract duration, relatively few country 2 firms can change prices. Even if the price of country 1 exports to 2 declined by the same extent as in Environment I, the lack of price movement by country 2 firms now leads to a larger initial deflation in 2.

A technology shock in country 2 generates responses of $e_{1,2}$ and $q_{1,2}$ that are both qualitatively and quantitatively similar to the responses under Environment I. Here, country 1's real interest rate increases upon impact (as under Environment I) because country 1 initially experiences some inflation. Country 1 and 3 firms increase prices, but due to sluggish price cuts from country 2 firms, the end result is inflation. The decline in country 2's real interest rate and the increase in country 1's real interest rate lead relative consumption levels and the exchange rate between 1 and 2 to respond similarly to the responses under Environment I.

The technology shock of primary interest is a shock to Country 3. Under Environment I, the favorable country 3 technology shock results in country 3 deflation and country 1 and 2 inflation. Here, country 1 firms raise prices while country 3 firms lower prices. There is, at least initially, relatively little price response by country 2 firms. Since country 2 firms do not raise prices on exports to 1 or 3 by much, the demand for 2's goods increase as does 2's output gap. Due to the endogenous monetary policy response, the real interest rate in country 2 is high relative to the rate in country 1. This raises country 1 consumption relative to country 2 and causes and $e_{1,2}$ and $q_{1,2}$ to increase. The impulse responses of the real interest rates to the country 3 shock are shown in the right panel of Figure 7.

Turning now to monetary policy shocks, the response of $e_{1,2}$ and $q_{1,2}$ to policy shocks originating in countries 1 and 2 are qualitatively the same as those obtained under the fully symmetric model of Environment I. However, a country 3 monetary tightening now generates a decline in $e_{1,2}$ and $q_{1,2}$. In the symmetric environment both r_1 and r_2 decrease by the same amount in response to 3's tightening. Here, with price-stickiness heterogeneities, r_2 declines by more than r_1, primarily because of differences in output gap responses. Because country 2's firm prices are the stickiest, it experiences the largest (negative) output gap whereas country 1 experiences the smallest gap. The endogenous monetary policy response in 1 and 2 is to lower the interest rate in 2 by more than in 1. Consumption in 2 is higher than consumption in 1 which implies a decline in $e_{1,2}$ and $q_{1,2}$.

The impulse response analysis has shown that direct and indirect pathways can lead third-country effects to have a measurable impact on the bilateral exchange rate. While country 3 shocks generally have a smaller effect than country 1 and 2 shocks, they are of similar orders of magnitude.

4.1.1 Exchange Rates and Third-Country Fundamentals

The impulse-response analysis shows how exogenous third-country shocks can generate bilateral exchange rate movements whereas the predictive regression evidence summarizes correlations among endogenous variables. This subsection revisits the predictive regression evidence and shows the extent to which our stylized three-country model can generate adjusted R^2 patterns that are found in the data.

We use the three-country model as the data-generating mechanism. We consider Environments II and III separately and also combine them (combining the managed float between Countries 2 and 3 with heterogeneity in price stickiness) in Environment IV. For each environment, we generate $N = 6000$ samples of length $T = 40$. The length of the time series is set to correspond to the number of quarterly observations used in our empirical work above. We compare the mean $\bar{R}^2$ from regressions of the exchange rate between 1 and 2 that condition only on variables from 1 and 2 and regressions that also condition on Country 3 variables.

Table 5 reports the results for deviation from PPP exchange rate regressions (eqs. (1) and (2)). At the one-period horizon, the $\bar{R}^2$ from regressing only on the deviation from PPP between countries 1 and 2, while not negative (as in the data), are relatively small and are in the neighborhood of the $\bar{R}^2$ for Korea and Sweden.

Augmenting the regression with the country 1 and 3 deviation from PPP raises the mean $\bar{R}^2$ by an average (across environments) from 0.014 to 0.06, which is about the same relative improvement as seen in the data. The results are not very sensitive to the environment.

At the four-period horizon, the $\bar{R}^2$ from eq.(1) that regresses only on the country 1 and 2 deviation from PPP conforms to the larger $\bar{R}^2$ values from the data (e.g., Great Britain, Indonesia, Japan, Korea, Sweden). The average value of 0.156 is somewhat larger than the 0.091 average value from the data. Augmenting the regression with the country 1 and 3 deviation from PPP raises the average $\bar{R}^2$ to 0.343, which is similar to the increased $\bar{R}^2$ found in the data. The results at the four-period horizon are also consistent across the different environments considered.

Table 6 shows the Monte Carlo results for the Taylor-rule fundamentals regressions (5) and (6). At the one-period horizon, the $\bar{R}^2$ of 0.061 in regressions that use only country 1 and 2 Taylor-rule fundamentals lies near the data under environment II. Under environments III and IV, the $\bar{R}^2$ values are somewhat larger and lie at the higher end of those observed in the data (e.g., Australia's $\bar{R}^2$ of 0.155). Augmenting the regression with country 3 Taylor-rule fundamentals achieves relatively larger increases in the $\bar{R}^2$ under environments III and IV. Averaging over the three environments, increases in the $\bar{R}^2$ from 0.11 to 0.16.

At the four-period horizon, the $\bar{R}^2$ also is smaller under Environment II and largest under Environment IV. Similarly, the gain in the $\bar{R}^2$ from augmenting the regression with country 3 variables is smallest under Environment II and largest under Environment IV. The average $\bar{R}^2$ from the unaugmented regressions of 0.32 is somewhat larger than the average value of 0.24 from the data. Augmenting the regression yields an average $\bar{R}^2$ value of 0.47.

5 Conclusion

This paper addresses the dimension of the exchange rate disconnect puzzle concerned with low explanatory power of macroeconomic fundamentals for exchange rate movements. We frame the puzzle in terms of the adjusted R^2 in exchange rate predictive regression. These regressions typically run the change in the (log) bilateral exchange rate between countries 1 and 2 on macro variables only from countries 1 and 2. The characteristicly low adjusted R^2 from these regressions could mean that exchange rate movements are inherently unexplainable (as argued in Engel and West (2005)) or that there are important omitted variables that have yet to be identified. Drawing on recent factor analyses on exchange rates, we consider the latter possibility and look to observations from third-country sources as potential omitted variables.

Third-country shocks were found to generate movements in the exchange rate between countries 1 and 2 if country level heterogeneity lead them to respond in different ways to the third-country shocks. We examined heterogeneity in monetary policy rules and in the duration of nominal price stickiness. Of course, there may be additional sources of cross-country heterogeneity that lead to third-country effects – differences in financial development, taxation, or labor market flexibility – not captured in our model. But even the limited menu of country heterogeneity that we consider the model to provides a reasonable accounting for the empirical evidence on the contribution of third-country effects.

Our analysis makes some progress towards resolving the disconnect puzzle, but is far from a complete solution (we do not obtain $\bar{R}^2$ values near 1). While we confine our study to the role of macroeconomic fundamentals in a very standard context, we acknowledge that there is room for microstructure considerations (e.g., Lyons, 2001) and non-fundamental influences (e.g., Mark and Wu, 1998, Jeanne and Rose, 2002, and Evans, 2012) to further improve our understanding of exchange rate movements.

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Table 1: $\bar{R}^2$ of Deviation from PPP Exchange Rate Regressions

	Deviation from PPP relative to			
	USD	USD & euro	USD & yen	USD & SF
	(1)	(2)	(3)	(4)
		A: One-period horizon		
Australia	-0.020	-0.040	-0.025	**0.018****
Brazil	-0.016	**0.097****	-0.036	**0.088****
Canada	-0.015	**-0.006**	-0.030	**0.013***
Denmark	-0.003	**0.000**	**0.124****	-0.024
Great Britain	0.042	0.024	**0.061****	0.024
Indonesia	-0.010	**-0.007**	-0.010	**0.027**
Japan	0.045	0.035		0.033
Korea	0.013	-0.006	-0.004	-0.004
New Zealand	-0.008	**0.111****	**0.093****	-0.026
Norway	-0.004	**0.033****	**0.008**	**0.054****
Philippines	-0.021	**0.038**	**0.070****	**0.029**
Singapore	-0.021	-0.029	**0.045****	-0.025
Sweden	0.016	0.010	**0.058****	**0.019**
Switzerland	-0.018	-0.033	**0.041****	
Thailand	-0.016	**0.000**	**0.065****	**-0.008**
		B: Four-period horizon		
Australia	-0.006	-0.024	**0.115***	**0.065***
Brazil	0.038	**0.488****	0.032	**0.429****
Canada	0.009	**0.042**	**0.013**	**0.100****
Denmark	0.070	**0.126**	**0.485****	0.062
Great Britain	0.250	**0.254**	**0.346****	0.243
Indonesia	0.216	**0.273**	**0.336****	**0.324***
Japan	0.199	**0.219**		**0.258****
Korea	0.244	0.243	**0.249**	**0.266**
New Zealand	0.057	**0.417****	**0.478****	0.067
Norway	0.070	0.061	**0.239****	0.064
Philippines	-0.008	**0.146****	**0.226****	**0.163****
Singapore	0.003	**0.095***	**0.147****	**0.124****
Sweden	0.211	**0.305****	**0.412****	**0.334****
Switzerland	0.017	-0.001	**0.256****	
Thailand	-0.013	**0.041**	**0.297****	**0.069**

Note: Bold face entries indicate that addition of third-country variables increase $\bar{R}^2$. * (**) indicates coefficient on third-country variable is significant at 10% (5%) level.

Table 2: $\bar{R}^2$ of Taylor-Rule Exchange Rate Regressions

	Taylor-Rule Fundamentals of Home Country and			
	US	US & Euro	US & Japan	US & Switz.
	(1)	(2)	(3)	(4)
		A: One-period horizon		
Australia	0.155	0.152	0.151	**0.248***
Brazil	0.068	0.060	0.030	**0.126***
Canada	0.079	**0.084**	**0.144**	**0.115***
Denmark	-0.041	**0.177***	-0.086	-0.045
Great Britain	0.099	**0.155***	0.078	**0.111**
Indonesia	0.023	-0.012	**0.078**	**0.041***
Japan	0.011	**0.038**		-0.030
Korea	0.142	0.120	**0.143***	**0.241****
New Zealand	0.136	**0.174****	**0.138**	**0.175***
Norway	0.066	**0.079**	0.058	0.027
Philippines	-0.047	**0.072***	-0.077	-0.051
Singapore	-0.043	**0.037***	-0.050	**0.046***
Sweden	0.023	**0.100****	0.002	**0.071***
Switzerland	-0.073	**0.169***	-0.114	
Thailand	0.054	0.045	0.034	0.021
		B: Four-period horizon		
Australia	0.356	**0.373**	0.355	**0.362***
Brazil	-0.017	**0.053**	**0.001**	**0.253***
Canada	0.491	**0.494**	0.468	**0.738***
Denmark	0.208	**0.278***	**0.265***	**0.249***
GreatBritain	0.270	**0.333***	**0.336***	**0.275**
Indonesia	0.396	0.382	0.386	0.380
Japan	-0.013	**0.045**		-0.063
Korea	0.479	0.457	**0.485****	**0.547***
New Zealand	0.381	**0.452***	**0.383**	**0.412***
Norway	0.323	**0.426***	0.309	**0.338**
Philippines	-0.003	**0.112***	**0.061**	**0.135**
Singapore	0.095	**0.157***	0.073	**0.164***
Sweden	0.315	0.287	0.291	**0.458****
Switzerland	0.085	**0.295***	**0.152***	
Thailand	0.182	0.167	0.160	**0.192**

Note: Bold face entries indicate that addition of third-country variables increase $\bar{R}^2$. * (**) indicates at least one coefficient on a third-country variable is significant at the 10% (5%) level.

Table 3: Cross-Rate Management.

Estimates of equation management coefficient in eq.(17).

Newey-West t-ratios in parentheses

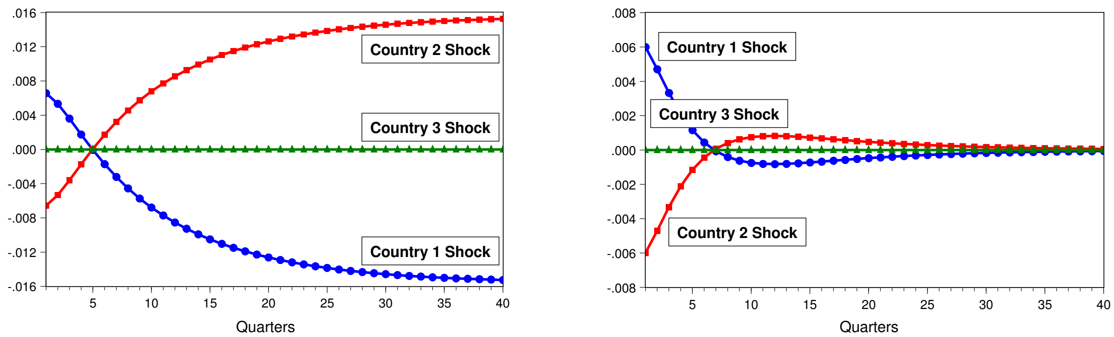
Country	Cross rate	σ	Country	Cross rate	σ
A.			Deviation from PPP		
Brazil	euro	0.301**	New Zealand	euro	0.295
		(3.664)			(1.124)
	SF	0.359**	Norway	yen	0.013*
		(4.573)			(1.885)
	yen	0.267**	Philippines	euro	0.103**
		(4.780)			(3.260)
Canada	euro	0.084*		yen	0.069**
		(1.849)			(2.399)
Denmark	euro	0.769**	Switzerland	euro	0.065**
		(4.616)			(3.141)
UK	yen	0.047*	Thailand	euro	0.124
		(1.844)			(1.531)
Indonesia	yen	0.072**	Singapore	yen	0.160
		(2.320)			(1.234)
Japan	euro	0.016**	Sweden	yen	0.034*
		(3.274)			(1.695)
Korea	euro	0.295			
		(1.124)			
B.			Real Depreciation		
Australia	euro	0.073**			
		(3.315)			

Notes: * (**) indicates significance at the 10 (5) percent level.

Table 4: Parameterization

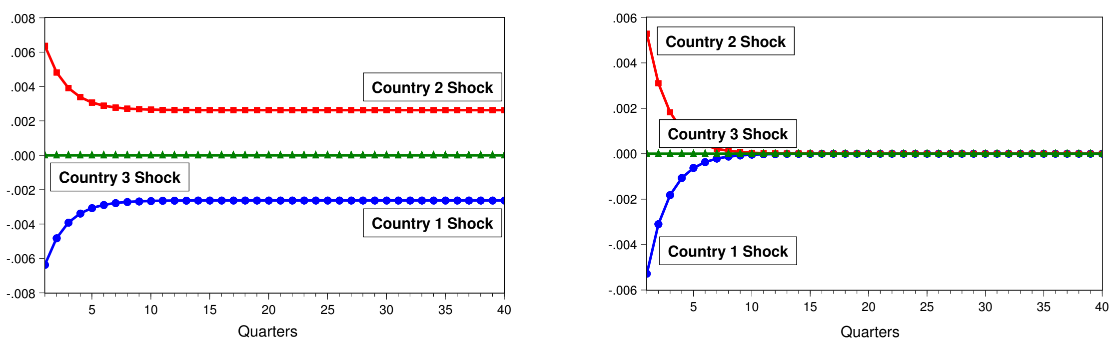
	Preferences			Symmetric Price Setting			Monetary Policy	
γ_1	2		α_j	0.75	$(j = 1, 2, 3)$	ρ	0.95
γ_2	2					λ	1.5
γ_3	2		Asymmetric Price Setting			ϕ	0.5
$d_{i,j}$	$\frac{1}{3}$	$(i, j = 1, 2, 3)$	α_1	0.407		σ	0.5
β	0.99		α_2	0.890			
			α_3	0.650		Technology	
						ψ	0.9

Figure 1. Exchange rate response to technology shock under Environment I (independent policy, symmetric stickiness)



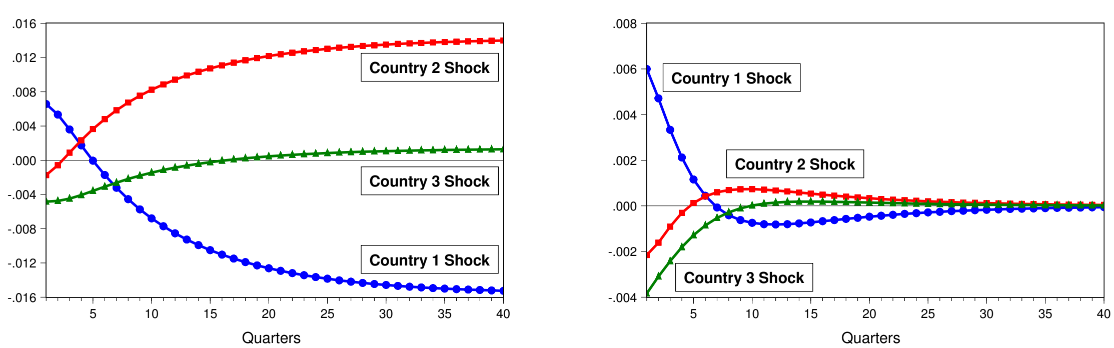
Nominal exchange rate ($e_{1,2}$) response Real exchange rate ($q_{1,2}$) response

Figure 2. Exchange rate response to monetary policy shock under Environment I (independent policy, symmetric stickiness)



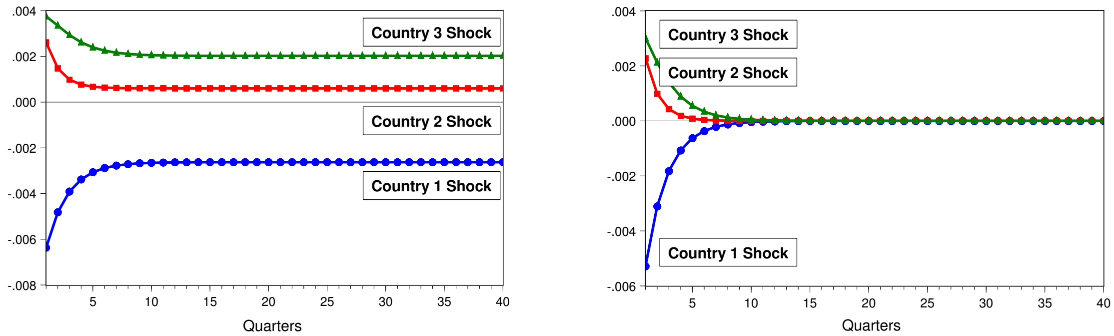
Nominal exchange rate ($e_{1,2}$) response Real exchange rate ($q_{1,2}$) response

Figure 3. Exchange rate response to technology shock under Environment II (managed float between 2 and 3, symmetric stickiness)



Nominal exchange rate ($e_{1,2}$) response Real exchange rate ($q_{1,2}$) response

Figure 4: Exchange rate response to policy shock under Environment II
(managed float between 2 and 3, symmetric stickiness)



Nominal exchange rate ($e_{1,2}$) response

Real exchange rate ($q_{1,2}$) response

Figure 5: Volatility of $\Delta e_{1,2}$ and relative volatility of Country 2 and 3 shocks

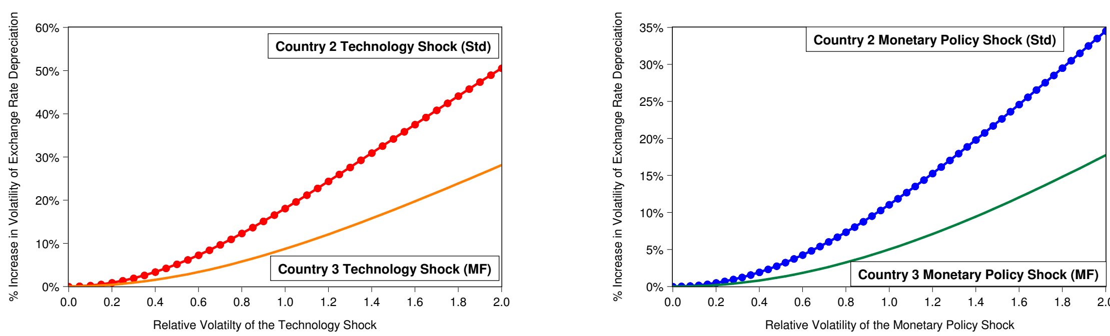
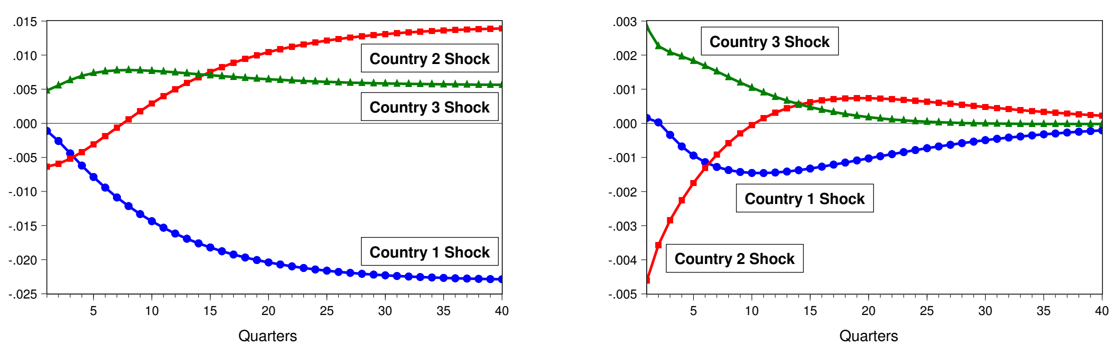


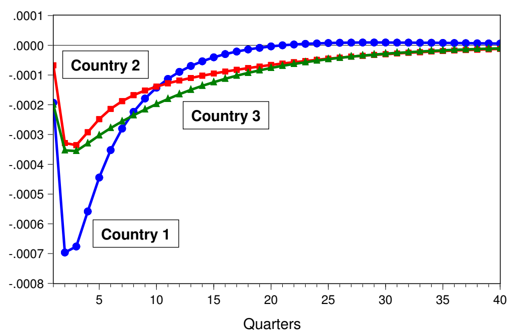
Figure 6: Exchange rate response to technology shocks under Environment III
(independent policy, heterogeneous stickiness by origin)



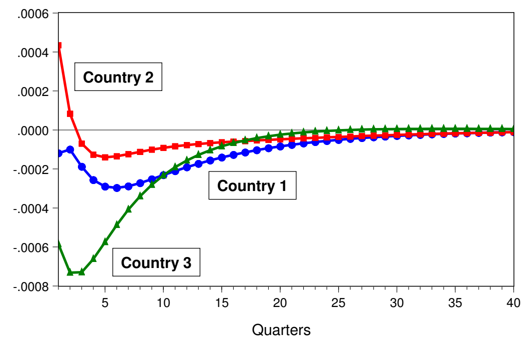
Nominal exchange rate ($e_{1,2}$) response

Real exchange rate ($q_{1,2}$) response

Figure 7: Real interest rate response to technology shock under Environment III (independent policy, heterogeneous stickiness by origin)

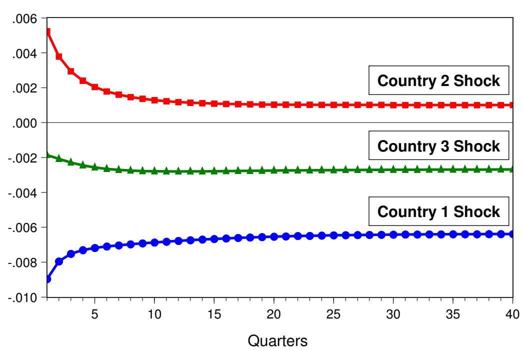


Response to Country 1 shock

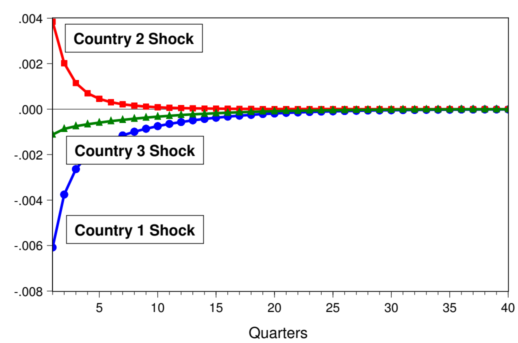


Response to Country 3 shock

Figure 8: Exchange rate response to monetary policy shocks under Environment III (independent policy, heterogeneous stickiness by origin)



Nominal exchange rate ($e_{1,2}$) response



Real exchange rate ($q_{1,2}$) response

Table 5: Monte Carlo Median $\bar{R}^2$ from PPP Exchange Rate Regressions

				Deviation from PPP relative to	
				Country 1	Countries 1 and 3
Horizon	Environment	Policy	Stickiness	$\bar{R}^2$	$\bar{R}^2$
1	II	managed float	symmetric	0.018	0.059
1	III	independent	asymmetric	0.014	0.057
1	IV	managed float	asymmetric	0.012	0.065
4	II	managed float	symmetric	0.171	0.349
4	III	independent	asymmetric	0.157	0.337
4	IV	managed float	asymmetric	0.140	0.345

Table 6: Monte Carlo Median $\bar{R}^2$ from Taylor-Rule Exchange-Rate Regressions

		Taylor-Rule Fundamentals of Country 2 and	
		Country 1	Countries 1 and 3
Horizon	Environment	$\bar{R}^2$	$\bar{R}^2$
1	II	0.061	0.089
1	III	0.122	0.170
1	IV	0.155	0.206
4	II	0.249	0.337
4	III	0.335	0.520
4	IV	0.369	0.538

Appendix (not intended for publication)

The first section of the appendix gives the derivations of the first-order approximation of the model around the deterministic and zero-inflation steady state. The second section reports reports estimates and robust t-ratios of all coefficients in monetary policy rules.

Equations of the Model

We begin with a listing of the equations of the model and their first-order approximations around the steady state. This is followed up by the derivations. To simplify the notation, we drop the explicit dependence on the state. For $i, j = 1, 2, 3$, we have from the consumer's problem,

$$\theta_2 (1 - n_{j,t})^{-\gamma_2} = \frac{W_{j,t}}{P_{j,t}} (C_{j,t})^{-\gamma_1} \tag{32}$$

$$\theta_3 \left(\frac{M_{j,t}}{P_{j,t}}\right)^{-\gamma_3} = \frac{i_{j,t}}{1 + i_{j,t}} (C_{j,t})^{-\gamma_1} \tag{33}$$

$$\frac{1}{1 + i_{j.t}} = \beta E_t \left(\frac{C_{j,t+1}}{C_{j,t}}\right)^{-\gamma_1} \left(\frac{P_{j,t}}{P_{j,t+1}}\right) \tag{34}$$

$$\frac{1}{1 + i_{1.t}} = \beta E_t \left(\frac{C_{j,t+1}}{C_{j,t}}\right)^{-\gamma_1} \left(\frac{P_{j,t}}{P_{j,t+1}}\right) \left(\frac{e_{i,j,t}}{e_{i,j,t+1}}\right) \tag{35}$$

$$q_{i,j.t} = \left(\frac{e_{i,j,t} P_{j,t}}{P_{i,t}}\right) = h_{i,j,0} \left(\frac{C_{j,t}}{C_{i,t}}\right)^{-\gamma_1} \tag{36}$$

$$e_{i,j,t} = \frac{q_{i,j,t} P_{i,t}}{P_{j,t}} \tag{37}$$

$$C_{j,t} = \left[\sum_{i=1}^{3} (d_{j,i})^{\frac{1}{\mu}} (C_{j,i,t})^{\left(\frac{\mu-1}{\mu}\right)}\right]^{\frac{\mu}{\mu-1}} \tag{38}$$

$$C_{i,j,t} = d_{i,j} \left(\frac{P_{i,j,t}}{P_{i,t}}\right)^{-\mu} C_{i,t} \tag{39}$$

$$c_{i,j,t}(\omega) = \begin{cases} \frac{d_{i,1}}{a_1} \left(\frac{p_{i,1,t}(\omega)}{P_{i,1,t}}\right)^{-\sigma} \left(\frac{P_{i,1,t}}{P_{i,t}}\right)^{-\mu} C_{i,t} & j = 1 \\ \frac{d_{i,2}}{a_2 - a_1} \left(\frac{p_{i,2,t}(\omega)}{P_{i,2,t}}\right)^{-\sigma} \left(\frac{P_{i,2,t}}{P_{i,t}}\right)^{-\mu} C_{i,t} & j = 2 \\ \frac{d_{i,3}}{1 - a_2} \left(\frac{p_{i,3,t}(\omega)}{P_{i,3,t}}\right)^{-\sigma} \left(\frac{P_{i,3,t}}{P_{i,t}}\right)^{-\mu} C_{i,t} & j = 3 \end{cases} \tag{40}$$

$$P_{j,t} = \left[\sum_{i=1}^{3} d_{j,i} P_{j,i}^{1-\mu}\right]^{\frac{1}{1-\mu}} \tag{41}$$

$$P_{i,j,t} = \begin{cases} \left(\frac{1}{a_1} \int_0^{a_1} p_{i,1}(\omega)^{(1-\sigma)} d\omega\right)^{\frac{1}{1-\sigma}} & j = 1 \\ \left(\frac{1}{a_2 - a_1} \int_{a_1}^{a_2} p_{i,2}(\omega)^{(1-\sigma)} d\omega\right)^{\frac{1}{1-\sigma}} & j = 2 \\ \left(\frac{1}{1 - a_2} \int_{a_2}^{1} p_{i,3}(\omega)^{(1-\sigma)} d\omega\right)^{\frac{1}{1-\sigma}} & j = 3 \end{cases} \tag{42}$$

From the firm's pricing problem, we have

$$
\begin{aligned}
& F_{i,j,t}\left(E_t\sum_{k=0}^{\infty}(\alpha_i\beta)^k C_{i,t+k}C_{j,t+k}^{-\gamma_1}\phi_{i,j,t+k}\left(\frac{P_{i,j,t+k}}{P_{i,j,t}}\right)^{\sigma-1}\left(\frac{P_{i,j,t+k}}{P_{i,t+k}}\right)q_{j,i,t+k}\right) \\
= \quad & \frac{\sigma}{(\sigma-1)}\left(E_t\sum_{k=0}^{\infty}(\alpha_i\beta)^k C_{i,t+k}C_{j,t+k}^{-\gamma_1}\phi_{i,j,t+k}\left(\frac{P_{i,j,t+k}}{P_{i,j,t}}\right)^{\sigma}\frac{W_{j,t+k}}{A_{j,t+k}P_{j,t+k}}\right) \qquad (43)
\end{aligned}
$$

where $F_{i,j,t}=\frac{p_{i,j,t}^*}{P_{i,j,t}}$.

$$
y_{j,t}(\omega)=A_{j,t}n_{j,t}(\omega) \qquad (44)
$$

We also have the market clearing conditions

$$
\begin{aligned}
Y_{1,t} &= \int_0^{a_1} y_{1,t}(\omega)\,d\omega = && C_{1,1,t}+C_{2,1,t}+C_{3,1,t} && (45)\\
Y_{2,t} &= \int_{a_1}^{a_2} y_{2,t}(\omega)\,d\omega = && C_{1,2,t,}+C_{2,2,t}+C_{3,2,t} && (46)\\
Y_{3,t} &= \int_{a_2}^{1} y_{3,t}(\omega)\,d\omega = && C_{1,3,t}+C_{2,3,t}+C_{3,3,t} && (47)
\end{aligned}
$$

$$
\sum_{j=1}^{3}Y_{j,t}=\sum_{j=1}^{3}C_{j,t} \qquad (48)
$$

$$
n_{j,t}=\begin{cases}\int_0^{a_1} n_{1,t}(\omega)\,d\omega & j=1\\ \int_{a_1}^{a_2} n_{2,t}(\omega)\,d\omega & j=2\\ \int_{a_2}^{1} n_{3,t}(\omega)\,d\omega & j=3\end{cases} \qquad (49)
$$

The first-order approximated model

$$
\begin{aligned}
\hat{W}_{j,t}-\hat{P}_{j,t} &= \gamma_1\hat{C}_{j,t}+\gamma_2\frac{n}{(1-n)}\hat{n}_{j,t} && (50)\\
\hat{M}_{j,t}-\hat{P}_{j,t} &= \frac{\gamma_1}{(1+\delta)\gamma_3}\hat{C}_{j,t}-\frac{(1-\delta)}{(1+\delta)\gamma_3}\hat{\imath}_{j,t} && (51)\\
\hat{C}_{j,t} &= E_t\left(\hat{C}_{j,t+1}-\frac{1}{\gamma_1}\left(\hat{\imath}_{j,t}-\hat{\pi}_{j,t+1}\right)\right) && (52)\\
\hat{C_{j,t}} &= E_t\left(\hat{C}_{j,t+1}-\frac{1}{\gamma_1}\left(\hat{\imath}_{1,t}-\Delta\hat{e}_{i,j,t+1}-\hat{\pi}_{j,t+1}\right)\right) && (53)
\end{aligned}
$$

$$
\hat{q}_{i,j,t}=-\gamma_1\left(\hat{C}_{j,t}-\hat{C_{i,t}}\right) \qquad (54)
$$

$$
\hat{e}_{i,j,t}=\hat{q}_{i,j,t}+\left(\hat{P}_{i,t}-\hat{P}_{j,t}\right) \qquad (55)
$$

$$
\hat{C}_{j,t}=\sum_{i=1}^{3}d_{j,i}\hat{C}_{j,i,t} \qquad (56)
$$

$$
\hat{C}_{i,j,t}=\mu\left(\hat{P}_{i,t}-\hat{P}_{i,j,t}\right)+\hat{C}_{i,t} \qquad (57)
$$

$$\hat{y}_{j,t} = \sum_{i=1}^{3} d_{i,j} \left(\mu \left(\hat{P}_{i,t} - \hat{P}_{i,j,t} \right) + \hat{C}_{i,t} \right) \tag{58}$$

$$\hat{P}_{j,t} = \sum_{i=1}^{3} d_{j,i} \hat{P}_{j,i,t} \tag{59}$$

$$\hat{y}_{j,t} = \hat{A}_{j,t} + \hat{n}_{j,t} \tag{60}$$

$$\pi_{i,j,t} = \frac{(1-\alpha_i)(1-\alpha_i\beta)}{\alpha_i} \left(\hat{m}_{j,t} - \hat{q}_{i,j,t} + \left(\hat{P}_{i,t} - \hat{P}_{i,j,t} \right) \right) + \beta E_t \pi_{i,j,t+1} \tag{61}$$

where $\hat{m}_{j,t} = \hat{W}_{j,t} - \hat{A}_{j,t} - \hat{P}_{j,t}$ is the first-order approximation of marginal cost for Country j firms.

$$\hat{P}_{i,t} - \hat{P}_{i,j,t} = \begin{cases} d_{i,2} \left(\hat{P}_{i,2,t} - \hat{P}_{i,3,t} \right) + (1-d_{i,1}) \left(\hat{P}_{i,3,t} - \hat{P}_{i,1,t} \right) & j = 1 \\ d_{i,1} \left(\hat{P}_{i,1,t} - \hat{P}_{i,3,t} \right) + (1-d_{i,2}) \left(\hat{P}_{i,3,t} - \hat{P}_{i,2,t} \right) & j = 2 \\ d_{i,1} \left(\hat{P}_{i,1,t} - \hat{P}_{i,3,t} \right) + d_{i,2} \left(\hat{P}_{i,2,t} - \hat{P}_{i,3,t} \right) & j = 3 \end{cases} \tag{62}$$

$$\left(\hat{P}_{i,j,t} - \hat{P}_{k,u,t} \right) = \left(\hat{P}_{i,j,t-1} - \hat{P}_{k,u,t-1} \right) + (\pi_{i,j,t} - \pi_{k,u,t}) \tag{63}$$

Household problem

Optimal behavior is characterized by the Euler equations

$$\theta_2 \left(1 - n_j \left(s^t\right)\right)^{-\gamma_2} = \frac{W_j \left(s^t\right)}{P_j \left(s^t\right)} C_j \left(s^t\right)^{-\gamma_1}, \tag{64}$$

$$\theta_3 \left(\frac{M_j \left(s^t\right)}{P_j \left(s^t\right)} \right)^{-\gamma_3} = \frac{i_j \left(s^t\right)}{1 + i_j \left(s^t\right)} C_j \left(s^t\right)^{-\gamma_1}, \tag{65}$$

$$Q \left(s_{t+1} | s^t\right) = \beta \pi \left(s_{t+1} | s^t\right) \left(\frac{C_j \left(s^{t+1}\right)}{C_j \left(s^t\right)} \right)^{-\gamma_1} \left(\frac{P_j \left(s^t\right)}{P_j \left(s^{t+1}\right)} \right) \left(\frac{e_{i,j} \left(s^t\right)}{e_{i,j} \left(s^{t+1}\right)} \right). \tag{66}$$

Because the state securities are denominated in Country 1 currency, using eq.(66), we obtain the consumption Euler equation,

$$\frac{1}{1 + i_j \left(s^t\right)} = \beta E_t \left(\frac{C_j \left(s^{t+1}\right)}{C_j \left(s^t\right)} \right)^{-\gamma_1} \left(\frac{P_j \left(s^t\right)}{P_j \left(s^{t+1}\right)} \right) \left(\frac{e_{1,j} \left(s^t\right)}{e_{1,j} \left(s^{t+1}\right)} \right). \tag{67}$$

Sum the pricing formula for the country 1 currency state-contingent bond (66) over states of nature to get (66) in the text,

$$\sum_{s_{t+1}} Q \left(s_{t+1} | s^t\right) = \frac{1}{1 + i_1 \left(s^t\right)} = \beta E_t \left(\frac{C_{j,t+1}}{C_{j,t}} \right)^{-\gamma_1} \left(\frac{P_{j,t}}{P_{j,t+1}} \right) \left(\frac{e_{i,j,t}}{e_{i,j,t+1}} \right).$$

For country 1, since $e_{11,t} = 1$, we have simply,

$$\frac{1}{1 + i_{1,t}} = \beta E_t \left(\frac{C_{1,t+1}}{C_{1,t}} \right)^{-\gamma_1} \left(\frac{P_{1,t}}{P_{1,t+1}} \right)$$

In addition, Countries 2 and 3 each have a non-traded, non-state contingent nominal bond in zero-net supply and the prices of these bonds are,

$$\begin{aligned}\frac{1}{1+i_{2,t}} &= \beta E_t\left(\frac{C_{2,t+1}}{C_{2,t}}\right)^{-\gamma_1}\left(\frac{P_{2,t}}{P_{2,t+1}}\right),\\ \frac{1}{1+i_{3,t}} &= \beta E_t\left(\frac{C_{3,t+1}}{C_{3,t}}\right)^{-\gamma_1}\left(\frac{P_{3,t}}{P_{3,t+1}}\right).\end{aligned}$$

The real exchange rate (**??**) is obtained by taking the price of the state-contingent security priced by agents in countries i and j

$$\left(\frac{C_i\left(s^{t+1}\right)}{C_i\left(s^t\right)}\right)^{-\gamma_1}\left(\frac{P_i\left(s^t\right)}{P_i\left(s^{t+1}\right)}\right)\left(\frac{e_{1,i}\left(s^t\right)}{e_{1,i}\left(s^{t+1}\right)}\right)=\left(\frac{C_j\left(s^{t+1}\right)}{C_j\left(s^t\right)}\right)^{-\gamma_1}\left(\frac{P_j\left(s^t\right)}{P_j\left(s^{t+1}\right)}\right)\left(\frac{e_{1,j}\left(s^t\right)}{e_{1,j}\left(s^{t+1}\right)}\right),$$

rearranging as

$$\left(\frac{e_{i,j}\left(s^{t+1}\right)P_j\left(s^{t+1}\right)}{P_i\left(s^{t+1}\right)}\right)\left(\frac{C_j\left(s^{t+1}\right)}{C_i\left(s^{t+1}\right)}\right)^{\gamma_1}=\left(\frac{e_{i,j}\left(s^t\right)P_j\left(s^t\right)}{P_i\left(s^t\right)}\right)\left(\frac{C_j\left(s^t\right)}{C_i\left(s^t\right)}\right)^{\gamma_1},$$

and performing repeated backward substitution to obtain

$$q_{i,j}\left(s^t\right)=\left(\frac{e_{i,j}\left(s^t\right)P_j\left(s^t\right)}{P_i\left(s^t\right)}\right)=h_{i,j,0}\left(\frac{C_j\left(s^t\right)}{C_i\left(s^t\right)}\right)^{-\gamma_1},$$

where $h_{i,j}\left(s^0\right)=\left(\frac{e_{i,j}\left(s^0\right)P_j\left(s^0\right)}{P_i\left(s^0\right)}\right)\left(\frac{C_j\left(s^0\right)}{C_i\left(s^0\right)}\right)^{\gamma_1}$.

The underlying demand system

The Lagrangian for the cost-minimization consumption allocation problem is

$$\begin{aligned}L &= \int_0^{a_1}p_{1,1}(\omega)\,c_{1,1}(\omega)+\int_{a_1}^{a_2}p_{1,2}(\omega)\,c_{1,2}(\omega)+\int_{a_2}^{1}p_{1,3}(\omega)\,c_{1,3}(\omega)\\ &\quad+\lambda\left[C_1-\left[(d_{1,1})^{\frac{1}{\mu}}(C_{1,1})^{\left(\frac{\mu-1}{\mu}\right)}+(d_{1,2})^{\frac{1}{\mu}}(C_{1,2})^{\left(\frac{\mu-1}{\mu}\right)}+(d_{1,3})^{\frac{1}{\mu}}(C_{1,3})^{\left(\frac{\mu-1}{\mu}\right)}\right]^{\frac{\mu}{\mu-1}}\right],\end{aligned}$$

with first-order conditions for optimality

$$\begin{aligned}\frac{\partial L}{\partial c_{1,1}(\omega)} &= p_{1,1}(\omega)-\lambda(d_{1,1})^{\frac{1}{\mu}}\left(\frac{1}{a_1}\right)^{\frac{1}{\sigma}}\left(\frac{C_1}{C_{1,1}}\right)^{\frac{1}{\mu}}\left(\frac{C_{1,1}}{c_{1,1}(\omega)}\right)^{\frac{1}{\sigma}}=0,\\ \frac{\partial L}{\partial c_{1,2}(\omega)} &= p_{1,2}(\omega)-\lambda(d_{1,2})^{\frac{1}{\mu}}\left(\frac{1}{a_2-a_1}\right)^{\frac{1}{\sigma}}\left(\frac{C_1}{C_{1,2}}\right)^{\frac{1}{\mu}}\left(\frac{C_{1,2}}{c_{1,2}(\omega)}\right)^{\frac{1}{\sigma}}=0,\\ \frac{\partial L}{\partial c_{1,3}(\omega)} &= p_{1,3}(\omega)-\lambda(d_{1,3})^{\frac{1}{\mu}}\left(\frac{1}{1-a_2}\right)^{\frac{1}{\sigma}}\left(\frac{C_1}{C_{1,3}}\right)^{\frac{1}{\mu}}\left(\frac{C_{1,3}}{c_{1,3}(\omega)}\right)^{\frac{1}{\sigma}}=0.\end{aligned}$$

Take ratios of the prices $p_{1,1}(\omega)$ and $p_{1,1}(\omega')$,

$$\frac{p_{1,1}(\omega)}{p_{1,1}(\omega')}=\left(\frac{c_{1,1}(\omega')}{c_{1,1}(\omega)}\right)^{\frac{1}{\sigma}}.$$

Solve for

$$c_{1,1}(\omega) = c_{1,1}(\omega')\left(\frac{p_{1,1}(\omega')}{p_{1,1}(\omega)}\right)^{\sigma}.$$

Substitute this into the definition of

$$\begin{aligned}
C_{1,1} &= \left(\frac{1}{a_1}\right)^{\frac{1}{\sigma-1}}\left(\int_0^{a_1} c_{1,1}(\omega)^{\frac{\sigma-1}{\sigma}}\,d\omega\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\frac{1}{a_1}\right)^{\frac{1}{\sigma-1}}\left(\int_0^{a_1}\left(c_{1,1}(\omega')\left(\frac{p_{1,1}(\omega')}{p_{1,1}(\omega)}\right)^{\sigma}\right)^{\frac{\sigma-1}{\sigma}}d\omega\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\frac{1}{a_1}\right)^{\frac{1}{\sigma-1}}\left(c_{1,1}(\omega')^{\frac{\sigma-1}{\sigma}}\,p_{1,1}(\omega')^{(\sigma-1)}\int_0^{a_1}\left(p_{1,1}(\omega)^{(1-\sigma)}\right)d\omega\right)^{\frac{\sigma}{\sigma-1}} \\
&= c_{1,1}(\omega')\,p_{1,1}(\omega')^{\sigma}\left(\frac{1}{a_1}\right)^{\frac{1}{\sigma-1}}\left(\int_0^{a_1} p_{1,1}(\omega)^{(1-\sigma)}\,d\omega\right)^{\left(\frac{\sigma}{\sigma-1}\right)} \\
&= c_{1,1}(\omega')\,p_{1,1}(\omega')^{\sigma}\left(\left(\frac{1}{a_1}\right)^{\frac{1}{\sigma}}\int_0^{a_1} p_{1,1}(\omega)^{(1-\sigma)}\,d\omega\right)^{\left(\frac{\sigma}{\sigma-1}\right)}
\end{aligned}$$

and now solve for

$$c_{1,1}(\omega') = \frac{C_{1,1}}{p_{1,1}(\omega')^{\sigma}\left(\left(\frac{1}{a_1}\right)^{\frac{1}{\sigma}}\int_0^{a_1} p_{1,1}(\omega)^{(1-\sigma)}\,d\omega\right)^{\left(\frac{\sigma}{\sigma-1}\right)}} \tag{68}$$

Substitute (68) into the expenditures for country 1 goods

$$\begin{aligned}
P_{1,1}C_{1,1} &= \int_0^{a_1} p_{1,1}(\omega)\,c_{1,1}(\omega) \\
&= \int_0^{a_1} p_{1,1}(\omega)\left(\frac{C_{1,1}}{p_{1,1}(\omega)^{\sigma}\left(\left(\frac{1}{a_1}\right)^{\frac{1}{\sigma}}\int_0^{a_1} p_{1,1}(\omega')^{(1-\sigma)}\,d\omega'\right)^{\left(\frac{\sigma}{\sigma-1}\right)}}\right)d\omega \\
&= \frac{C_{1,1}\left(\int_0^{a_1} p_{1,1}(\omega)^{(1-\sigma)}\,d\omega\right)}{\left(\left(\frac{1}{a_1}\right)^{\frac{1}{\sigma}}\int_0^{a_1} p_{1,1}(\omega')^{(1-\sigma)}\,d\omega'\right)^{\left(\frac{\sigma}{\sigma-1}\right)}}
\end{aligned}$$

Cancel out the $C_{1,1}$ on both sides,

$$\begin{aligned}
P_{1,1} &= \frac{\left(\int_0^{a_1} p_{1,1}(\omega)^{(1-\sigma)}\,d\omega\right)}{\left(\left(\frac{1}{a_1}\right)^{\frac{1}{\sigma}}\int_0^{a_1} p_{1,1}(\omega')^{(1-\sigma)}\,d\omega'\right)^{\left(\frac{\sigma}{\sigma-1}\right)}} \\
&= \frac{A}{\left(\left(\frac{1}{a_1}\right)^{\frac{1}{\sigma}}A\right)^{\frac{\sigma}{\sigma-1}}} = \frac{A^{1-\frac{\sigma}{\sigma-1}}}{\left(\frac{1}{a_1}\right)^{\frac{1}{\sigma}\frac{\sigma}{\sigma-1}}} = \frac{A^{\frac{-1}{\sigma-1}}}{\left(\frac{1}{a_1}\right)^{\frac{1}{\sigma-1}}} \\
&= \left(\frac{1}{a_1}\int_0^{a_1} p_{1,1}(\omega)^{(1-\sigma)}\,d\omega\right)^{\frac{1}{1-\sigma}}.
\end{aligned} \tag{69}$$

By symmetric arguments, obtain

$$
\begin{aligned}
c_{1,2}\left(\omega'\right) &= \frac{C_{1,2}}{p_{1,2}\left(\omega'\right)^{\sigma}\left(\left(\frac{1}{a_2-a_1}\right)^{\frac{1}{\sigma}}\int_{a_1}^{a_2} p_{1,2}\left(\omega\right)^{(1-\sigma)}d\omega\right)^{\left(\frac{\sigma}{\sigma-1}\right)}} \\
c_{1,3}\left(\omega'\right) &= \frac{C_{1,3}}{p_{1,3}\left(\omega'\right)^{\sigma}\left(\left(\frac{1}{1-a_2}\right)^{\frac{1}{\sigma}}\int_{a_2}^{1} p_{1,3}\left(\omega\right)^{(1-\sigma)}d\omega\right)^{\left(\frac{\sigma}{\sigma-1}\right)}}
\end{aligned}
$$

$$
\begin{aligned}
P_{1,2} &= \left(\frac{1}{a_2-a_1}\int_{a_1}^{a_2} p_{1,2}\left(\omega\right)^{(1-\sigma)}d\omega\right)^{\frac{1}{1-\sigma}} \\
P_{1,3} &= \left(\frac{1}{1-a_2}\int_{a_2}^{1} p_{1,3}\left(\omega\right)^{(1-\sigma)}d\omega\right)^{\frac{1}{1-\sigma}}
\end{aligned}
$$

Next, we obtain the demand functions (21). We work through the case for Country 1. Derivations for Countries 2 and 3 are analogous. If utility is the CES index of composite goods

$$
C_1 = \left[\left(d_{1,1}\right)^{\frac{1}{\mu}}\left(C_{1,1}\right)^{\left(\frac{\mu-1}{\mu}\right)} + \left(d_{1,2}\right)^{\frac{1}{\mu}}\left(C_{1,2}\right)^{\left(\frac{\mu-1}{\mu}\right)} + \left(d_{1,3}\right)^{\frac{1}{\mu}}\left(C_{1,3}\right)^{\left(\frac{\mu-1}{\mu}\right)}\right]^{\frac{\mu}{\mu-1}}
$$

it's well-known (e.g., Varian, p95, ed 1) the demand functions for the composite goods are

$$
\begin{aligned}
C_{1,1} &= d_{1,1}\left(\frac{P_{1,1}}{P_1}\right)^{-\mu}C_1 \qquad (70) \\
C_{1,2} &= d_{1,2}\left(\frac{P_{1,2}}{P_1}\right)^{-\mu}C_1 \\
C_{1,3} &= d_{1,3}\left(\frac{P_{1,3}}{P_1}\right)^{-\mu}C_1
\end{aligned}
$$

with price index

$$
P_1 = \left(d_{1,1}P_{1,1}^{1-\mu} + d_{1,2}P_{1,2}^{1-\mu} + d_{1,3}P_{1,3}^{1-\mu}\right)^{\frac{1}{1=\mu}}
$$

Substitute the expression for the price index $P_{1,1}$ (69) into the demand for $c_{1,1}\left(\omega'\right)$ (68),

$$
\begin{aligned}
c_{1,1}\left(\omega'\right) &= \frac{C_{1,1}}{p_{1,1}\left(\omega'\right)^{\sigma}\left(\left(\frac{1}{a_1}\right)^{\frac{1}{\sigma}}\int_{0}^{a_1} p_{1,1}\left(\omega\right)^{(1-\sigma)}d\omega\right)^{\left(\frac{\sigma}{\sigma-1}\right)}} \\
&= \frac{C_{1,1}}{p_{1,1}\left(\omega'\right)^{\sigma}\left(a_1\right)\left(P_{1,1}\right)^{-\sigma}} \\
&= \left(\frac{p_{1,1}\left(\omega'\right)}{P_{1,1}}\right)^{-\sigma}\frac{C_{1,1}}{a_1}
\end{aligned}
$$

Now use the expression for the demand for the composite good C_{11} (70) to obtain the individual good ω demand

$$
c_{1,1}\left(\omega\right) = \frac{d_{1,1}}{a_1}\left(\frac{p_{1,1}\left(\omega\right)}{P_{1,1}}\right)^{-\sigma}\left(\frac{P_{1,1}}{P_1}\right)^{-\mu}C_1
$$

This is eq. (21) in the text. Analogous steps deliver

$$
\begin{aligned}
c_{1,2}(\omega) &= \frac{d_{1,2}}{a_2 - a_1}\left(\frac{p_{1,2}(\omega)}{P_{1,2}}\right)^{-\sigma}\left(\frac{P_{1,2}}{P_1}\right)^{-\mu} C_1 \\
c_{1,3}(\omega) &= \frac{d_{1,3}}{1 - a_2}\left(\frac{p_{1,3}(\omega)}{P_{1,3}}\right)^{-\sigma}\left(\frac{P_{1,3}}{P_1}\right)^{-\mu} C_1
\end{aligned}
$$

Country 1 Firm's Problem

We solve the problem facing Country 1 firms. Analogous steps are used to obtain solutions for Country 2 and 3 firms. Firm ω has access to the technology

$$
y_{1,t}(\omega) = A_{1,t} n_{1,t}(\omega).
$$

The firm's output $y_{1,t}(\omega)$ is demand determined,

$$
y_{1,t}(\omega) = \underbrace{c_{1,1,t}(\omega)}_{\text{1's demand}} + \underbrace{c_{2,1,t}(\omega)}_{\text{2's demand}} + \underbrace{c_{3,1,t}(\omega)}_{\text{3's demand}}.
$$

We can write the firm's nominal revenues, in Country 1 currency units, as

$$
\text{Revenues} = p_{1,1,t}(\omega) c_{1,1,t}(\omega) + e_{1,2,t} p_{2,1,t}(\omega) c_{2,1,t}(\omega) + e_{1,3,t} p_{3,1,t}(\omega)
$$

Under this linear technology that depends only on labor, the firm's marginal cost function is a constant. The cost of one more worker is W_1/P_1. That worker produces A_1 units of output so the unit marginal cost (the cost of making one more unit of output) is $\frac{W_{1,t}}{A_{1,t}P_{1,t}}$. Hence, real profits are

$$
\begin{aligned}
\Pi_{1,t} &= \frac{e_{1,1,t}}{P_{1,t}} p_{1,1,t}(\omega) c_{1,1,t}(\omega) + \frac{e_{1,2,t}}{P_{1,t}} p_{2,1,t}(\omega) c_{2,1,t}(\omega) + \frac{e_{1,3,t}}{P_{1,t}} p_{3,1,t}(\omega) - \frac{W_{1,t}}{P_{1,t}} n_{1,t}(\omega) \\
&= \frac{e_{1,1,t}}{P_{1,t}} p_{1,1,t}(\omega) c_{1,1,t}(\omega) + \frac{e_{1,2,t}}{P_{1,t}} p_{2,1,t}(\omega) c_{2,1,t}(\omega) + \frac{e_{1,3,t}}{P_{1,t}} p_{3,1,t}(\omega) \\
&\quad - \frac{W_{1,t}}{P_{1,t}} \frac{\left(c_{1,1,t}(\omega) + c_{2,1,t}(\omega) + c_{3,1,t}(\omega)\right)}{A_{1,t}}
\end{aligned}
$$

Substitute the demand formulations (21) into the preceding to express time $t+k$ profits as

$$
\begin{aligned}
\Pi_{t+k}(\omega) &= \frac{e_{1,1,t+k}}{P_{1,t+k}} p_{1,1,t+k}(\omega) \phi_{1,1,t+k}\left(\frac{p_{1,1,t+k}(\omega)}{P_{1,1,t+k}}\right)^{-\sigma} C_{1,t+k} \\
&\quad - \frac{W_{1,t+k}}{A_{1,t+k}P_{1,t+k}} \phi_{1,1,t+k}\left(\frac{p_{1,1,t+k}(\omega)}{P_{1,1,t+k}}\right)^{-\sigma} C_{1,t+k} \\
&\quad + \frac{e_{1,2,t+k}}{P_{1,t+k}} p_{2,1,t+k}(\omega) \phi_{2,1,t+k}\left(\frac{p_{2,1,t+k}(\omega)}{P_{2,1,t+k}}\right)^{-\sigma} C_{2,t+k} \\
&\quad - \frac{W_{1,t+k}}{A_{1,t+k}P_{1,t+k}} \phi_{2,1,t+k}\left(\frac{p_{2,1,t+k}(\omega)}{P_{2,1,t+k}}\right)^{-\sigma} C_{2,t+k} \\
&\quad + \frac{e_{1,3,t+k}}{P_{1,t+k}} p_{3,1,t+k}(\omega) \phi_{3,1,t+k}\left(\frac{p_{3,1,t+k}(\omega)}{P_{3,1,t+k}}\right)^{-\sigma} C_{3,t+k} \\
&\quad - \frac{W_{1,t+k}}{A_{1,t+k}P_{1,t+k}} \phi_{3,1,t+k}\left(\frac{p_{3,1,t+k}(\omega)}{P_{3,1,t+k}}\right)^{-\sigma} C_{3,t+k}
\end{aligned}
$$

Calvo pricing. Firms are subject to Calvo pricing. α is the probability that the firm is stuck with last period's price, $(1-\alpha)$ is the probability that the firm can reset its price. We allow reset probabilities to vary across countries with α_1, α_2 and α_3 in countries 1,2, and 3, respectively. The firm waits to see if it is chosen to reset prices for products sent to country j. If chosen to reset prices in country 1, the expected present value of future profits from that price resetting decision is

$$\begin{aligned}
& E_t \sum_{k=0}^{\infty} (\alpha_1\beta)^k C_{1,t+k}^{-\gamma_1} \left(\begin{array}{c} \frac{e_{1,1,t+k}}{P_{1,t+k}} p_{1,1,t}^{*}(\omega)\, \phi_{1,1,t+k} \left(\frac{p_{1,1,t}^{*}(\omega)}{P_{1,1,t+k}}\right)^{-\sigma} C_{1,t+k} \\ -\frac{W_{1,t+k}}{A_{1,t+k}P_{1,t+k}} \phi_{1,1,t+k} \left(\frac{p_{1,1,t}^{*}(\omega)}{P_{1,1,t+k}}\right)^{-\sigma} C_{1,t+k} \end{array} \right) \\
= \; & E_t \sum_{k=0}^{\infty} (\alpha_1\beta)^k C_{1,t+k}^{1-\gamma_1} \phi_{1,1,t+k} P_{1,1,t+k}^{\sigma} \left(\begin{array}{c} \frac{e_{1,1,t+k}}{P_{1,t+k}} p_{1,1,t}^{*}(\omega)^{1-\sigma} \\ -\frac{W_{1,t+k}}{A_{1,t+k}P_{1,t+k}} p_{1,1,t}^{*}(\omega)^{-\sigma} \end{array} \right)
\end{aligned}$$

For country 2 price reset,

$$\begin{aligned}
& E_t \sum_{k=0}^{\infty} (\alpha_2\beta)^k C_{1,t+k}^{-\gamma_1} \left(\begin{array}{c} \frac{e_{1,2,t+k}}{P_{1,t+k}} p_{2,1,t}^{*}(\omega)\, \phi_{2,1,t+k} \left(\frac{p_{2,1,t}^{*}(\omega)}{P_{2,1,t+k}}\right)^{-\sigma} C_{2,t+k} \\ -\frac{W_{1,t+k}}{A_{1,t+k}P_{1,t+k}} \phi_{2,1,t+k} \left(\frac{p_{2,1,t}^{*}(\omega)}{P_{2,1,t+k}}\right)^{-\sigma} C_{2,t+k} \end{array} \right) \\
= \; & E_t \sum_{k=0}^{\infty} (\alpha_2\beta)^k C_{2,t+k} C_{1,t+k}^{-\gamma_1} \phi_{2,1,t+k} P_{2,1,t+k}^{\sigma} \left(\begin{array}{c} \frac{e_{1,2,t+k}}{P_{1,t+k}} p_{2,1,t}^{*}(\omega)^{1-\sigma} \\ -\frac{W_{1,t+k}}{A_{1,t+k}P_{1,t+k}} p_{2,1,t}^{*}(\omega)^{-\sigma} \end{array} \right)
\end{aligned}$$

and for country 3 price reset,

$$\begin{aligned}
& E_t \sum_{k=0}^{\infty} (\alpha_3\beta)^k C_{1,t+k}^{-\gamma_1} \left(\begin{array}{c} \frac{e_{1,3,t+k}}{P_{1,t+k}} p_{3,1,t}^{*}(j)\, \phi_{3,1,t+k} \left(\frac{p_{3,1,t}^{*}(j)}{P_{3,1,t+k}}\right)^{-\sigma} C_{2,t+k} \\ -\frac{W_{1,t+k}}{A_{1,t+k}P_{1,t+k}} \phi_{3,1,t+k} \left(\frac{p_{3,1,t}^{*}(j)}{P_{3,1,t+k}}\right)^{-\sigma} C_{3,t+k} \end{array} \right) \\
= \; & E_t \sum_{k=0}^{\infty} (\alpha_3\beta)^k C_{3,t+k} C_{1,t+k}^{-\gamma_1} \phi_{3,1,t+k} P_{3,1,t+k}^{\sigma} \left(\begin{array}{c} \frac{e_{1,3,t+k}}{P_{1,t+k}} p_{3,1,t}^{*}(j)^{1-\sigma} \\ -\frac{W1,t+k}{A_{1,t+k}P_{1,t+k}} p_{3,1,t}^{*}(j)^{-\sigma} \end{array} \right) \qquad (71)
\end{aligned}$$

Illustrate price reset by Country 1 firms

To fix ideas, we'll look at the firm's decision to reset prices on exports to Country 3. Differentiate (71) with respect to $p_{3,1,t}^{*}(w)$ and set result to 0 to obtain

$$0 = E_t \sum_{k=0}^{\infty} (\alpha_3\beta)^k C_{3,t+k} C_{1,t+k}^{-\gamma_1} \phi_{3,1,t+k} P_{3,1,t+k}^{\sigma} \left(\begin{array}{c} (1-\sigma) \frac{e_{1,3,t+k}}{P_{1,t+k}} p_{3,1,t}^{*}(j)^{-\sigma} \\ +\sigma \frac{W_{1,t+k}}{A_{1,t+k}P_{1,t+k}} p_{3,1,t}^{*}(j)^{-\sigma-1} \end{array} \right),$$

which rearranges to

$$\begin{aligned}
& p_{3,1,t}^{*} \left(E_t \sum_{k=0}^{\infty} (\alpha_3\beta)^k C_{3,t+k} C_{1,t+k}^{-\gamma_1} \phi_{3,1,t+k} P_{3,1,t+k}^{\sigma} \frac{e_{1,3,t+k}}{P_{1,t+k}} \right) \\
= \; & \frac{\sigma}{(\sigma-1)} \left(E_t \sum_{k=0}^{\infty} (\alpha_3\beta)^k C_{3,t+k} C_{1,t+k}^{-\gamma_1} \phi_{3,1,t+k} P_{3,1,t+k}^{\sigma} \frac{W_{1,t+k}}{A_{1,t+k}P_{1,t+k}} \right). \qquad (72)
\end{aligned}$$

Call the left hand side of eq.(72) lhs. Then,

$$\begin{aligned}
lhs &= p^*_{3,1,t}\left(E_t\sum_{k=0}^{\infty}(\alpha_3\beta)^k C_{3,t+k}C_{1,t+k}^{-\gamma_1}\phi_{3,1,t+k}P^{\sigma}_{3,1,t+k}\frac{e_{1,3,t+k}}{P_{1,t+k}}\right)\\
&= p^*_{3,1,t}\left(E_t\sum_{k=0}^{\infty}(\alpha_3\beta)^k C_{3,t+k}C_{1,t+k}^{-\gamma_1}\phi_{3,1,t+k}P^{\sigma}_{3,1,t+k}\frac{e_{1,3,t+k}}{P_{1,t+k}}\left(\frac{P_{3,1,t+k}}{P_{3,1,t+k}}\frac{P_{3,t+k}}{P_{3,t+k}}\right)\right)\\
&= p^*_{3,1,t}\left(E_t\sum_{k=0}^{\infty}(\alpha_3\beta)^k C_{3,t+k}C_{1,t+k}^{-\gamma_1}\phi_{3,1,t+k}P^{\sigma-1}_{3,1,t+k}\frac{P_{3,t+k}e_{1,3,t+k}}{P_{1,t+k}}\left(\frac{P_{3,1,t+k}}{P_{3,t+k}}\right)\right)\\
&= p^*_{3,1,t}\left(E_t\sum_{k=0}^{\infty}(\alpha_3\beta)^k C_{3,t+k}C_{1,t+k}^{-\gamma_1}\phi_{3,1,t+k}P^{\sigma-1}_{3,1,t+k}\left(\frac{P_{3,1,t+k}}{P_{3,t+k}}\right)q_{1,3,t+k}\right)
\end{aligned}$$

Put this back together with the right hand side of eq. (72),

$$\begin{aligned}
&p^*_{3,1,t}\left(E_t\sum_{k=0}^{\infty}(\alpha_3\beta)^k C_{3,t+k}C_{1,t+k}^{-\gamma_1}\phi_{3,1,t+k}P^{\sigma-1}_{3,1,t+k}\left(\frac{P_{3,1,t+k}}{P_{3,t+k}}\right)q_{1,3,t+k}\right)\\
= &\frac{\sigma}{(\sigma-1)}\left(E_t\sum_{k=0}^{\infty}(\alpha_3\beta)^k C_{3,t+k}C_{1,t+k}^{-\gamma_1}\phi_{3,1,t+k}P^{\sigma}_{3,1,t+k}\frac{W_{1,t+k}}{A_{1,t+k}P_{1,t+k}}\right)
\end{aligned}$$

Multiply both sides by $P^{-\sigma}_{3,1,t}$:

$$\begin{aligned}
&p^*_{3,1,t}P^{-\sigma}_{3,1,t}\left(E_t\sum_{k=0}^{\infty}(\alpha_3\beta)^k C_{3,t+k}C_{1,t+k}^{-\gamma_1}\phi_{3,1,t+k}P^{\sigma-1}_{3,1,t+k}\left(\frac{P_{3,1,t+k}}{P_{3,t+k}}\right)q_{1,3,t+k}\right)\\
= &\frac{\sigma}{(\sigma-1)}P^{-\sigma}_{3,1,t}\left(E_t\sum_{k=0}^{\infty}(\alpha_3\beta)^k C_{3,t+k}C_{1,t+k}^{-\gamma_1}\phi_{3,1,t+k}P^{\sigma}_{3,1,t+k}\frac{W_{1,t+k}}{A_{1,t+k}P_{1,t+k}}\right)
\end{aligned}$$

Multiply and divide the left hand side by $P_{3,1,t}$

$$\begin{aligned}
&p^*_{3,1,t}P^{-\sigma}_{3,1,t}\left(\frac{P_{3,1,t}}{P_{3,1,t}}\right)\left(E_t\sum_{k=0}^{\infty}(\alpha_3\beta)^k C_{3,t+k}C_{1,t+k}^{-\gamma_1}\phi_{3,1,t+k}P^{\sigma-1}_{3,1,t+k}\left(\frac{P_{3,1,t+k}}{P_{3,t+k}}\right)q_{1,3,t+k}\right)\\
= &\frac{\sigma}{(\sigma-1)}P^{-\sigma}_{3,1,t}\left(E_t\sum_{k=0}^{\infty}(\alpha_3\beta)^k C_{3,t+k}C_{1,t+k}^{-\gamma_1}\phi_{3,1,t+k}P^{\sigma}_{3,1,t+k}\frac{W_{1,t+k}}{A_{1,t+k}P_{1,t+k}}\right)\\
&\frac{p^*_{3,1,t}}{P_{3,1,t}}\left(E_t\sum_{k=0}^{\infty}(\alpha_3\beta)^k C_{3,t+k}C_{1,t+k}^{-\gamma_1}\phi_{3,1,t+k}\left(\frac{P_{3,1,t+k}}{P_{3,1,t}}\right)^{\sigma-1}\left(\frac{P_{3,1,t+k}}{P_{3,t+k}}\right)q_{1,3,t+k}\right)\\
= &\frac{\sigma}{(\sigma-1)}\left(E_t\sum_{k=0}^{\infty}(\alpha_3\beta)^k C_{3,t+k}C_{1,t+k}^{-\gamma_1}\phi_{3,1,t+k}\left(\frac{P_{3,1,t+k}}{P_{3,1,t}}\right)^{\sigma}\frac{W_{1,t+k}}{A_{1,t+k}P_{1,t+k}}\right)
\end{aligned}$$

To lighten the notation, define $F_{3,1,t}=\frac{p^*_{3,1,t}}{P_{3,1,t}}$. This gives

$$\begin{aligned}
&F_{3,1,t}\left(E_t\sum_{k=0}^{\infty}(\alpha_3\beta)^k C_{3,t+k}C_{1,t+k}^{-\gamma_1}\phi_{3,1,t+k}\left(\frac{P_{3,1,t+k}}{P_{3,1,t}}\right)^{\sigma-1}\left(\frac{P_{3,1,t+k}}{P_{3,t+k}}\right)q_{1,3,t+k}\right)\\
= &\frac{\sigma}{(\sigma-1)}\left(E_t\sum_{k=0}^{\infty}(\alpha_3\beta)^k C_{3,t+k}C_{1,t+k}^{-\gamma_1}\phi_{3,1,t+k}\left(\frac{P_{3,1,t+k}}{P_{3,1,t}}\right)^{\sigma}\frac{W_{1,t+k}}{A_{1,t+k}P_{1,t+k}}\right) \qquad (73)
\end{aligned}$$

Steady State

In the steady state, for Countries $j = 1, 2, 3$,

$$A_j = 1$$

$$P_j = P_{j,1} = P_{j,2} = P_{j,3} = p_{j,1}(\omega) = p_{j,2}(\omega) = p_{j,3}(\omega)$$

$$\begin{aligned}
i_j &= \delta \\
Y_j &= C_j = n_j \\
C_{j,k} &= d_{j,k} C_j = d_{j,k} n_j \\
\frac{W_j}{P_j} &= \theta_2 (1 - n_j)^{-\gamma_2} (n_j)^{\gamma_1} \\
\left(\frac{M_j}{P_j}\right)^{-\gamma_3} &= \frac{\delta}{1+\delta} \frac{(n_j)^{-\gamma_1}}{\theta_3}
\end{aligned}$$

The real exchange rate

$$q_{i,j} = \left(\frac{C_j}{C_i}\right)^{-\gamma_1} \left(q_{i,j,0} \left(\frac{C_{i,0}}{C_{j,0}}\right)^{-\gamma_1}\right) = \left(\frac{Y_j}{Y_i}\right)^{-\gamma_1} \left(q_0 \left(\frac{Y_{i,0}}{Y_{j,0}}\right)^{-\gamma_1}\right)$$

depends on initial conditions. We assume $\left(q_0 \left(\frac{Y_{i,0}}{Y_{j,0}}\right)^{-\gamma_1}\right) = 1$. Hence, with equal sized countries, the steady state value of the real exchange rate is 1.

The steady-state price of any good in units of the home currency is,

$$p^*_{i,j} \left(\frac{e_{j,i}}{P_j}\right) = \frac{\sigma}{(\sigma - 1)} \left(\frac{W_j}{P_j}\right)$$

which is a constant markup over nominal marginal cost. Since all firms in Country j will charge the same steady-state price on goods destined for Country i, we have

$$\left(\frac{W_j}{A_j P_j}\right) = \left(\frac{e_{j,3} p^*_{3,j}}{P_j}\right) \frac{(\sigma - 1)}{\sigma} = \left(\frac{e_{j,2} p^*_{2,j}}{P_j}\right) \frac{(\sigma - 1)}{\sigma} = \left(\frac{p^*_{j,j}}{P_j}\right) \frac{(\sigma - 1)}{\sigma}.$$

This implies all countries have identical steady-state marginal cost

$$\left(\frac{W_i}{A_i P_i}\right) = \frac{(\sigma - 1)}{\sigma}.$$

First-order approximation around the steady state

We omit the steps to obtain the first-order approximations of the household's Euler equations since they have been presented in the literature and the steps are straightforward. We provide more details on the steps to approximate the price-reset decisions by firms. The notation here is to use a hat to denote the log approximate deviation from the steady state, so if x is the steady state value of $x_{t,}$ we write $\hat{x}_t = \frac{(x_t - x)}{x}$. In obtaining the approximation, we will exploit two results.

Result 1. The first result is just the equivalence between a forward-looking first-order stochastic difference equation and its present value formulation,

$$x_t = (1-\beta) E_t \sum_{k=0} \beta^k y_{t+k} = (1-\beta) y_t + \beta E_t x_{t+1}. \tag{74}$$

Result 2.

$$\hat{F}_{3,1,t} = \frac{\alpha_3}{1-\alpha_3} \pi_{3,1,t}$$

To obtain this result, since $F_{3,1,t} = \frac{p^*_{3,1,t}}{P_{3,1,t}}$, it follows that

$$\hat{F}_{3,1,t} = \hat{p}^*_{3,1,t} - \hat{P}_{3,1,t} \tag{75}$$

Notice that

$$\begin{aligned}
P^{1-\sigma}_{3,1,t} &= \alpha P^{1-\sigma}_{3,1,t-1} + (1-\alpha) p^{*(1-\sigma)}_{3,1,t} \\
(1-\sigma) P^{1-\sigma}_{3,1} \hat{P}_{3,1,t} &= \alpha (1-\sigma) P^{1-\sigma}_{3,1} \hat{P}_{3,1,t-1} + (1-\alpha)(1-\sigma) p^*_{3,1} \hat{p}^*_{3,1,t} \\
\hat{P}_{3,1,t} &= \alpha \hat{P}_{3,1,t-1} + (1-\alpha) \hat{p}^*_{3,1,t} \\
(1-\alpha) \hat{p}^*_{3,1,t} &= \hat{P}_{3,1,t} - \alpha \hat{P}_{3,1,t-1} + \alpha \hat{P}_{3,1,t} - \alpha \hat{P}_{3,1,t} \\
&= (1-\alpha) \hat{P}_{3,1,t} + \alpha \pi_{3,1,t} \\
\hat{p}^*_{3,1,t} - \hat{P}_{3,1,t} &= \frac{\alpha}{1-\alpha} \pi_{3,1,t}
\end{aligned}$$

The result is then obtained by substituting this last line into (75).

To obtain the approximation for eq. (73) around the steady state, recall that

$$\begin{aligned}
\phi_{i,1,t} &= \left(\frac{d_{i,1}}{a_1}\right) \left(\frac{P_{i,1,t}}{P_{i,t}}\right)^{-\mu}, \\
\phi_{i,2,t} &= \left(\frac{d_{i,2}}{a_2 - a_1}\right) \left(\frac{P_{i,2,t}}{P_{i,t}}\right)^{-\mu}, \\
\phi_{i,3,t} &= \left(\frac{d_{i,3}}{1 - a_2}\right) \left(\frac{P_{i,3,t}}{P_{i.t}}\right)^{-\mu}.
\end{aligned}$$

Substitute these expressions into eq. (73) to obtain

$$\begin{aligned}
& F_{3,1,t} \left(E_t \sum_{k=0}^{\infty} (\alpha_3 \beta)^k C_{3,t+k} C^{-\gamma_1}_{1,t+k} \phi_{3,1,t+k} \left(\frac{P_{3,1,t+k}}{P_{3,1,t}}\right)^{\sigma-1} \left(\frac{P_{3,1,t+k}}{P_{3,t+k}}\right) q_{1,3,t+k} \right) \\
= \; & \frac{\sigma}{(\sigma-1)} \left(E_t \sum_{k=0}^{\infty} (\alpha_3 \beta)^k C_{3,t+k} C^{-\gamma_1}_{1,t+k} \phi_{3,1,t+k} \left(\frac{P_{3,1,t+k}}{P_{3,1,t}}\right)^{\sigma} \frac{W_{1,t+k}}{A_{1,t+k} P_{1,t+k}} \right).
\end{aligned}$$

Beginning with a typical term on the left hand side,

$$
\begin{aligned}
& C_{3,t+k} C_{1,t+k}^{-\gamma_1} \phi_{3,1,t+k} \left(\frac{P_{3,1,t+k}}{P_{3,1,t}} \right)^{\sigma-1} \left(\frac{P_{3,1,t+k}}{P_{3,t+k}} \right) q_{1,3,t+k} \\
= & \left(C_3 C_1^{-\gamma_1} \phi_{3,1} q_{1,3} \right) \left(\begin{array}{c} \hat{q}_{1,3,t+k} + \hat{\phi}_{3,1,t+k} + \hat{C}_{3,t+k} - \gamma_1 \hat{C}_{1,t+k} \\ + (\sigma - 1) \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,1,t} \right) \\ + \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k} \right) \end{array} \right) \\
= & \left(C_3 C_1^{-\gamma_1} \phi_{3,1} q_{1,3} \right) \left(\begin{array}{c} \hat{q}_{1,3,t+k} - \mu \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k} \right) + \hat{C}_{3,t+k} - \gamma_1 \hat{C}_{1,t+k} \\ + (\sigma - 1) \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,1,t} \right) + \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k} \right) \end{array} \right),
\end{aligned}
$$

where

$$
\hat{\phi}_{i,1} = -\mu \left(\hat{P}_{i,1,t} - \hat{P}_{i,t} \right).
$$

A typical term on the right hand side approximates to

$$
\begin{aligned}
& \frac{\sigma}{(\sigma - 1)} \left(C_{3,t+k} C_{1,t+k}^{-\gamma_1} \phi_{3,1,t+k} \left(\frac{P_{3,1,t+k}}{P_{3,1,t}} \right)^{\sigma} \frac{W_{1,t+k}}{A_{1,t+k} P_{1,t+k}} \right) \\
= & \frac{\sigma}{(\sigma - 1)} \left(C_{3,t+k} C_{1,t+k}^{-\gamma_1} \phi_{3,1,t+k} \left(\frac{P_{3,1,t+k}}{P_{3,1,t}} \right)^{\sigma} m_{1,t+k} \right) \\
= & \frac{\sigma}{(\sigma - 1)} \left(C_3 C_1^{-\gamma_1} \phi_{3,1} m_1 \right) \left(\hat{\phi}_{3,1,t+k} + \hat{C}_{3,t+k} - \gamma_1 \hat{C}_{1,t+k} + \hat{m}_{1,t+k} + \sigma \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,1,t} \right) \right) \\
= & \frac{\sigma}{(\sigma - 1)} \left(C_3 C_1^{-\gamma_1} \phi_{3,1} m_1 \right) \left(\hat{C}_{3,t+k} - \gamma_1 \hat{C}_{1,t+k} + \hat{m}_{1,t+k} + \sigma \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,1,t} \right) - \mu \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k} \right) \right)
\end{aligned}
$$

Putting them together gives,

$$
\begin{aligned}
& \sum_{k=0}^{\infty} (\alpha_3 \beta)^k \left(C_3 C_1^{-\gamma_1} \phi_{3,1} q_{1,3} \right) \hat{F}_{3,1,t} \\
& + \left(C_3 C_1^{-\gamma_1} \phi_{3,1} q_{1,3} \right) E_t \sum_{k=0}^{\infty} (\alpha_3 \beta)^k \left(\begin{array}{c} \hat{q}_{1,3,t+k} - \mu \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k} \right) + \hat{C}_{3,t+k} - \gamma_1 \hat{C}_{1,t+k} \\ + (\sigma - 1) \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,1,t} \right) + \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k} \right) \end{array} \right) \\
= & \frac{\sigma}{(\sigma - 1)} \left(C_3 C_1^{-\gamma_1} \phi_{3,1} m_1 \right) E_t \sum_{k=0}^{\infty} (\alpha_3 \beta)^k \left(\begin{array}{c} \hat{C}_{3,t+k} - \gamma_1 \hat{C}_{1,t+k} + \hat{m}_{1,t+k} \\ + \sigma \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,1,t} \right) - \mu \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k} \right) \end{array} \right)
\end{aligned}
$$

To lighten notation, define

$$
\begin{aligned}
b_q &= \left(C_3 C_1^{-\gamma_1} \phi_{3,1} q_{1,3} \right) \\
b_m &= \left(C_3 C_1^{-\gamma_1} \phi_{3,1} m_1 \right) \\
b &= \frac{b_m}{b_q} = \bar{m}_1 = \frac{(\sigma - 1)}{\sigma} \\
\alpha &= \alpha_3 \\
(b - 1) &= -\frac{1}{\sigma}
\end{aligned}
$$

Then

$$
\begin{aligned}
& \frac{\hat{F}_{3,1,t}}{1-\alpha\beta} \\
& +E_t \sum_{k=0}^{\infty} (\alpha_3\beta)^k \begin{pmatrix} \hat{q}_{1,3,t+k} - \mu\left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k}\right) + \hat{C}_{3,t+k} - \gamma_1 \hat{C}_{1,t+k} \\ +(\sigma-1)\left(\hat{P}_{3,1,t+k} - \hat{P}_{3,1,t}\right) + \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k}\right) \end{pmatrix} \\
= \; & \frac{\sigma}{(\sigma-1)} E_t b \sum_{k=0}^{\infty} (\alpha_3\beta)^k \begin{pmatrix} \hat{C}_{3,t+k} - \gamma_1 \hat{C}_{1,t+k} + \hat{m}_{1,t+k} \\ +\sigma\left(\hat{P}_{3,1,t+k} - \hat{P}_{3,1,t}\right) - \mu\left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k}\right) \end{pmatrix}.
\end{aligned}
$$

Since

$$
\frac{\sigma}{(\sigma-1)} b = 1,
$$

It follows that,

$$
\begin{aligned}
\frac{\hat{F}_{3,1,t}}{1-\alpha\beta} &= E_t \sum_{k=0}^{\infty} (\alpha_3\beta)^k \left(\hat{C}_{3,t+k} - \gamma_1 \hat{C}_{1,t+k} + \hat{m}_{1,t+k} + \sigma\left(\hat{P}_{3,1,t+k} - \hat{P}_{3,1,t}\right) - \mu\left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k}\right)\right) \\
&\quad - \sum_{k=0}^{\infty} (\alpha_3\beta)^k \begin{pmatrix} \hat{q}_{1,3,t+k} - \mu\left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k}\right) + \hat{C}_{3,t+k} - \gamma_1 \hat{C}_{1,t+k} \\ +(\sigma-1)\left(\hat{P}_{3,1,t+k} - \hat{P}_{3,1,t}\right) + \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k}\right) \end{pmatrix} \\
&= E_t \sum_{k=0}^{\infty} (\alpha_3\beta)^k \begin{pmatrix} \hat{C}_{3,t+k} - b\gamma_1 \hat{C}_{1,t+k} + b\hat{m}_{1,t+k} \\ +\sigma\left(\hat{P}_{3,1,t+k} - \hat{P}_{3,1,t}\right) - \mu\left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k}\right) \\ -\begin{pmatrix} \hat{q}_{1,3,t+k} - \mu\left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k}\right) + \hat{C}_{3,t+k} - \gamma_1 \hat{C}_{1,t+k} \\ +(\sigma-1)\left(\hat{P}_{3,1,t+k} - \hat{P}_{3,1,t}\right) + \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k}\right) \end{pmatrix} \end{pmatrix} \\
&= E_t \sum_{k=0}^{\infty} (\alpha_3\beta)^k \left(\hat{m}_{1,t+k} + \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,1,t}\right) - \hat{q}_{1,3,t+k} - \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k}\right)\right) \\
&= E_t \sum_{k=0}^{\infty} (\alpha_3\beta)^k \begin{pmatrix} \underbrace{\hat{m}_{1,t+k} - \hat{q}_{1,3,t+k}}_{x_{t+k}} \\ +\hat{P}_{3,1,t+k} - \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k}\right) \end{pmatrix} - \hat{P}_{3,1,t}\left(\sum_{k=0}^{\infty} (\alpha_3\beta)^k\right)
\end{aligned}
$$

To lighten notation again, let

$$
x_{t+k} = \hat{m}_{1,t+k} - \hat{q}_{1,3,t+k}.
$$

Then

$$
\begin{aligned}
\frac{\hat{F}_{3,1,t}}{1-\alpha\beta} &= E_t \sum_{k=0}^{\infty} (\alpha_3\beta)^k \left(x_{t+k} + \hat{P}_{3,1,t+k} - \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k}\right)\right) \\
&\quad - \frac{1}{1-\alpha\beta} \hat{P}_{3,1,t}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\hat{F}_{3,1,t}}{1-\alpha\beta} + \frac{1}{1-\alpha\beta}\hat{P}_{3,1,t} &= E_t \sum_{k=0}^{\infty} (\alpha_3\beta)^k \left(x_{t+k} + \underbrace{\hat{P}_{3,1,t+k} - \left(\hat{P}_{3,1,t+k} - \hat{P}_{3,t+k}\right)}_{y_{t+k}}\right) \\
y_{t+k} &= \hat{P}_{3,t+k}
\end{aligned}
$$

Using Results 1 and 2 here gives

$$\begin{aligned}
\frac{\hat{F}_{3,1,t}}{1-\alpha\beta}+\frac{1}{1-\alpha\beta}\hat{P}_{3,1,t} &= E_t\sum_{k=0}^{\infty}(\alpha_3\beta)^k\left(x_{t+k}+\hat{P}_{3,t+k}\right)\\
\hat{F}_{3,1,t}+\hat{P}_{3,1,t} &= (1-\alpha\beta)\,E_t\sum_{k=0}^{\infty}(\alpha\beta)^k\left(x_{t+k}+\hat{P}_{3,t+k}\right)\\
\frac{\alpha}{1-\alpha}\pi_{3,1,t}+\hat{P}_{3,1,t} &= (1-\alpha\beta)\left(x_t+\hat{P}_{3,t}\right)+(\alpha\beta)\,E_t\left(\frac{\alpha}{1-\alpha}\pi_{3,1,t}+\hat{P}_{3,1,t}\right)
\end{aligned}$$

$$\begin{aligned}
\frac{\alpha}{1-\alpha}\pi_{3,1,t} &= (1-\alpha\beta)\left(x_t+\hat{P}_{3,t}\right)+(\alpha\beta)\,E_t\left(\frac{\alpha}{1-\alpha}\pi_{3,1,t+1}+\hat{P}_{3,1,t+1}\right)-\hat{P}_{3,1,t}\\
&\quad+(\alpha\beta)\,\hat{P}_{3,1,t}-(\alpha\beta)\,\hat{P}_{3,1,t}\\
&= (1-\alpha\beta)\left(x_t+\hat{P}_{3,t}\right)+(\alpha\beta)\,E_t\frac{\alpha}{1-\alpha}\pi_{3,1,t+1}+(\alpha\beta)\,E_t\hat{P}_{3,1,t+1}\\
&\quad-(\alpha\beta)\,\hat{P}_{3,1,t}-\left(\hat{P}_{3,1,t}-(\alpha\beta)\,\hat{P}_{3,1,t}\right)\\
&= (1-\alpha\beta)\left(x_t+\hat{P}_{3,t}\right)+(\alpha\beta)\,E_t\frac{\alpha}{1-\alpha}\pi_{3,1,t+1}+(\alpha\beta)\left(E_t\hat{P}_{3,1,t+1}-\hat{P}_{3,1,t}\right)\\
&\quad-(1-\alpha\beta)\,\hat{P}_{3,1,t}\\
&= (1-\alpha\beta)\left(x_t+\hat{P}_{3,t}\right)+(\alpha\beta)\,E_t\frac{\alpha}{1-\alpha}\pi_{3,1,t+1}\\
&\quad+(\alpha\beta)\,E_t\left(\pi_{3,1,t+1}\right)-(1-\alpha\beta)\,\hat{P}_{3,1,t}\\
&= (1-\alpha\beta)\left(x_t+\hat{P}_{3,t}\right)+(\alpha\beta)\,E_t\left[\frac{\alpha}{1-\alpha}+1\right]\pi_{3,1,t+1}\\
&\quad-(1-\alpha\beta)\,\hat{P}_{3,1,t}\\
&= (1-\alpha\beta)\left(x_t+\hat{P}_{3,t}-\hat{P}_{3,1,t}\right)+(\alpha\beta)\,E_t\left[\frac{\alpha}{1-\alpha}+1\right]\pi_{3,1,t+1}
\end{aligned}$$

$$\begin{aligned}
\pi_{3,1,t} &= \frac{(1-\alpha)\,(1-\alpha\beta)}{\alpha}\left(x_t+\hat{P}_{3,t}-\hat{P}_{3,1,t}\right)+(\alpha\beta)\,E_t\left[1+\frac{(1-\alpha)}{\alpha}\right]\pi_{3,1,t+1}\\
x_t &= \hat{m}_{1,t}-\hat{q}_{1,3,t}
\end{aligned}$$

$$\begin{aligned}
\pi_{3,1,t} &= \frac{(1-\alpha)\,(1-\alpha\beta)}{\alpha}\left((\hat{m}_{1,t}-\hat{q}_{1,3,t})-\left(\hat{P}_{3,1,t}-\hat{P}_{3,t}\right)\right)\\
&\quad+\beta E_t\pi_{3,1,t+1}
\end{aligned}$$

The end result is

$$\pi_{3,1,t}=\frac{(1-\alpha)\,(1-\alpha\beta)}{\alpha}\left((\hat{m}_{1,t}-\hat{q}_{1,3,t})+\left(\hat{P}_{3,t}-\hat{P}_{3,1,t}\right)\right)+\beta E_t\pi_{3,1,t+1}. \tag{76}$$

By analogous arguments, we have in the general case,

$$\pi_{i,j,t}=\frac{(1-\alpha)\,(1-\alpha\beta)}{\alpha}\left((\hat{m}_{j,t}-\hat{q}_{i,j,t})+\left(\hat{P}_{i,t}-\hat{P}_{i,j,t}\right)\right)+\beta E_t\pi_{i,j,t+1}.$$

Approximation for price levels

$$\begin{aligned}\hat{P}_{i,t} &= d_{i,1}\hat{P}_{i,1,t}+d_{i,2}\hat{P}_{i,2,t}+d_{i,3}\hat{P}_{i,3,t}\\ &= d_{i,1}\hat{P}_{i,1,t}+d_{i,2}\hat{P}_{i,2,t}+\left(1-d_{i,1}-d_{i,2}\right)\hat{P}_{i,3,t}\\ \hat{P}_{i,t}-\hat{P}_{i,1,t} &= \left(d_{i,1}-1\right)\hat{P}_{i,1,t}+d_{i,2}\left(\hat{P}_{i,2,t}-\hat{P}_{i,3,t}\right)+\left(1-d_{i,1}\right)\hat{P}_{i,3,t}\end{aligned}$$

It follows that

$$\begin{aligned}\hat{P}_{i,t}-\hat{P}_{i,1,t} &= d_{i,2}\left(\hat{P}_{i,2,t}-\hat{P}_{i,3,t}\right)+\left(1-d_{i,1}\right)\left(\hat{P}_{i,3,t}-\hat{P}_{i,1,t}\right)\\ \hat{P}_{i,t}-\hat{P}_{i,2,t} &= d_{i,1}\left(\hat{P}_{i,1,t}-\hat{P}_{i,3,t}\right)+\left(1-d_{i,2}\right)\left(\hat{P}_{i,3,t}-\hat{P}_{i,2,t}\right)\\ \hat{P}_{i,t}-\hat{P}_{i,3,t} &= d_{i,1}\left(\hat{P}_{i,1,t}-\hat{P}_{i,3,t}\right)+d_{i,2}\left(\hat{P}_{i,2,t}-\hat{P}_{i,3,t}\right)\end{aligned}$$

So for this particular case at hand,

$$\left(\hat{P}_{3,t}-\hat{P}_{3,1,t}\right)=d_{3,2}\left(\hat{P}_{3,2,t}-\hat{P}_{3,3,t}\right)+\left(1-d_{3,1}\right)\left(\hat{P}_{3,3,t}-\hat{P}_{3,1,t}\right)$$

Substitute into (76) to get,

$$\pi_{3,1,t}=\frac{\left(1-\alpha\right)\left(1-\alpha\beta\right)}{\alpha}\left(\left(\hat{m}_{1,t}-\hat{q}_{1,3,t}\right)+\left(d_{3,2}\left(\hat{P}_{3,2,t}-\hat{P}_{3,3,t}\right)+\left(1-d_{3,1}\right)\left(\hat{P}_{3,3,t}-\hat{P}_{3,1,t}\right)\right)\right)+\beta E_t\pi_{3,1,t+1},$$

and by definition, we also have

$$\begin{aligned}\left(\hat{P}_{3,2,t}-\hat{P}_{3,3,t}\right) &= \left(\hat{P}_{3,2,t-1}-\hat{P}_{3,3,t-1}\right)+\left(\pi_{3,2,t}-\pi_{3,3,t}\right)\\ \left(\hat{P}_{3,3,t}-\hat{P}_{3,1,t}\right) &= \left(\hat{P}_{3,3,t-1}-\hat{P}_{3,1,t-1}\right)+\left(\pi_{3,3,t}-\pi_{3,1,t}\right).\end{aligned}$$

We'll have to write this out for each i,j combination. Before we do this, we obtain the first-order approximation for the market clearing conditions, which we reproduce here for convenience.

$$\begin{aligned}Y_{1,t} &= C_{1,1,t}+C_{2,1,t}+C_{3,1,t}\\ Y_{2,t} &= C_{1,2,t,}+C_{2,2,t}+C_{3,2,t}\\ Y_{1,t}+Y_{2,t}+Y_{3,t} &= C_{1,t}+C_{2,t}+C_{3,t}.\end{aligned}$$

We illustrate by approximating $Y_{1,t}$. Noting that the approximation of $C_{i,j,t}=d_{i,j}\left(\frac{P_{i,j,t}}{P_{i,t}}\right)^{-\mu}C_{i,t}$ is

$$\hat{C}_{i,j,t}=\mu\left(\hat{P}_{i,t}-\hat{P}_{i,j,t}\right)+\hat{C}_{i,t}, \tag{77}$$

it follows that

$$\hat{y}_{1,t}=\frac{C_{1,1}}{y_1}\hat{C}_{1,1,t}+\frac{C_{2,1}}{y_1}\hat{C}_{2,1,t}+\frac{C_{3,1}}{y_1}\hat{C}_{3,1,t}. \tag{78}$$

In the steady state, $C_{1,1} = d_{1,1}C_1 = d_{1,1}Y_1$. $C_{2,1} = d_{2,1}C_2 = d_{2,1}Y_2$, $C_{3,1} = d_{3,1}C_3 = d_{3,1}Y_3$. This, together with (77) gives

$$\begin{aligned}
\hat{Y}_{1,t} &= d_{1,1}\hat{C}_{1,1,t} + \left(\frac{d_{2,1}y_2}{y_1}\right)\hat{C}_{2,1,t} + \left(\frac{d_{3,1}y_3}{y_1}\right)\hat{C}_{3,1,t} \\
&= d_{1,1}\left(\mu\left(\hat{P}_{1,t} - \hat{P}_{1,1,t}\right) + \hat{C}_{1,t}\right) + \left(\frac{d_{2,1}Y_2}{Y_1}\right)\left(\mu\left(\hat{P}_{2,t} - \hat{P}_{2,1,t}\right) + \hat{C}_{2,t}\right) \\
&\quad + \left(\frac{d_{3,1}Y_3}{Y_1}\right)\left(\mu\left(\hat{P}_{3,t} - \hat{P}_{3,1,t}\right) + \hat{C}_{3,t}\right) \\
&= \mu\left[d_{1,1}\left(\hat{P}_{1,t} - \hat{P}_{1,1,t}\right) + \left(\frac{d_{2,1}Y_2}{Y_1}\right)\left(\hat{P}_{2,t} - \hat{P}_{2,1,t}\right) + \left(\frac{d_{3,1}Y_3}{Y_1}\right)\left(\hat{P}_{3,t} - \hat{P}_{3,1,t}\right)\right] \\
&\quad + d_{1,1}\hat{C}_{1,t} + \left(\frac{d_{2,1}Y_2}{Y_1}\right)\hat{C}_{2,t} + \left(\frac{d_{3,1}Y_3}{Y_1}\right)\hat{C}_{3,t}
\end{aligned}$$

If the countries are equal in size such that in the steady state $Y_1 = Y_2 = Y_3$, we have

$$\begin{aligned}
\hat{y}_{1,t} &= \mu\left(\psi_{1,1}\left(\hat{P}_{1,t} - \hat{P}_{1,1,t}\right) + d_{2,1}\left(\hat{P}_{2,t} - \hat{P}_{2,1,t}\right) + d_{3,1}\left(\hat{P}_{3,t} - \hat{P}_{3,1,t}\right)\right) \\
&\quad + d_{1,1}\hat{C}_{1,t} + d_{2,1}\hat{C}_{2,t} + d_{3,1}\hat{C}_{3,t} \qquad (79)
\end{aligned}$$

Finally, we can use the decompositions

$$\begin{aligned}
\hat{P}_{1,t} - \hat{P}_{1,1,t} &= d_{1,2}\left(\hat{P}_{1,2,t} - \hat{P}_{1,3,t}\right) + (1 - d_{1,1})\left(\hat{P}_{1,3,t} - \hat{P}_{1,1,t}\right) \\
\hat{P}_{2,t} - \hat{P}_{2,1,t} &= d_{2,2}\left(\hat{P}_{2,2,t} - \hat{P}_{2,3,t}\right) + (1 - d_{2,1})\left(\hat{P}_{2,3,t} - \hat{P}_{2,1,t}\right) \\
\hat{P}_{3,t} - \hat{P}_{3,1,t} &= d_{3,2}\left(\hat{P}_{3,2,t} - \hat{P}_{3,3,t}\right) + (1 - d_{3,1})\left(\hat{P}_{3,3,t} - \hat{P}_{3,1,t}\right)
\end{aligned}$$

which come from

$$\begin{aligned}
\hat{P}_{1,t} &= d_{1,1}\hat{P}_{1,1,t} + d_{1,2}\hat{P}_{1,2,t} + (1 - d_{1,1} - d_{1,2})\,\hat{P}_{1,3,t} \\
\hat{P}_{2,t} &= d_{2,1}\hat{P}_{2,1,t} + d_{2,2}\hat{P}_{2,2,t} + (1 - d_{2,1} - d_{2,2})\,\hat{P}_{2,3,t} \\
\hat{P}_{3,t} &= d_{3,1}\hat{P}_{3,1,t} + d_{3,2}\hat{P}_{3,2,t} + (1 - d_{3,1} - d_{3,2})\,\hat{P}_{3,3,t}
\end{aligned}$$

in (79).

The complete first-order approximated model

We can now state the complete model.

$$\begin{aligned}
\pi_{1,1,t} &= \frac{(1-\alpha_1)(1-\alpha_1\beta)}{\alpha_1}\left(\hat{m}_{1,t} + \left(\hat{P}_{1,t} - \hat{P}_{1,1,t}\right)\right) + \beta E_t \pi_{1,1,t+1} \\
\pi_{1,2,t} &= \frac{(1-\alpha_1)(1-\alpha_1\beta)}{\alpha_1}\left((\hat{m}_{2,t} - \hat{q}_{2,1,t}) + \left(\hat{P}_{1,t} - \hat{P}_{1,2,t}\right)\right) + \beta E_t \pi_{1,2,t+1} \\
\pi_{1,3,t} &= \frac{(1-\alpha_1)(1-\alpha_1\beta)}{\alpha_1}\left((\hat{m}_{3,t} - \hat{q}_{3,1,t}) + \left(\hat{P}_{1,t} - \hat{P}_{1,3,t}\right)\right) + \beta E_t \pi_{1,3,t+1}
\end{aligned}$$

$$\begin{aligned}
\pi_{2,1,t} &= \frac{(1-\alpha_2)(1-\alpha_2\beta)}{\alpha_2}\left((\hat{m}_{1,t}-\hat{q}_{1,2,t})+\left(\hat{P}_{2,t}-\hat{P}_{2,1,t}\right)\right)+\beta E_t\pi_{2,1,t+1}\\
\pi_{2,2,t} &= \frac{(1-\alpha_2)(1-\alpha_2\beta)}{\alpha_2}\left(\hat{m}_{2,t}+\left(\hat{P}_{2,t}-\hat{P}_{2,2,t}\right)\right)+\beta E_t\pi_{2,2,t+1}\\
\pi_{2,3,t} &= \frac{(1-\alpha_2)(1-\alpha_2\beta)}{\alpha_2}\left((\hat{m}_{3,t}-\hat{q}_{3,2,t})+\left(\hat{P}_{2,t}-\hat{P}_{2,3,t}\right)\right)+\beta E_t\pi_{2,3,t+1}
\end{aligned}$$

$$\begin{aligned}
\pi_{3,1,t} &= \frac{(1-\alpha_3)(1-\alpha_3\beta)}{\alpha_3}\left((\hat{m}_{1,t}-\hat{q}_{1,3,t})+\left(\hat{P}_{3,t}-\hat{P}_{3,1,t}\right)\right)+\beta E_t\pi_{3,1,t+1}\\
\pi_{3,2,t} &= \frac{(1-\alpha_3)(1-\alpha_3\beta)}{\alpha_3}\left((\hat{m}_{2,t}-\hat{q}_{2,3,t})+\left(\hat{P}_{3,t}-\hat{P}_{3,2,t}\right)\right)+\beta E_t\pi_{3,2,t+1}\\
\pi_{3,3,t} &= \frac{(1-\alpha_3)(1-\alpha_3\beta)}{\alpha_3}\left(\hat{m}_{3,t}+\left(\hat{P}_{3,t}-\hat{P}_{3,3,t}\right)\right)+\beta E_t\pi_{3,3,t+1}
\end{aligned}$$

$$\begin{aligned}
\hat{Y}_{1,t} &= \mu\left(d_{1,1}\left(\hat{P}_{1,t}-\hat{P}_{1,1,t}\right)+d_{2,1}\left(\hat{P}_{2,t}-\hat{P}_{2,1,t}\right)+d_{3,1}\left(\hat{P}_{3,t}-\hat{P}_{3,1,t}\right)\right)\\
&\quad +d_{1,1}\hat{C}_{1,t}+d_{2,1}\hat{C}_{2,t}+d_{3,1}\hat{C}_{3,t}\\
\hat{Y}_{2,t} &= \mu\left(d_{1,2}\left(\hat{P}_{1,t}-\hat{P}_{1,2,t}\right)+d_{2,2}\left(\hat{P}_{2,t}-\hat{P}_{2,2,t}\right)+d_{3,2}\left(\hat{P}_{3,t}-\hat{P}_{3,2,t}\right)\right)\\
&\quad +d_{1,2}\hat{C}_{1,t}+d_{2,2}\hat{C}_{2,t}+d_{3,2}\hat{C}_{3,t}\\
\hat{Y}_{3,t} &= \mu\left(d_{1,3}\left(\hat{P}_{1,t}-\hat{P}_{1,3,t}\right)+d_{2,3}\left(\hat{P}_{2,t}-\hat{P}_{2,3,t}\right)+d_{3,3}\left(\hat{P}_{3,t}-\hat{P}_{3,3,t}\right)\right)\\
&\quad +d_{1,3}\hat{C}_{1,t}+d_{2,3}\hat{C}_{2,t}+d_{3,3}\hat{C}_{3,t}
\end{aligned}$$

$$\begin{aligned}
\hat{Y}_{1,t} &= \hat{A}_{1,t}+\hat{n}_{1,t}\\
\hat{Y}_{2,t} &= \hat{A}_{2,t}+\hat{n}_{2,t}\\
\hat{Y}_{3,t} &= \hat{A}_{3,t}+\hat{n}_{3,t}
\end{aligned}$$

$$\begin{aligned}
\hat{P}_{1,t}-\hat{P}_{1,1,t} &= d_{1,2}\left(\hat{P}_{1,2,t}-\hat{P}_{1,3,t}\right)+(1-d_{1,1})\left(\hat{P}_{1,3,t}-\hat{P}_{1,1,t}\right)\\
\hat{P}_{2,t}-\hat{P}_{2,1,t} &= d_{2,2}\left(\hat{P}_{2,2,t}-\hat{P}_{2,3,t}\right)+(1-d_{2,1})\left(\hat{P}_{2,3,t}-\hat{P}_{2,1,t}\right)\\
\hat{P}_{3,t}-\hat{P}_{3,1,t} &= d_{3,2}\left(\hat{P}_{3,2,t}-\hat{P}_{3,3,t}\right)+(1-d_{3,1})\left(\hat{P}_{3,3,t}-\hat{P}_{3,1,t}\right)
\end{aligned}$$

$$\begin{aligned}
\hat{P}_{1,t}-\hat{P}_{1,2,t} &= d_{1,1}\left(\hat{P}_{1,1,t}-\hat{P}_{1,3,t}\right)+(d_{1,2}-1)\left(\hat{P}_{1,2,t}-\hat{P}_{1,3,t}\right)\\
\hat{P}_{2,t}-\hat{P}_{2,2,t} &= d_{2,1}\left(\hat{P}_{2,1,t}-\hat{P}_{2,3,t}\right)+(d_{2,2}-1)\left(\hat{P}_{2,2,t}-\hat{P}_{2,3,t}\right)\\
\hat{P}_{3,t}-\hat{P}_{3,2,t} &= d_{3,1}\left(\hat{P}_{3,1,t}-\hat{P}_{3,3,t}\right)+(d_{3,2}-1)\left(\hat{P}_{3,2,t}-\hat{P}_{3,3,t}\right)
\end{aligned}$$

$$
\begin{aligned}
\hat{P}_{1,t,} - \hat{P}_{1,3,t} &= d_{1,1}(\hat{P}_{1,1,t} - \hat{P}_{1,3,t}) + d_{1,2}\left(\hat{P}_{1,2,t} - \hat{P}_{1,3,t}\right) \\
\hat{P}_{2,t} - \hat{P}_{2,3,t} &= d_{2,1}\left(\hat{P}_{2,1,t} - \hat{P}_{2,3,t}\right) + d_{2,2}\left(\hat{P}_{2,2,t} - \hat{P}_{2,3,t}\right) \\
\hat{P}_{3,t} - \hat{P}_{3,3,t} &= d_{3,1}\left(\hat{P}_{3,1,t} - \hat{P}_{3,3,t}\right) + d_{3,2}\left(\hat{P}_{3,2,t} - \hat{P}_{3,3,t}\right)
\end{aligned}
$$

$$
\begin{aligned}
\hat{W}_{1,t} - \hat{P}_{1,t} &= \gamma_1 \hat{C}_{1,t} + \gamma_2 \frac{n}{(1-n)} \hat{n}_{1,t} && (80) \\
\hat{W}_{2,t} - \hat{P}_{2,t} &= \gamma_1 \hat{C}_{2,t} + \gamma_2 \frac{n}{(1-n)} \hat{n}_{2,t} && (81) \\
\hat{W}_{3,t} - \hat{P}_{3,t} &= \gamma_1 \hat{C}_{3,t} + \gamma_2 \frac{n}{(1-n)} \hat{n}_{3,t} && (82)
\end{aligned}
$$

$$
\begin{aligned}
\hat{M}_{1,t} - \hat{P}_{1,t} &= \frac{\gamma_1}{(1+i)\,\gamma_3} \hat{C}_{1,t} - \frac{(1-i)}{(1+i)\,\gamma_3} \hat{\imath}_{1,t} && (83) \\
\hat{M}_{2,t} - \hat{P}_{2,t} &= \frac{\gamma_1}{(1+i)\,\gamma_3} \hat{C}_{2,t} - \frac{(1-i)}{(1+i)\,\gamma_3} \hat{\imath}_{2,t} \\
\hat{M}_{3,t} - \hat{P}_{3,t} &= \frac{\gamma_1}{(1+i)\,\gamma_3} \hat{C}_{3,t} - \frac{(1-i)}{(1+i)\,\gamma_3} \hat{\imath}_{3,t}
\end{aligned}
$$

$$
\begin{aligned}
\hat{C}_{1,t} &= E_t\left(\hat{C}_{1,t+1} - \frac{1}{\gamma_1}\left(\hat{\imath}_{1,t} - \hat{\pi}_{1,t+1}\right)\right) \\
\hat{C}_{2,t} &= E_t\left(\hat{C}_{2,t+1} - \frac{1}{\gamma_1}\left(\hat{\imath}_{2,t} - \hat{\pi}_{2,t+1}\right)\right) \\
\hat{C}_{3,t} &= E_t\left(\hat{C}_{3,t+1} - \frac{1}{\gamma_1}\left(\hat{\imath}_{3,t} - \hat{\pi}_{3,t+1}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
\hat{q}_{1,2,t} &= -\gamma_1\left(\hat{C}_{2,t} - \hat{C}_{1,t}\right) \\
\hat{q}_{1,3,t} &= -\gamma_1\left(\hat{C}_{3,t} - \hat{C}_{1,t}\right) \\
\hat{q}_{2,3,t} &= -\gamma_1\left(\hat{C}_{3,t} - \hat{C}_{2,t}\right)
\end{aligned}
$$

Monetary Policy Rule Estimates

It is not the purpose of this work to paint a complete picture of monetary policy around the world, and we are not arguing that any country in particular does or does not violate the Taylor principle, for example, in its conduct of monetary policy–that is acts with a policy feedback rule with $\lambda < 1$.

Table 7: Cross-Rate Management.

Estimates of equation (17). Newey-West t-ratios in parentheses

Country	Cross rate	ρ	λ	ϕ	σ
Australia[1]	euro	0.811	0.200	0.376	0.073**
		(11.187)	(0.809)	(0.689)	(3.315)
Brazil	euro	0.791	-0.181	0.330	0.301**
		(13.552)	(-0.383)	(0.900)	(3.664)
	SF	0.766	-1.432	0.595	0.359**
		(15.753)	(-2.043)	(2.477)	(4.573)
	yen	0.736	-1.219	0.467	0.267**
		(11.129)	(-1.933)	(1.684)	(4.780)
Canada	euro	0.733	-0.349	1.576	0.084*
		(8.930)	(-1.289)	(7.944)	(1.849)
Denmark	euro	0.769	-0.676	0.269	0.769**
		(11.008)	(-1.569)	(5.574)	(4.616)
UK	yen	0.810	0.196	-0.231	0.047*
		(8.975)	(1.854)	(-0.676)	(1.844)
Indonesia	yen	0.695	0.087	–	0.072**
		(28.259)	(0.669)	–	(2.320)
Japan	euro	0.730	0.034	0.000	0.016**
		(7.857)	(0.266)	(0.028)	(3.274)
Korea	euro	0.962	1.963	4.466	0.295
		(30.813)	(1.047)	(1.273)	(1.124)
New Zealand	euro	0.962	1.963	4.466	0.295
		(30.813)	(1.047)	(1.273)	(1.124)
Norway	yen	0.880	0.077	–	0.013*
		(13.324)	(0.234)	–	(1.885)
Philippines	euro	0.794	-0.386	–	0.103**
		(25.674)	(-1.606)	–	(3.260)
	yen	0.818	0.044	–	0.069**
		(23.397)	(0.302)	–	(2.399)
Singapore	yen	0.947	-0.012	–	0.160
		(18.500)	(-0.019)	–	(1.234)
Sweden	yen	0.883	0.430	0.895	0.034*
		(17.885)	(3.659)	(2.091)	(1.695)
Switzerland	euro	0.606	0.227	-0.231	0.065**
		(9.396)	(6.605)	(-1.849)	(3.141)
Thailand	euro	0.654	-0.220	–	0.124
		(2.862)	(-1.195)	–	(1.531)

Notes: * (**) indicates significance of the cross-rate management term at the 10 (5) percent level.