Uncertainty, Long-Run, and Monetary Policy Risks in a Two-Country Macro Model

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Abstract

We study international currency risk and the forward premium bias in a two-country New Keynesian model with recursive utility, production, but no physical capital. Monetary policy follows a Taylor-type interest rate feedback rule and exogenous total factor productivity growth follows a long-run risk process with stochastic volatility, which we estimate from data. Variation in the currency risk premium is driven primarily by shocks to stochastic volatility. Under complete markets, cross-country differentials in an uncertainty-risk factor determine the currency risk premium. Under incomplete markets, differentials in both uncertainty and non-uncertainty factors may be required to understand the currency risk premium. Alternative export pricing conventions of local, producer, and dominant currency pricing alter response dynamics to shocks, but they do not affect aggregate uncertainty faced by agents.

Keywords: Currency risk, uncertainty-risk factors, uncertainty shocks
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Introduction

We study the international currency risk premium (the deviation from uncovered interest parity) and the forward premium bias in a two-country dynamic stochastic general equilibrium New Keynesian model with recursive utility, production, but no physical capital. Monetary policy follows a Taylor-type interest rate feedback rule and exogenous total factor productivity growth follows a long-run risk process with stochastic volatility, which we estimate from data. We examine complete and incomplete market environments and export pricing conventions of local, producer, and dominant currency pricing.

Under complete markets, the currency risk premium has the analytic representation as the foreign-home differential of a series expansion of the higher-ordered conditional cumulants of the nominal stochastic discount factor (SDF) (Backus et al. (2001)). It is convenient to view these higher-ordered cumulants as an uncertainty-risk factor in the pricing of the nominal interest rate and also, as the key driver of precautionary saving.[1] Foreign precautionary saving increases in response to an increase in the foreign uncertainty-risk factor which drives down the foreign interest rate. A positive currency risk premium incentivizes home agents to borrow from foreign to satisfy foreign's excess precautionary saving. A systematic differential in the foreign-home uncertainty-risk factor is generated by cross-country differences in monetary policy rules and productivity growth processes. One might think that the country with lower uncertainty risk is safe and its currency would serve as a hedge asset, earning a negative currency risk premium. Instead, in our framework and perhaps a bit unintuitively, the high uncertainty country 'pays' the currency risk premium, as in Ready et al. (2017).

The forward premium bias refers to a slope coefficient that lies below 1 in the regression of the one-step ahead depreciation of country 1's currency on the interest rate differential between countries 1 and 2. The forward premium anomaly is when the slope coefficient is negative.[2] Under complete markets, the forward premium anomaly requires two things. First, a strong negative covariance between the future currency depreciation and the foreign-home uncertainty factor differential, and second, that the uncertainty-factor differential dominates the variation in the interest rate differential.[3] Backus et al. (2013) are able to generate a negative slope coefficient in a complete markets endowment economy model, but we find this to be much more challenging in a production model. Our assessment is that a forward premium anomaly is unlikely to emerge in a production model without additional shocks such as taste shocks or noise trader shocks.

[1] We refer to 'uncertainty' not in the Knightian sense but in the modern macro sense of high dispersion in the underlying probability distributions. Our analysis of the uncertainty factor pays particular attention to the second conditional cumulant of the nominal SDF. It serves as a useful, preference-based measure of economic uncertainty, that accounts for attitudes toward risk and the psychological ease of intertemporal consumption substitution.

[2] Popularized by Fama (1984), the forward premium bias refers to a slope coefficient, that lies between zero and one. We refer to this regression as the Fama regression and the slope as β_F.

[3] The forward premium bias and the currency risk premium are distinctly different phenomena and this was explored in a different context in Hassan and Mano (2019).

In the incomplete markets environment, non-state contingent nominal bonds are the only internationally traded assets. While this may be an extreme assumption, the operation of actual economies are likely bracketed between complete and incomplete markets. Under incomplete markets, the analytical representation of the currency risk premium is unavailable and the analysis relies on simulations. For a given set of structural differences in monetary policy rules and/or productivity growth processes, the foreign-home uncertainty-risk factor differentials are much larger under incomplete markets, due to reduced opportunities for risk sharing. However, compared to the complete markets currency risk premium, we find no exceptional differences in sign, magnitude, or dynamical response to shocks. The complete-markets insight–that the currency risk premium reflects the foreign-home uncertainty-risk factor differential–can break down under incomplete markets, especially when that differential is modest in size. In these situations, it is necessary to also take into account non-uncertainty-risk factors, which are measured by the first conditional cumulant of the nominal SDF. These are first-order factors that determine foreign-home differentials in national saving associated with consumption smoothing and intertemporal substitution motives.

Across the four exogenous shocks—productivity growth, long-run risk, stochastic volatility, and monetary policy—we find that stochastic volatility shocks generate the most variation in the uncertainty-risk factors and in the currency risk premium, and generate the smallest variation in output. Unlike the impacts from productivity growth, long-run risk, and monetary policy shocks, stochastic volatility shocks also generate persistent variations in the underlying probability distributions. Monetary policy shocks, on the other hand, generate the most variability in the exchange rate.

The export pricing convention matters primarily for trade-related variables in response to monetary policy shocks. This is because the exchange rate is most sensitive to monetary policy shocks and to the currency in which export prices are set. This is why the dominant currency pricing analysis in Gopinath et al. (2020) focuses on the effect of monetary policy shocks on trade variables. However, the alternative export pricing conventions have an unremarkable effect on the overall amount of uncertainty faced by agents. As a result, the systematic currency risk premium is, by and large, insensitive to how export prices are set.

The remainder of the paper is as follows. The next section discusses related literature. Section 2 provides a brief presentation of the model and Section 3 discusses how the currency risk premium and the forward premium bias emerges in our setup. Section 4 reports the model parameterization. The main results from the model are presented in Section 5 and Section 6. Section 7 concludes.

1 Related Literature

Our paper is part of an open economy modeling literature that features recursive utility in production models. In our model, productivity growth is subject to three shocks—a direct shock, a shock to a long-run risk component, and a shock to a stochastic volatility component. In contrast, productivity is a cointegrated random walk in Tretvoll (2018), Mumtaz and Theodoridis (2017), Berg and Mark (2019), and Kollmann (2019). Productivity growth in Colacito et al. (2018b) has long-run risk but no stochastic volatility. In Benigno et al. (2012), productivity growth has a stochastic volatility component in a common global productivity component but no long-run risk, and Gourio et al. (2013) have a disaster shock in productivity with recursive utility.

Research on international finance topics often combine recursive preferences with long-run risk and stochastic volatility processes for consumption growth in endowment economies. Bansal and Shaliastovich (2012) incorporate long-run risk and stochastic volatility in consumption growth and inflation to study bond prices but do not study the currency risk premium or the forward premium bias/anomaly. David et al. (2016) employ a similar structure to study average returns to capital in emerging markets. Kollmann (2016) models a stochastic volatility component in consumption growth to study international risk sharing. Colacito et al. (2018a) is a multi-country endowment model where consumption growth is a long-run risk and stochastic volatility process and is used to explain how the cross-section of currency risk premia emerge from cross-country variation in exposure to global endowment shocks.[4]

As in this paper, Backus et al. (2013) studies the role of cross-country monetary policy heterogeneity in determining the currency risk premium. They do so in a two-country endowment economy model under complete markets where log consumption is a random walk with stochastic volatility. They present two key results. The country whose monetary policy is relatively a) more procyclical or b) more accommodating to inflation pays a positive risk premium. Results from our general equilibrium model are consistent with their first result, but not the second.

Other research that studies the implications of real structural heterogeneity across countries include Benigno et al. (2012) and Ready et al. (2017). Under complete markets, Benigno et al. (2012) assume recursive utility and embed stochastic volatility in a global productivity factor, but they do not have long-run risk. They employ their model to study the effect of stochastic volatility and monetary policy shocks on the forward premium bias. Ready et al. (2017) present a two-country model with asymmetries in production and trade structure to emphasize the role of macroeconomic instability and precautionary saving in driving the currency risk premium.

Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021) explain currency risk and the forward premium bias in general equilibrium with noise traders operating in segmented markets

[4]Our paper also makes contact with Dou and Verdelhan (2018), who also emphasize the importance of cross-country heterogeneity, but they do not study the currency risk premium or forward premium bias and do not consider long-run risk and stochastic volatility processes.

and financial frictions.[5] Chen et al. (2021) implement a reduced form version of the Itskhoki and Mukhin (2021) financial frictions with an exogenous shock that creates a wedge between exchange rate depreciation and the relative log stochastic discount factors in an otherwise complete markets specification. Like us, they are unable to generate a forward premium anomaly in a model with production.

2 The Model

We consider both complete and incomplete markets in a two-country New Keynesian model. We sometimes refer to country 1 as the home country and country 2 as the foreign country. Labor is the only input into production and prices are sticky in the sense of Calvo (1983). The presentation of the model is in its nonstationary form due to a unit-root in the log-level of productivity. The numerical solution, which we obtain by perturbation of a third-order approximation around a nonstochastic steady state, requires a stationary representation of the model. We do this by dividing the nonstationary variables by the one-period lag of the productivity level. Simulations of the model are implemented after pruning. The details of the stationarity inducing transformation are suppressed from the text. Since models in this class are well known and familiar to most readers, the text provides only a sketch of the model.

The main presentation assumes local-currency pricing of exports (LCP). We also consider producer currency pricing (PCP) and dominant currency pricing (DCP), but only provide a quick sketch for the setting of export prices under those rules. Early research, branching from the Mundell (1963)-Fleming (1962) tradition (e.g., Obstfeld and Rogoff (1995)), assumed both countries set export prices by PCP whereby the law-of-one price holds for every traded good. Questions about the appropriateness of this implication led to the development of models under LCP (Betts and Devereux (2000)). Recently, Gopinath et al. (2020) report evidence that the practice of DCP, with the U.S. dollar as the dominant currency, is widespread and pervasive. In our two-country setup, DCP results when country 1 (home) sets export prices by PCP and country 2 (foreign) sets by LCP.

2.1 Households

Let $c_{k,t}$ be household consumption in country k at time t and $\ell_{k,t}$ be labor input from country k at time t where $k \in \{1, 2\}$. Households have recursive utility,

$$V_{k,t} = (1-\beta)\left(\ln\left(c_{k,t}\right) - \eta\frac{\ell_{k,t}^{1+\chi}}{1+\chi}\right) - \frac{\beta}{\phi}\ln\left[\mathrm{E}_t\left(e^{-\phi V_{k,t+1}}\right)\right], \tag{1}$$

[5] Mark and Wu (1998) and Jeanne and Rose (2002) show how the forward premium anomaly emerges in partial equilibrium models with noise trading.

where E_t is the conditional expectation operator, $\beta \in (0,1)$ is the subjective discount factor, $\eta > 0$, $\chi > 0$, and $\phi \in R$ are parameters. $1/\chi$ is the Frisch elasticity of labor supply. This particular logarithmic form of utility constrains the intertemporal elasticity of substitution to be 1 and was introduced by Swanson (2019). Swanson (2019) shows that relative risk aversion is $RRA = \phi + \left(1 + \frac{\eta}{\chi}\right)^{-1}$. The intertemporal marginal rate of substitution, or equivalently, the real SDF is.

$$M_{k,t+1} = \beta \left(\frac{c_{k,t}}{c_{k,t+1}}\right) \left(\frac{e^{-\phi V_{k,t+1}}}{\mathrm{E}_t \left(e^{-\phi V_{k,t+1}}\right)}\right), \tag{2}$$

The nominal SDF is $N_{k,t+1} = M_{k,t+1} e^{-\pi_{k,t+1}}$ where $\pi_{k,t}$ is the inflation rate in country k.

2.1.1 Complete Markets

To lighten the notation under complete markets, we suppress the functional dependence on the state. Under complete markets, households in both countries have access to a full set of nominal state-contingent securities, each paying one unit of country 1's currency if the state occurs. Let $B_{k,t}$ be the number of these securities held by country k households with nominal price Λ_t. Households receive flow resources from real labor income, real firm profits and state-contingent bond payoffs. Shares of firms are not internationally traded. Households spend their resources on consumption and a portfolio of state-contingent bonds. Let $S_{k,j,t}$ be the nominal country k currency price of a unit of country j currency, $P_{k,t}$ be the price level, $w_{k,t}$ be the real wage, and $\Pi_{k,t}$ be real firm profits in country k. The household budget constraint in k is,

$$c_{k,t} + \frac{E_t \left(\Lambda_{t+1} B_{k,t+1}\right)}{S_{1,k,t} P_{k,t}} = w_{k,t} \ell_{k,t} + \Pi_{k,t} + \frac{B_{k,t}}{S_{1,k,t} P_{k,t}}. \tag{3}$$

where $(S_{k,k,t} = 1)$. The optimality conditions for the household are the labor supply equation,

$$w_{k,t} = \eta c_{k,t} \ell_{k,t}^{\chi}. \tag{4}$$

and the Euler equation for the nominal state-contingent bond,

$$\Lambda_{t+1} = E_t \left[N_{k,t+1} \left(\frac{S_{1,k,t}}{S_{1,k,t+1}}\right)\right]. \tag{5}$$

Summing over the Euler equations for prices of each nominal state-contingent bond gives the price of the nominal risk-free bond,

$$\frac{1}{1 + i_{k,t}} = E_t \left[N_{k,t+1} \left(\frac{S_{1,k,t}}{S_{1,k,t+1}}\right)\right], \tag{6}$$

where $i_{k,t}$ is the nominal interest rate. It follows that the nominal exchange rate depreciation is

$$\frac{S_{1,2,t+1}}{S_{1,2,t}} = \frac{N_{2,t+1}}{N_{1,t+1}}. \tag{7}$$

2.1.2 Incomplete Markets

Under incomplete markets, each country issues a nominal, non-state contingent discount bond denominated in their own currency. The issue price is one unit of the country k currency and the time $t+1$ payoff is $1+i_{k,t}$ units of the country k currency for $k \in \{1,2\}$. These are the only internationally traded assets. Let $B_{k,j,t} > 0$ be the number of currency j bonds held by country k agents ($k, j \in \{1,2\}$). There are no short-sale constraints, so if country k agents have shorted the bond, then $B_{k,j,t} < 0$. Let $b_{k,j,t}$ be the country j real value of those bonds.

To keep bond holdings stationary, we adopt the Schmitt-Grohe and Uribe (2003) method of imposing a small fee (τ) on residents on either their long or short positions on foreign currency denominated bonds. The real cost to a country k household for taking a position in the currency j bond is $\Gamma\left(b_{k,j,t}\right) = \frac{\tau}{2}\left(Q_{k,j,t}b_{k,j,t}/\sqrt{A_{k,t-1}}\right)^2$, where $Q_{k,j,t} = (S_{k,j,t}P_{j,t})/P_{k,t}$ is the real exchange rate. In the steady state, for any $\tau > 0$, households will want $b_{k,j} = b_{j,k} = 0$. Because the level of productivity ($A_{k,t}$) is nonstationary, we normalize the model by the one-period lagged productivity level ($A_{k,t-1}$) to induce stationarity in the quantities. The term $A_{k,t-1}$ enters the bond tax formula in anticipation of the normalization.

Households own the firms only of their own country. Household resources consists of real firm profits, real labor income, and real bond payoffs. These resources are spent on consumption and a new bond portfolio. Let $r_{k,t}$ be the real interest rate. Then the gross real bond return is $(1+r_{k,t-1}) = (1+i_{k,t-1})\,e^{-\pi_{k,t}}$. The real budget constraint for the country k household is

$$c_{k,t} + b_{k,k,t} + Q_{k,j,t}b_{k,j,t} + \Gamma\left(b_{k,j,t}\right) = \left(1+r_{k,t-1}\right)b_{k,k,t-1} + \left(1+r_{j,t-1}\right)Q_{k,j,t}b_{k,j,t-1} + w_{k,t}\ell_{k,t} + \Pi_{k,t}. \tag{8}$$

The bond choice Euler equations for a country k household are

$$\text{Domestic Bond: } \frac{1}{(1+i_{k,t})} = \mathrm{E}_t\left[N_{k,t+1}\right], \tag{9}$$

$$\text{Non-Domestic Bond: } \left(\frac{1}{1+i_{j,t}}\right)\left(1+\frac{\tau Q_{k,j,t}b_{k,j,t}}{A_{k,t-1}}\right) = \mathrm{E}_t\left[N_{k,t+1}\left(\frac{Q_{k,j,t+1}}{Q_{k,j,t}}\right)\right], \tag{10}$$

where $k \neq j$. The labor supply optimality condition is not affected by the change to incomplete markets and is described by eq. (4). In equilibrium, we require zero net bonds outstanding. Hence, for $k, j \in \{1,2\}$ and $k \neq j$,

$$0 = b_{k,k,t} + b_{j,k,t}. \tag{11}$$

The remainder of the model that follows holds under both complete and incomplete markets.

2.2 Goods Demand

In each country, a continuum of firms indexed by $f \in [0,1]$ each produce a differentiated product. Let λ be the elasticity of substitution between varieties f. $c_{k,j,t}(f)$ are goods produced by firm f in country j and consumed in country k, and $p_{k,j,t}(f)$ is the price in currency k (LCP) of that product. The index of imports ($k \neq j$) or domestic demand ($k = j$) and the associated price index are

$$c_{k,j,t} = \left[\int_0^1 c_{k,j,t}(f)^{\frac{\lambda-1}{\lambda}} df\right]^{\frac{\lambda}{\lambda-1}}, \tag{12}$$

$$P_{k,j,t} = \left[\int_0^1 p_{k,j,t}(f)^{1-\lambda} df\right]^{\frac{1}{1-\lambda}}. \tag{13}$$

Aggregate demand of country k and the associated price level under LCP are

$$c_{k,t} = \left(d^{\frac{1}{\mu}} c_{k,k,t}^{\frac{\mu-1}{\mu}} + (1-d)^{\frac{1}{\mu}} c_{k,j,t}^{\frac{\mu-1}{\mu}}\right)^{\frac{\mu}{\mu-1}}, \tag{14}$$

$$P_{k,t} = \left[dP_{k,k,t}^{1-\mu} + (1-d) P_{k,j,t}^{1-\mu}\right]^{\frac{1}{1-\mu}}, \tag{15}$$

where d is the degree of home bias and μ is the elasticity of substitution between the domestically and non-domestically produced goods

2.3 Firms

Under LCP, firm $f \in [0,1]$ can distinguish between domestic and non-domestic shoppers and is able to charge them different prices. The production function for a firm in country k where $k \in \{1,2\}$ is

$$y_{k,t}(f) = A_{k,t}\ell_{k,t}(f), \tag{16}$$

where $A_{k,t}$ is the productivity level. The firm's real total costs are $w_{k,t}\ell_{k,t}(f)$. Output is demand determined, $y_{k,t}(f) = c_{k,k,t}(f) + c_{j,k,t}(f)$, where $k \neq j$. Domestic and non-domestic demands are, respectively,

$$c_{k,k,t}(f) = d\left(\frac{p_{k,k,t}(f)}{P_{k,k,t}}\right)^{-\lambda}\left(\frac{P_{k,k,t}}{P_{k,t}}\right)^{-\mu} c_{k,t}, \tag{17}$$

$$c_{j,k,t}(f) = (1-d)\left(\frac{p_{j,k,t}(f)}{P_{j,k,t}}\right)^{-\lambda}\left(\frac{P_{j,k,t}}{P_{j,t}}\right)^{-\mu} c_{j,t}. \tag{18}$$

It follows that labor employed by firm f is

$$\ell_{k,t}(f) = \frac{c_{k,k,t}(f) + c_{j,k,t}(f)}{A_{k,t}}. \tag{19}$$

Prices are sticky in the sense of Calvo (1983). Each period, the firm is allowed to change its price with probability $1-\alpha$. LCP means firms in country 1 set export prices in country 2 currency while firms in country 2 set export prices in country 1 currency. Price setting goes as follows. If a firm in country k, ($k, j \in \{1,2\}$ and $k \neq j$), is chosen to reset prices, it adjusts both the currency k price for the domestic market ($p_{k,k,t}(f)$) and the currency j price for exports ($p_{j,k,t}(f)$). Prices are set to maximize the expected present value of future real profits with prices fixed at the optimal choices. Let $M_{k,t,t+h} = \prod_{z=0}^{h} M_{k,t+z}$ be the $h-$period real SDF where $M_{k,t} = 1$. Formally, the problem for price resetting is to maximize

$$\mathrm{E}_t \sum_{h=0}^{\infty} (\alpha)^h M_{k,t,t+h} \left[\frac{p_{k,k,t}(f)}{P_{k,t+h}} c_{k,k,t+h}(f) + \frac{Q_{k,j,t+h} p_{j,k,t}(f)}{P_{j,t+h}} c_{j,k,t+h}(f) - w_{k,t+h} \ell_{k,t+h}(f) \right], \quad (20)$$

subject to the output demand eqs. (17) and (18) and the labor demand eq. (19).

Under PCP, firms in country 1 set export prices in country 1 currency while firms in country 2 set export prices in country 2 currency, where $P_{k,j,t}$ is now denominated in country j's currency. The price level in eq. (15) becomes $P_{k,t} = \left[dP_{k,k,t}^{1-\mu} + (1-d)(S_{k,j,t} P_{k,j,t})^{1-\mu} \right]^{\frac{1}{1-\mu}}$. Domestic output demand $c_{k,k,t}(f)$ is again given by eq. (17), but non-domestic demand is

$$c_{j,k,t}(f) = (1-d) \left(\frac{p_{j,k,t}(f)}{P_{j,k,t}} \right)^{-\lambda} \left(\frac{S_{j,k,t} P_{j,k,t}}{P_{j,t}} \right)^{-\mu} c_{j,t}, \quad (21)$$

where $k \neq j$. The firm's price setting problem is to choose prices to maximize

$$E_t \sum_{h=0}^{\infty} (\alpha)^h M_{k,t,t+h} \left[\frac{p_{k,k,t}(f)}{P_{k,t+h}} c_{k,k,t+h}(f) + \frac{p_{j,k,t}(f)}{P_{k,t+h}} c_{j,k,t+h}(f) - w_{k,t+h} \ell_{k,t+h}(f) \right]. \quad (22)$$

Under DCP, firms in country 1 set export prices in country 1 currency (they engage in PCP) and firms in country 2 also set export prices in country 1 currency (they engage in LCP).

2.4 Monetary Policy

The monetary authorities set the interest rate according to a Taylor (1993)-type feedback rule that responds to inflation deviations from its steady state level and to the output gap. For country $k \in \{1,2\}$, we follow Swanson (2019) by setting the natural (log) level of output to be an infinite-dimensional moving average of output,

$$\ln(\bar{y}_{k,t}) = \rho_{y_k} \ln(\bar{y}_{k,t-1}) + (1-\rho_{y_k}) \ln(y_{k,t}). \quad (23)$$

The output gap is then the deviation between $\ln(y_{k,t})$ and $\ln(\bar{y}_{k,t})$. The monetary authorities set the short-term interest rate, with interest rate smoothing, according to

$$i_{k,t} = (1-\delta_k)\bar{\imath}_k + \delta_k i_{k,t-1} + (1-\delta_k)\left[\xi_k\left(\pi_{k,t} - \bar{\pi}_k\right) + \zeta_k\left(\ln(y_{k,t}) - \ln(\bar{y}_{k,t})\right)\right] + e_{k,t}, \tag{24}$$

where δ_k, ξ_k, and ζ_k are parameters, $\bar{\imath}_k$ is the steady state interest rate, $\pi_{k,t}$ is the inflation rate, $\bar{\pi}_k$ is the steady state inflation rate, and $e_{k,t} \overset{iid}{\sim} N\left(0, \sigma^2_{e_k}\right)$.

2.5 Aggregation and Equilibrium

Aggregate demand for goods produced in country k is given by equating firm f's supply to demand,

$$A_{k,t}\ell_{k,t}(f) = d\left(\frac{P_{k,k,t}}{P_{k,t}}\right)^{-\mu}\left(\frac{p_{k,k,t}(f)}{P_{k,k,t}}\right)^{-\lambda} c_{k,t} + (1-d)\left(\frac{p_{j,k,t}(f)}{P_{j,k,t}}\right)^{-\lambda}\left(\frac{P_{j,k,t}}{P_{j,t}}\right)^{-\mu} c_{j,t}, \tag{25}$$

then integrating eq. (25) to obtain,

$$A_{k,t}\ell_{k,t} = c_{k,k,t}v^p_{k,k,t} + c_{j,k,t}v^p_{j,k,t}, \tag{26}$$

where $\ell_{k,t} = \int_0^1 \ell_{k,t}(f)\,df$ is total country k employment,

$$c_{k,k,t} = d\left(\frac{P_{k,k,t}}{P_{k,t}}\right)^{-\mu} c_{k,t} = \left(\int_0^1 c_{k,k,t}(f)^{\frac{\lambda-1}{\lambda}}\,df\right)^{\frac{\lambda}{\lambda-1}}, \tag{27}$$

is aggregate domestic demand, and

$$c_{j,k,t} = (1-d)\left(\frac{P_{j,k,t}}{P_{j,t}}\right)^{-\mu} c_{j,t} = \left(\int_0^1 c_{j,k,t}(f)^{\frac{\lambda-1}{\lambda}}\,df\right)^{\frac{\lambda}{\lambda-1}}, \tag{28}$$

is aggregate export demand. In eq. (26), $v^p_{k,k,t} \equiv \int_0^1 \left(\frac{p_{k,k,t}(f)}{P_{k,k,t}}\right)^{-\lambda} df$ is a measure of price dispersion for goods in the domestic market and $v^p_{j,k,t} \equiv \int_0^1 \left(\frac{p_{j,k,t}(f)}{P_{j,k,t}}\right)^{-\lambda} df$ is import price dispersion in country j. The recursive representation for the price dispersion terms $v^p_{k,j,t}$ $(k, j \in \{1,2\})$, is obtained by noting that a fraction α of these firms are stuck with last period's price, $p_{k,j,t-1}(f)$. Since there are a large number of firms charging the same price as last period, it will also be the case that $\int_0^\alpha p_{k,j,t-1}(f)^{-\lambda}\,df = \alpha P^{-\lambda}_{k,j,t-1}$. The complementary measure of firms $(1-\alpha)$ are able to reset the prices for exports and for the domestic market. They all reset to the same price,

$p^*_{k,j,t}$. The result is the recursive representation,

$$v^p_{k,j,t} = (1-\alpha)\left(\frac{p^*_{k,j,t}}{P_{k,j,t}}\right)^{-\lambda} + \alpha\left(\frac{P_{k,j,t-1}}{P_{k,j,t}}\right)^{-\lambda} v^p_{k,j,t-1}. \tag{29}$$

3 Uncertainty Risk and Currency Risk

In this section, we assume complete markets to exploit available analytical representations for the currency risk premium and the forward premium bias. Backus et al. (2001) show that the currency risk premium is the foreign–home (country 2–country 1) differential of a series expansion of the nominal SDF's higher-ordered conditional cumulants. We view these higher-ordered cumulants as an uncertainty-risk factor which not only determine the currency risk premium but also the degree of precautionary saving. We illustrate the fundamental difference between the currency risk premium and the forward premium bias, and the particular challenge for complete markets models to explain the forward premium anomaly. Unless required to prevent confusion, this section suppresses the country-specific notation.

3.1 Uncertainty Risk

Assume complete markets and let κ_{jt} be the $j-$th conditional cumulant of the nominal SDF N_{t+1}.[6] Backus et al. (2001) observed that $\ln(E_t N_{t+1})$ is the conditional cumulant generating function of N_{t+1} evaluated at 1 and has the series expansion, $\ln(E_t N_{t+1}) = \sum_{j=1}^{\infty} \kappa_{jt}/j!$.

Taking the logarithm of both sides of the bond-pricing eq. (6) gives the nominal interest rate,

$$i_t = -\sum_{j=1}^{\infty} \frac{\kappa_{jt}}{j!}. \tag{30}$$

The higher-ordered conditional cumulants (κ_{jt} for $j \geq 2$) are agents' subjective assessments of uncertainty. They also represent the effect of precautionary saving on the interest rate. Higher uncertainty raises precautionary saving, drives bond prices up and interest rates down. It is

[6] Let $\psi(t) = E\left(e^{tX}\right)$ be the moment generating function of X. Then the $k-th$ derivative evaluated at $t=0$, is the $k-th$ moment of X, $\psi^k(0) = E\left(X^k\right)$. The logarithm of the moment generating function is the cumulant generating function, $\phi(t) = \ln(\psi(t)) = \ln\left(E\left(e^{tX}\right)\right)$. Just as the moment generating function can be expanded, the cumulant generating function has the expansion

$$\phi(t) = \ln\left(E\left(e^{tX}\right)\right) = t\kappa_1 + \frac{t^2\kappa_2}{2!} + \frac{t^3\kappa_3}{3!} + \cdots$$

The $k-$th derivative evaluated at $t=0$, $\phi^k(0)$ is the $k-$th cumulant of X. Letting $X = \ln(N_{t+1})$ gives the result in the text. The first 3 conditional cumulants of X are also its first 3 central moments. That is, $\kappa_1 \equiv \phi^1(0) = E(X)$, $\kappa_2 \equiv \phi^2(0) = E(X - E(X))^2$, and $\kappa_3 \equiv \phi^3(0) = E(X - E(X))^3$. Cumulants of order 4 and higher are complicated functions of the moments.

convenient to think of these higher-ordered cumulants as an uncertainty-risk factor. Note that eq. (30) holds under both complete and incomplete markets.

κ_{1t} is a non-uncertainty-risk factor, and is key in setting national saving associated with consumption smoothing and intertemporal substitution motives. An increase in $\kappa_{1t} = E_t(\ln N_{t+1})$ means increased value of future cash flows. This results in higher saving, higher bond prices, and lower interest rates. In a world of certainty, κ_{1t} completely determines the interest rate since $\kappa_{jt} = 0$ for $j \geq 2$.

3.2 Currency Risk Premium

The currency risk premium (deviation from uncovered interest parity) is the expected excess return from borrowing country 2's currency and lending country 1's currency,

$$rp_t = \mathrm{E}_t \left(i_{1,t} - i_{2,t} - \Delta \ln(S_{1,2,t+1}) \right). \tag{31}$$

Let κ_{jt}^c be the $j-$th conditional cumulant for country $c \in \{1, 2\}$, and $\kappa_{jt}^* \equiv \kappa_{jt}^{(2)} - \kappa_{jt}^{(1)}$ be the foreign-home differential. Define the differential in the foreign-home uncertainty-risk factor to be[7]

$$\mathrm{BFT}_t = \sum_{j=2}^{\infty} \frac{\kappa_{jt}^*}{j!}, \tag{32}$$

Under complete markets, Backus et al. (2001) showed that the currency risk premium is,

$$rp_t = \mathrm{BFT}_t. \tag{33}$$

which follows from substituting eqs. (7) and (30) into eq. (31). Under incomplete markets, eq. (33) does not hold because eq. (7) does not hold.

As seen from eqs. (32) and (33), fluctuations in the currency risk premium are driven entirely by time-variation in the foreign-home uncertainty-risk factor differential. Non-uncertainty-risk factors (κ_{1t}^c) are irrelevant. The sign of rp_t is determined by assessments of relative economic uncertainty, which in turn drive the relative strength of precautionary saving. If $rp_t > 0$, uncertainty is assessed to be higher in country 2 as is country 2's desired precautionary saving. The positive rp_t induces country 1 to borrow (short) the country 2 currency and to lend (go long) the country 1 currency. Importantly, under complete markets, the currency risk premium is a function only of variables that determine the interest rate differential. The exchange rate does not play an explicit role.[8]

[7] We call this variable BFT to acknowledge the contribution of Backus et al. (2001).

[8] Although the mechanisms under complete and incomplete markets are different, we will investigate through simulations the extent to which the complete markets insights carry through to incomplete markets. To understand the currency risk premium under incomplete markets, particularly when there is heterogeneity across countries, it

3.3 Forward Premium Bias

There is a forward premium bias when the slope coefficient in the regression ($\Delta \ln(S_{1,2,t+1}) = \beta_0 + \beta_F(i_{1,t} - i_{2,t}) + e_{t+1}$), of the future currency 1 depreciation on the current interest rate differential is less than 1. We refer to the slope in question as the 'Fama coefficient,' and denote it by β_F. There is a forward premium anomaly when the slope coefficient is negative ($\beta_F < 0$). These issues were prominently discussed in Fama (1984).

Under complete markets, we employ eqs. (7) and (30) to re-express the Fama regression as

$$\ln\left(\frac{N_{2,t+1}}{N_{1,t+1}}\right) = \beta_0 + \beta_F \left(\sum_{j=1}^{\infty} \frac{\kappa_{jt}^*}{j!}\right) + e_{t+1}. \tag{34}$$

To get a forward premium anomaly ($\beta_F < 0$), some of the κ_{jt}^* need to be negatively correlated with the foreign-home log nominal SDF differential. This is highly unlikely for the first component because $\kappa_{1t}^* = E_t \ln(N_{2,t+1}/N_{1,t+1})$ is its conditional expectation. The forward premium anomaly would appear to require the foreign-home uncertainty-risk factor differential, BFT_t, to be negatively correlated with $\ln(N_{2,t+1}/N_{1,t+1})$, and to be more variable than (κ_{1t}^*). To the extent that BFT_t is negatively correlated with κ_{1t}^*, times of high relative uncertainty risk (high BFT_t) are also times of high relative expected inflation, high relative expected consumption growth, and high relative uncertainty of future utility (low κ_{1t}^*).[9]

To summarize, under complete markets, the forward premium bias/anomaly is a phenomenon of low/negative covariance between the foreign-home log nominal SDF differential and the foreign-home conditional cumulants differential. The currency risk premium reflects the foreign-home relative strength of precautionary saving.

4 Model Parameterization

In this section, we outline the parameterization of the model with symmetric monetary policy and productivity growth processes. We first describe the estimation of productivity growth, modeled as a long-run risk and stochastic volatility process, using data for the United States. We also report the remaining parameters in the model.

may be necessary also to take into account κ_{1t}^*.

[9]For simplicity, let $e^{-\phi V_{t+1}}$ be conditionally log-normally distributed. Then,

$$\kappa_{1t} = -E_t \pi_{t+1} - E_t \Delta \ln(c_{t+1}) - \left(\frac{\phi^2 Var_t(V_{t+1})}{2}\right) + \ln(\beta).$$

4.1 Productivity Growth Process

Typically, long-run risk in international macroeconomics and finance is used to model consumption growth in endowment models (Bansal and Yaron (2004), Bansal and Shaliastovich (2012), Backus et al. (2013), and Colacito et al. (2018a)). Productivity shocks with stochastic volatility is more extensively studied in closed economy macro models (see the review article by Fernández-Villeverde and Guerrón-Quintana (2020)). Our productivity growth specification largely mimics that used for consumption growth in long-run risk models of asset pricing with the stochastic volatility process following Fernández-Villeverde and Guerrón-Quintana (2020) to keep the stochastic volatility component (σ_t) positive.

Notation to differentiate parameter values across countries is suppressed. Let $a_t = \ln(A_t)$ be log productivity, x_t be the long-run risk component, and σ_t be the stochastic volatility component. Productivity growth is governed by,

$$\Delta a_t = \mu_g + x_{t-1} + e^{\theta}\sigma_{t-1}\epsilon_t, \tag{35}$$

$$x_t = \rho_x x_{t-1} + \sigma_{t-1}u_t, \tag{36}$$

$$\ln(\sigma_t) = (1-\rho_\sigma)\,\mu_\sigma + \rho_\sigma \ln(\sigma_{t-1}) + e^{\eta}v_t, \tag{37}$$

where $\epsilon_t \overset{nid}{\sim} (0,1)$, $u_t \overset{nid}{\sim} (0,1)$, and $v_t \overset{nid}{\sim} (0,1)$.

We begin by studying the symmetric model where both countries have the same productivity growth process. To choose sensible parameter values, we estimate the productivity growth process using U.S. total factor productivity (TFP). To construct TFP, we first use quarterly GDP, investment, and employment data from *Datastream* and FRED. The capital stock is constructed by the perpetual inventory method. From this, we construct quarterly total factor productivity.

Table 1: Posterior Means–Productivity Growth Process for the United States

	Posterior Mean	5%	95%
μ_g	0.002	0.000	0.003
θ	1.131	0.648	1.684
ρ_x	0.726	0.647	0.809
ρ_σ	0.842	0.765	0.921
η	-1.353	-3.855	1.167
μ_σ	-6.257	-6.711	-5.793

Notes: The productivity growth process for the United States is governed by eqs. (35)–(37). Posterior means and the upper and lower 5% bands from a Bayesian estimation are reported.

We employ the posterior means from Bayesian estimation as the parameter values in the long-run risk and stochastic volatility process for productivity growth. The posterior means and the upper and lower 5% bands are shown in Table 1.

Table 2: Volatility of Productivity Growth and Components for the United States

	Data	Simulated					
		Unadjusted			Adjusted		
	$\sigma(\Delta a_t)$	$\sigma(\Delta a_t)$	$\sigma(x_t)$	$\sigma(\sigma_t)$	$\sigma(\Delta a_t)$	$\sigma(x_t)$	$\sigma(\sigma_t)$
United States	2.605	3.433	1.467	0.453	2.603	1.106	0.350

Notes: $\sigma(\bullet)$ is the volatility or standard deviation of the variable stated as percent per annum. a_t is log productivity, x_t is the long-run risk component, and σ_t is the stochastic volatility component.

Table 2 shows the volatility of the process components $(\Delta a_t, x_t, \sigma_t)$ implied by the estimates. Volatility of simulated TFP growth generated by the estimated process (3.433) overstates volatility in the data (2.605). In the ensuing analysis, we scale the innovations (ϵ_t, u_t, v_t) in eqs. (35)-(37) by 2.605/3.433 to match the volatility in the data. The volatility of simulated TFP growth, long-run risk component, and stochastic volatility component after adjustment are shown in Table 2 under 'Adjusted'. As can be seen, the adjustment produces a close match between the volatility of the simulated process and the data.

Figure 1: United States Productivity Data and Realized Simulation

Productivity Growth

Log Productivity

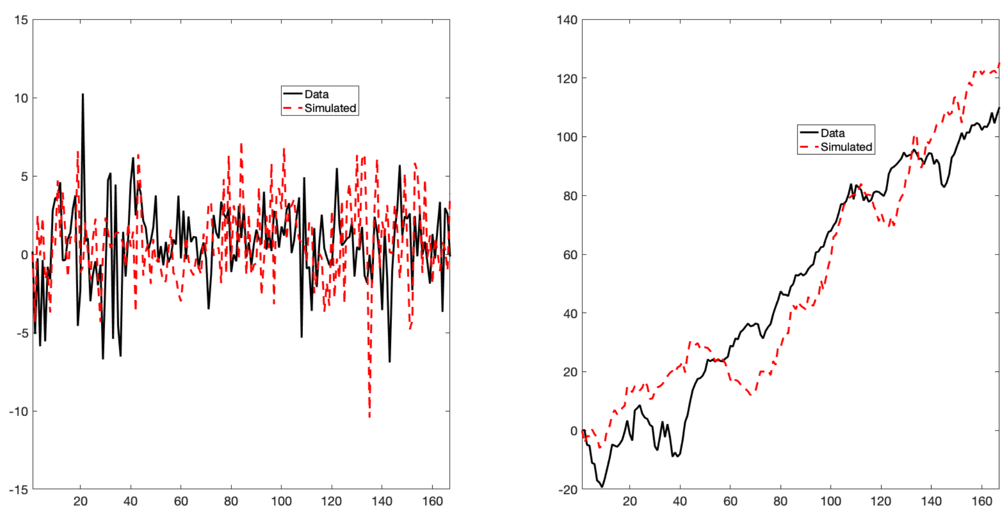


Figure 1 plots productivity growth and log levels from the data as well as a realized simulation of the adjusted process. The model is seen to do a reasonable job of capturing major features of the productivity data.

Note that log TFP is modeled as a unit-root process. The two-country model solution requires that the two countries' log TFP be cointegrated. To achieve this, we employ the modified specification,

$$\Delta a_{1,t} = x_{1,t-1} + e^{\theta_1}\sigma_{1,t-1}\epsilon_{1,t} + \psi(a_{1,t-1} - a_{2,t-1}) \tag{38}$$

$$\Delta a_{2,t} = x_{2,t-1} + e^{\theta_2}\sigma_{2,t-1}\epsilon_{2,t} + \psi(a_{2,t-1} - a_{1,t-1}) \tag{39}$$

with $\psi = 0.0005$ and $\mu_g = 0$. This makes log productivity in countries 1 and 2 to be driftless and cointegrated, but not strongly so.

4.2 Remaining Parameterization

Monetary policy for both countries follow the benchmark Taylor rule where coefficients for the inflation response, the output gap response, and interest rate smoothing are $\xi = 1.5$, $\zeta = 0.5$, and $\delta = 0.7$, respectively. In regard to preference parameters, $\beta = 0.9925$, $\eta = 0.545$, $\chi = 3$, and $\phi = 40$. We refer to ϕ as the risk-aversion coefficient, since it is the dominant parameter (For $\phi = 40$, relative risk aversion is $\phi + 1/(1 + \eta/\chi) = 40.84$.). Remaining parameters of the model are $d = 0.85$, $\tau = 0.001$, $\lambda = 10$, $\mu = 1.5$, $\alpha = 0.8$, and $\rho_y = 0.96$.

5 Dynamics of Uncertainty Risk and Currency Risk

In this section, we study the dynamics of uncertainty-risk factors and international currency risk implied by the model through impulse response analysis and variance decompositions. The analysis is done for complete and incomplete markets under symmetry in monetary policy and productivity growth across countries.[10]

A key finding of the paper is that the currency in which export prices are set is not central for understanding systematic currency risk premium. Export pricing may matter for specific impulse responses, especially with respect to trade-related variables, but its effect on the aggregate uncertainty in the economy is unremarkable. As a result, unless otherwise noted, LCP is the export pricing convention. For the most part, the qualitative responses under DCP and PCP are approximately the same. The DCP and PCP results are reported in the appendix.

[10] As a reference, it may be useful to classify the shocks as aggregate demand or aggregate supply based on their impact effect. If we say that, upon impact, aggregate demand (AD) shocks cause output and inflation to move in the same direction and aggregate supply (AS) shocks cause output and inflation to move in opposite directions, then productivity shocks are reliably and well-known to be AS shocks and monetary policy shocks are reliably AD shocks. The appendix shows that under all export pricing schemes, the stochastic volatility shock is an AD shock, as in Xu (2016) and Leduc and Liu (2016). Interestingly, the long-run risk shock is also classified as an AD shock.

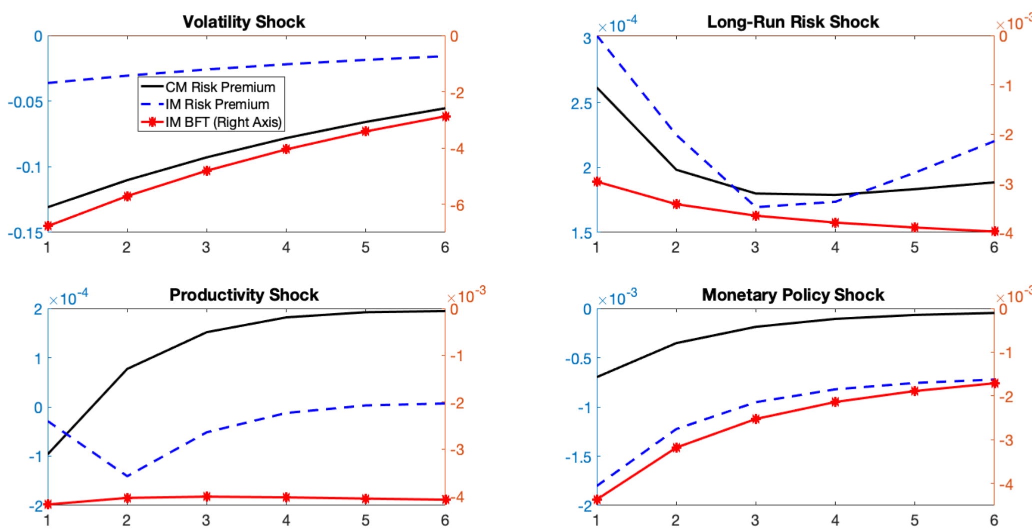
5.1 Impulse Response Function Analysis

The impulse responses are to a positive country 1 shock and are reported in percent per annum. We focus on key model variables that help to understand the currency risk premium.

Risk Premium and Uncertainty-Risk Factor

Figure 2 plots the impulse responses of the currency risk premium (rp_t) under complete and incomplete markets (left axis) and of BFT_t under incomplete markets (right axis). Here, and in all subsequent analyses, $\text{BFT}_t = \kappa_{2t}^*/2! + \kappa_{3t}^*/3!$ is truncated at the third order. Recall, under complete markets, BFT_t and the currency risk premium are one and the same, but not under incomplete markets. We make three points regarding Figure 2.

Figure 2: BFT_t and Risk Premium (rp_t) Impulse Responses under LCP



Notes: The impulse responses are to positive country 1 shocks under LCP and are reported in percent per annum. $rp_t = E_t\left(i_{1,t} - i_{2,t} - \Delta ln(S_{1,2,t+1})\right)$ is the currency risk premium and $\text{BFT}_t = \kappa_{2t}^*/2! + \kappa_{3t}^*/3!$. CM represents complete markets and IM represents incomplete markets. Parameterization follows from Section 4.

First, we see substantial variation in the response magnitudes across the different shocks. rp_t and BFT_t responses are dominated by stochastic volatility shocks under both complete and incomplete markets. Under incomplete markets, the BFT_t response (red line with stars–right scale) is much larger than the rp_t response (dashed blue line–left scale). The foreign-home uncertainty-risk factor differential BFT_t, under incomplete markets is much larger than under complete markets, but these differences are not matched by the size of the rp_t response. In contrast to the stochastic volatility shock, long-run risk, productivity growth, and monetary policy shocks generate very small rp_t and BFT_t responses under both complete and incomplete markets.

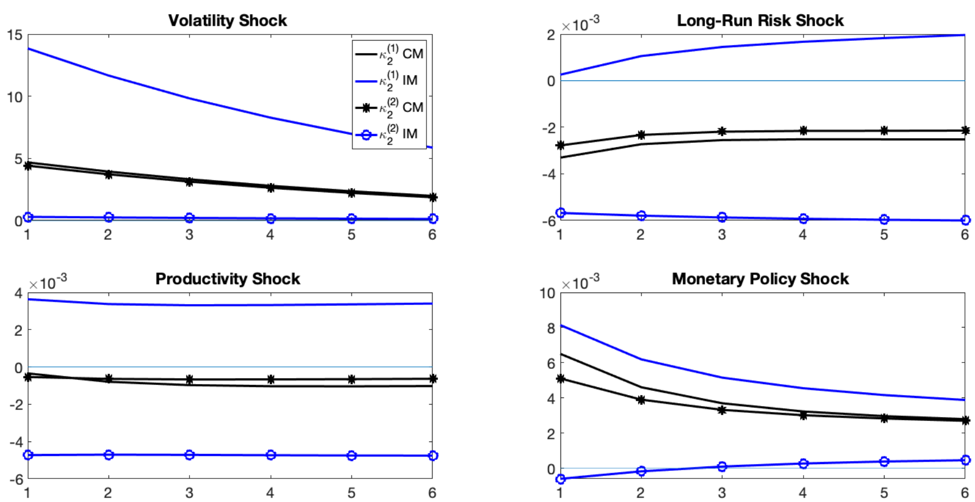
Second, we comment on the direction of the responses. Under complete and incomplete markets, stochastic volatility, productivity growth, and monetary policy shocks from country 1 raise uncertainty in country 1 relative to country 2, as seen by a decline in BFT_t. These shocks cause rp_t to initially decline under both complete and incomplete markets. Interestingly, the country 1 long-run risk shock causes rp_t to increase under both complete and incomplete markets. Under incomplete markets, every country 1 shock lowers BFT_t and precautionary saving becomes stronger in country 1 relative to country 2. But BFT_t does not necessarily 'explain' rp_t in the sense that these variables respond in opposite directions to the long-run risk shock, although the magnitudes here are tiny.

Third, we note that monetary policy shocks generate BFT_t responses of similar magnitude under both complete and incomplete markets. For the other three shocks, the incomplete market BFT_t response is much larger (more negative) than the complete market response.

Transmission/Sharing of Uncertainty

Figure 3 plots impulse responses of $\kappa_{2t}^{(1)}$ and $\kappa_{2t}^{(2)}$. We focus on κ_{2t} here because variation in κ_{2t}, which is large relative to κ_{3t}, dominates variation in the foreign-home uncertainty differential. Here, we are primarily interested in three things.

Figure 3: $\kappa_{2t}^{(1)}$ and $\kappa_{2t}^{(2)}$ Impulse Responses under LCP



Notes: Impulse responses are to positive country 1 shocks under LCP and are reported in percent per annum. $\kappa_{2t}^{(1)}$ and $\kappa_{2t}^{(2)}$ are the second conditional cumulant of country 1's and country 2's nominal SDF respectively. CM represents complete markets and IM represents incomplete markets. Parameterization follows from Section 4.

First, which shocks are most important in generating country-specific uncertainty? Second, to what extent is risk and uncertainty shared or transmitted from one country to the other? Third, how does the sharing of uncertainty differ under complete and incomplete markets?

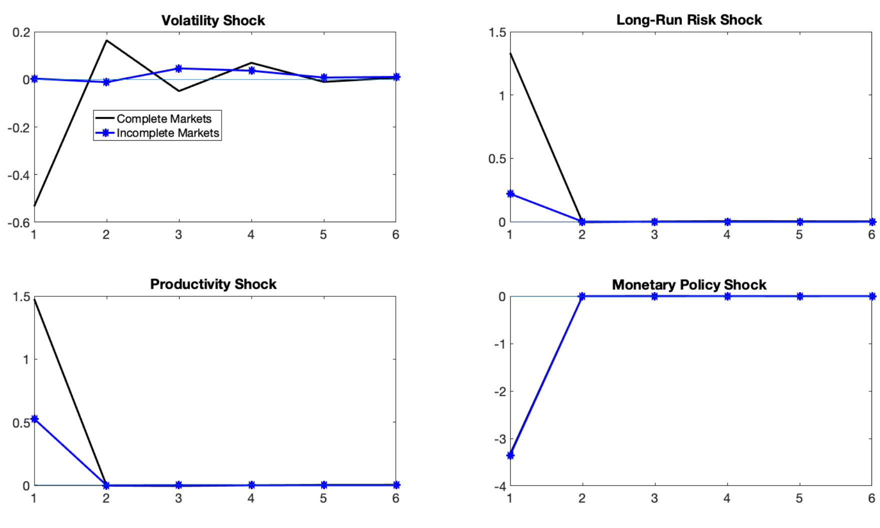
Under complete markets (black lines), there is a near complete transmission or sharing of uncertainty. Positive country 1 shocks raise $\kappa_{2t}^{(1)}$ and $\kappa_{2t}^{(2)}$ by nearly the same amount. Interestingly, long-run risk shocks and productivity growth shocks lower $\kappa_{2t}^{(1)}$ and $\kappa_{2t}^{(2)}$, but the magnitudes are trivial.

Under incomplete markets (blue lines), relative uncertainty is always higher in country 1 after a shock, because the financial structure is effective at insulating (or ineffective at sharing) country 2 uncertainty from country 1 shocks. $\kappa_{2t}^{(2)}$ is barely affected by the country 1 stochastic volatility shock. Clearly, stochastic volatility shocks are the most important in affecting the country-level uncertainty risk factor.

Uncertainty Persistence

Figure 4 shows the impulse responses of the exchange rate forecast error $(\ln(S_{1,2,t}) - E_{t-1}\ln(S_{1,2,t}))$, to country 1 shocks under complete and incomplete markets. Long-run risk and productivity growth shocks generate a one-period unanticipated country 1 exchange rate depreciation whereas monetary policy shocks generate a one period unanticipated country 1 exchange rate appreciation. Except for monetary policy shocks, the forecast errors are larger under complete markets.

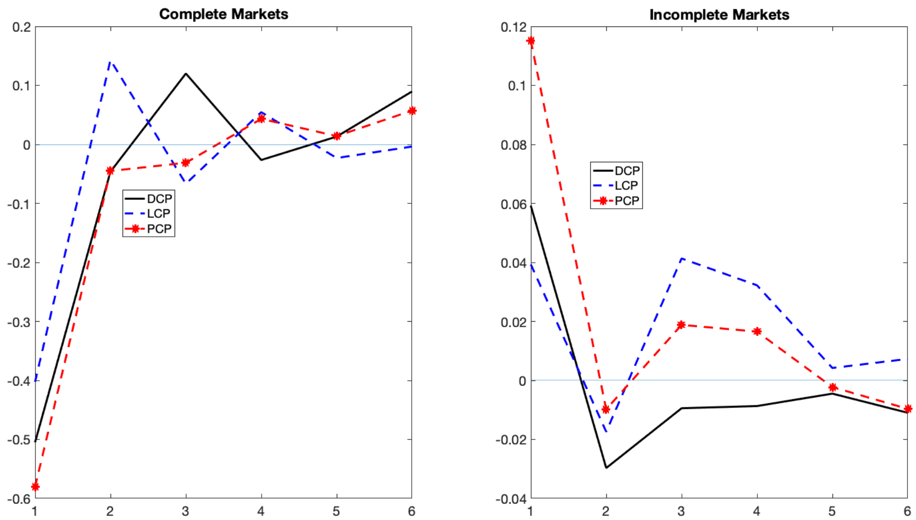
Figure 4: Exchange Rate Forecast Error Impulse Responses under LCP



Notes: The impulse responses are to positive country 1 shocks under LCP and are reported in percent per annum. The exchange rate forecast error is $\ln(S_{1,2,t}) - E_{t-1}ln(S_{1,2,t})$. Parameterization follows from Section 4.

In contrast, the stochastic volatility shock generates persistent exchange rate forecast errors. The direction of the initial errors depends on whether markets are complete or incomplete. There is an initial unanticipated country 1 appreciation under complete markets and an initial unanticipated country 1 depreciation under incomplete markets. The magnitude of the initial exchange rate forecast error is larger under complete markets.

Figure 5: Exchange Rate Forecast Error Impulse Responses to Stochastic Volatility Shock Under Alternative Export Pricing



Notes: The impulse responses are to a positive country 1 stochastic volatility shock under DCP, LCP, and PCP and are reported in percent per annum. The exchange rate forecast error is $ln\left(S_{1,2,t}\right) - E_{t-1}ln\left(S_{1,2,t-1}\right)$. Parameterization follows from Section 4.

Figure 5 zooms in on the forecast error responses to stochastic volatility shocks under complete and incomplete markets. Because the exchange rate response can vary with the export pricing convention, we show the responses under LCP, DCP, and PCP. Under complete markets, the exchange rate forecast error shows an initial unanticipated country 1 appreciation under LCP, PCP, and DCP. Under incomplete markets, the country 1 stochastic volatility shock causes an initial unanticipated country 1 depreciation under all three pricing schemes.

Because the shock results in a persistent elevation of stochastic volatility, there is also a persistent evolution in the underlying conditional distributions. Evidently, the continual variation in the probability distributions interferes with the precision of agents' expectations and suggests why the currency risk premium is so responsive to stochastic volatility shocks.

5.2 Variance Decomposition

Table 3 provides another perspective on the sources of uncertainty and contributions to the currency risk premium. The table reports simulated variance decompositions for some key variables.

For both complete and incomplete markets, only stochastic volatility shocks cause variation in $\kappa_{2t}^{(1)}$ and $\kappa_{2t}^{(2)}$. Most of the variation is from the response to 'own' country stochastic volatility shocks, but the relative importance of the own shock is much greater under incomplete markets.

Table 3: Simulated Variance Decomposition under LCP

	Volatility		Long-Run Risk		Productivity		Monetary		
	$\nu_{1,t}$	$\nu_{2,t}$	$u_{1,t}$	$u_{2,t}$	$\epsilon_{1,t}$	$\epsilon_{2,t}$	$e_{1,t}$	$e_{2,t}$	Total
Complete Markets									
$\kappa_{2t}^{(1)}$	54.208	46.830	0.000	0.000	0.000	0.000	0.000	0.000	101.044
$\kappa_{2t}^{(2)}$	48.398	52.645	0.000	0.000	0.000	0.000	0.000	0.000	101.043
rp_t	50.289	48.859	0.223	0.092	0.070	0.091	0.000	0.000	99.626
$\Delta ln(S_{1,2,t})$	0.774	0.764	4.858	4.992	7.115	6.977	32.675	32.287	90.444
er_t	0.725	0.716	5.719	4.205	5.806	4.394	35.863	35.573	92.993
Incomplete Markets									
$\kappa_{2t}^{(1)}$	99.948	0.044	0.000	0.000	0.000	0.000	0.000	0.000	99.994
$\kappa_{2t}^{(2)}$	0.044	99.960	0.000	0.000	0.000	0.000	0.000	0.000	100.008
rp_t	43.976	43.437	2.069	2.489	1.904	1.517	0.280	0.242	95.914
BFT_t	50.747	50.233	0.000	0.000	0.000	0.000	0.000	0.000	100.983
$\Delta ln(S_{1,2,t})$	0.441	0.438	1.049	0.993	1.078	1.085	47.406	46.477	98.968
er_t	0.030	0.030	0.232	0.230	1.035	1.044	48.921	48.176	99.696

Notes: $\kappa_{2t}^{(1)}$ is the second conditional cumulant of country 1's nominal SDF, $\kappa_{2t}^{(2)}$ is the second conditional cumulant of country 2's nominal SDF, $rp_t = E_t\left(i_{1,t} - i_{2,t} - \Delta ln(S_{1,2,t+1})\right)$ is the currency risk premium, $S_{1,2,t}$ is the nominal exchange rate, $er_t = i_{1,t-1} - i_{2,t-1} - \Delta ln(S_{1,2,t})$ is the ex post currency excess return, and $\text{BFT}_t = \frac{\kappa_{2t}^*}{2!} + \frac{\kappa_{3t}^*}{3!}$. Parameterization follows from Section 4. Averages over 10 replications of 5,000 periods. Numbers may not add up to 100 due to i) non-zero correlation of simulated shocks in small samples and ii) nonlinearity.

Most of the variation in rp_t is generated by stochastic volatility shocks. Long-run risk, productivity growth, and monetary policy shocks are relatively more important sources of rp_t variation under incomplete markets than under complete markets. Under both complete and incomplete markets, variation in the ex post currency excess return (er_t) is driven primarily by the exchange rate depreciation, which in turn is driven primarily by monetary policy shocks.

The message here again, is that stochastic volatility shocks drive the uncertainty-risk factor with non-own shocks mattering more under complete markets than incomplete markets. Stochastic volatility shocks also dominate variation in the currency risk premium, but variation in the ex post currency excess return, whose movements are dominated by the exchange rate, is dominated by monetary policy shocks.

6 Unconditional Moments

This section reports implied unconditional moments from simulations of the model. The purpose here is to examine implied systematic international currency risk (non-zero meaned currency risk premia) and the forward premium bias. Additionally, while quantitative moment matching is not the objective of this paper, it is useful to show those dimensions where the model performs well and where it falls short. We report results only under LCP and relegate PCP and DCP results to an appendix.

6.1 Symmetric Countries

This section shows, under symmetric monetary policy and productivity growth, the cross-country correlations and implied volatility of most of the model variables are reasonable and largely invariant to the degree of risk aversion. The main effect of increasing risk aversion is to raise the uncertainty-risk factor. With regard to the macroeconomic variables, the main shortcoming of the model is insufficient exchange rate volatility.

Table 4 reports implied unconditional moments for risk aversion coefficient (ϕ) values of $4, 40$, and 60. Entries above the line show that implied volatility of the macroeconomic variables in the model are plausible.[11] Under both complete and incomplete markets, cross-country consumption growth correlations are close to zero and lie below output growth correlations. The correlation between log nominal SDFs are near one under complete markets, and are about a quarter that size under incomplete markets. The correlation between the nominal and real exchange rate depreciation is over 0.9. These features of the model are reasonably satisfactory, and none are very sensitive to risk aversion.

Entries below the line identify challenges to the model. While the real exchange rate depreciation is about as volatile as the nominal depreciation, both volatilities are low (about one-third the level in the data). Under complete markets, the reason is because there is too much risk sharing. Examination of the variance of the nominal exchange rate depreciation,

$$\sigma^2(\Delta \ln(S_{1,2,t+1})) = \sigma^2(\ln(N_{2,t+1})) + \sigma^2(\ln(N_{1,t+1})) - 2\text{Cov}(\ln(N_{1,t+1}), \ln(N_{2,t+1})), \tag{40}$$

finds that we have the Brandt et al. (2006) problem where the high covariance between the natural log nominal SDFs, $\text{Cov}(\ln(N_{1,t+1}), \ln(N_{2,t+1}))$, depresses exchange rate depreciation volatility. While eq. (40) does not apply under incomplete markets, exchange rate depreciation volatility is similarly understated even though the log nominal SDFs are much less correlated. Due to the symmetry across countries, this model produces only a trivial mean currency risk premium and

[11] Since the countries are symmetric, we only report the volatility and the first autocorrelation coefficients of the country 1 variables. We compare model implied volatility and the data in the next section when we consider productivity heterogeneity across countries.

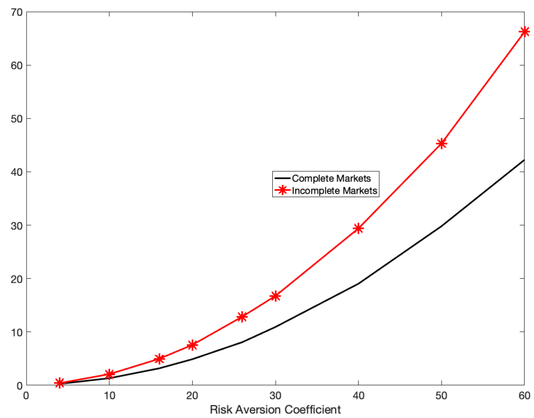
trivial forward premium bias.

Table 4: Unconditional Moments and Risk Aversion Coefficient under LCP

	Complete Markets			Incomplete Markets		
Risk Aversion Coefficient	4	40	60	4	40	60
$\sigma(\Delta \ln y_{1,t})$	2.928	2.959	2.937	3.031	3.058	3.111
$\sigma(\Delta \ln c_{1,t})$	3.218	3.215	3.303	3.509	3.536	3.636
$\sigma(i_{1,t})$	1.361	1.537	1.499	1.184	1.254	1.379
$\sigma(r_{1,t})$	1.460	1.467	1.466	1.461	1.459	1.484
$\sigma(\pi_{1,t})$	1.462	1.637	1.594	1.121	1.193	1.308
$\rho(\Delta \ln y_{1,t}, \Delta \ln y_{2,t})$	0.317	0.330	0.338	0.178	0.182	0.169
$\rho(\Delta \ln c_{1,t}, \Delta \ln c_{2,t})$	0.051	0.054	0.037	-0.073	-0.079	-0.104
$\rho(\ln N_{1,t}, \ln N_{2,t})$	0.993	0.999	0.999	0.232	0.267	0.237
$\rho(\Delta \ln Q_{1,2,t}, \Delta \ln S_{1,2,t})$	0.913	0.907	0.904	0.968	0.964	0.959
$\Delta \ln y_{1,t}$–AR(1)	0.038	0.028	0.0327	-0.021	-0.031	-0.0445
$\Delta \ln c_{1,t}$–AR(1)	-0.119	-0.125	-0.132	-0.083	-0.090	-0.104
$\pi_{1,t}$–AR(1)	0.760	0.751	0.748	0.605	0.572	0.574
$\sigma(\Delta \ln S_{1,2,t})$	5.732	6.085	6.157	4.986	5.081	5.151
$\sigma(\Delta \ln Q_{1,2,t})$	5.361	5.731	5.767	4.364	4.398	4.436
$\mu(rp_t)$	-0.001	0.004	-0.024	0.000	0.006	-0.008
$\mu(er_t)$	0.010	0.032	0.000	0.000	0.002	-0.021
β_F	1.013	1.011	1.019	0.992	1.001	0.965

Notes: $\mu(\bullet)$ is the mean of the variable stated, $\sigma(\bullet)$ is the volatility or standard deviation of the variable stated, $\rho(\bullet)$ is the correlation of the variables stated, and AR(1) is the first-order autocorrelation coefficient of the variable stated. $y_{1,t}$ is output in country 1, $c_{1,t}$ is consumption in country 1, $i_{1,t}$ is the nominal interest rate in country 1, $r_{1,t}$ is the real interest rate in country 1, $\pi_{1,t}$ is inflation in country 1, $N_{1,t}$ and $N_{2,t}$ are the nominal SDFs in countries 1 and 2, $Q_{1,2,t}$ is the real exchange rate, $S_{1,2,t}$ is the nominal exchange rate, $rp_t = E_t(i_{1,t} - i_{2,t} - \Delta ln(S_{1,2,t+1}))$ is the currency risk premium, $er_t = i_{1,t-1} - i_{2,t-1} - \Delta ln(S_{1,2,t})$ is the ex post currency excess return, and β_F is the Fama coefficient. Parameterization follows from Section 4 except for the risk aversion coefficient (ϕ). Averages over 10 replications of 5,000 periods.

Figure 6: $\mu(\kappa_{2t}^{(1)})$ and Risk Aversion Coefficient under LCP



Notes: $\mu(\kappa_{2t}^{(1)})$ is the mean of the second conditional cumulant of country 1's nominal SDF and is stated in percent per annum. Parameterization follows from Section 4 except for the risk aversion coefficient (ϕ). Averages over 10 replications of 5,000 periods.

Significantly, the risk aversion coefficient has almost no effect on the unconditional moments in Table 4. What the risk aversion coefficient does affect is the uncertainty-risk factor. Figure 6 shows, for a given risk aversion coefficient, perception of uncertainty is higher under incomplete markets than complete markets, but in both cases, $\mu(\kappa_{2t}^{(1)})$ is increasing in the risk aversion coefficient.[12]

Table 5: Autocorrelation of the Risk Premium (rp_t) and BFT_t under LCP

	Complete Markets rp_t	Incomplete Markets rp_t	Incomplete Markets BFT_t
Autocorrelation	0.840	0.866	0.851
t-ratio	(109.463)	(122.373)	(114.654)
R^2	0.706	0.750	0.725
SER	0.185	0.051	9.603

Notes: $rp_t = E_t(i_{1,t} - i_{2,t} - \Delta \ln(S_{1,2,t+1}))$ is the currency risk premium and $\text{BFT}_t = \frac{\kappa_{2t}^*}{2} + \frac{\kappa_{3t}^*}{6}$. T-ratios in parentheses. SER is the standard error of the regression in percent per annum. Parameterization follows from Section 4. The sample size is 5,000 periods.

Next, we look into why there is no forward premium bias even though there is persistent (but zero-meaned) variation in the currency risk premium.

[12]Since the countries are symmetric, we only report the values for country 1.

Looking at Table 5, the first-order autocorrelation coefficients of rp_t and BFT_t, imply a fairly high degree of persistence. The standard error of the regression (SER) for the incomplete markets BFT_t shows relatively high variability. Recall, whether markets are complete or incomplete, the interest rate differential is $i_{1,t} - i_{2,t} = (\kappa^*_{1t} + \kappa^*_{2t}/2! + \kappa^*_{3t}/3!)$.[13] Regressing $\Delta \ln(S_{1,2,t+1})$ on the interest rate differential $(\kappa^*_{1t} + \kappa^*_{2t}/2! + \kappa^*_{3t}/3!)$ gives β_F in Table 4.

Table 6: Unpacking the Fama Regression under LCP

A. Relative Conditional Cumulant Moments				
	$\sigma(\kappa^*_{1t})$	$\sigma(\kappa^*_{2t}/2)$	$\sigma(\kappa^*_{3t}/6)$	$\rho(\kappa^*_{1t}, \kappa^*_{2t})$
Complete Markets	2.503	0.332	9.44e-8	-0.254
Incomplete Markets	17.519	17.966	1.62e-9	-0.996
B. Eq. (41) Regression				
	Complete Markets		Incomplete Markets	
	Coeff	t-ratio	Coeff	t-ratio
β_{F1}	0.987	29.461	1.064	24.674
β_{F2}	0.181	0.325	1.060	25.219
β_{F3}	8.11e-5	0.943	4.21e-7	1.001

Notes: $\sigma(\bullet)$ is the volatility or standard deviation of the variable stated and $\rho(\bullet)$ is the correlation of the variables stated. $\kappa^*_{jt} = \kappa^{(2)}_{jt} - \kappa^{(1)}_{jt}$ for $j \in \{1, 2, 3\}$. Eq. (41) is $\Delta \ln(S_{1,2,t+1}) = \beta_0 + \beta_{F1}\kappa^*_{1t} + \beta_{F2}\frac{\kappa^*_{2t}}{2} + \beta_{F3}\frac{\kappa^*_{3t}}{6} + e_{t+1}$. Coeff represents coefficient. Parameterization follows from Section 4. The sample size is 5,000 periods.

Under complete markets, because $\kappa^*_{1t} = E_t(\Delta \ln S_{1,2,t+1})$, a forward premium anomaly would seem to require a rather large negative covariance between κ^*_{1t} and κ^*_{2t}, and for κ^*_{2t} to dominate variation in the interest differential. To investigate some properties of the interest rate differential's factor components, Panel A of Table 6 shows that κ^*_{2t} and κ^*_{1t} are mildly negatively correlated under complete markets and nearly perfectly negatively correlated under incomplete markets. Periods of high κ^*_{2t} are associated with periods of low κ^*_{1t} either because of higher relative conditional variance of future utility or higher relative expected consumption growth, the latter of which is consistent with higher relative current saving which reinforces an increase in relative precautionary saving.

Now suppose we allow the components of the interest rate differential to enter separately in the Fama regression,[14]

$$\Delta \ln(S_{1,2,t+1}) = \beta_0 + \beta_{F1}\kappa^*_{1t} + \beta_{F2}\frac{\kappa^*_{2t}}{2} + \beta_{F3}\frac{\kappa^*_{3t}}{6} + e_{t+1}. \quad (41)$$

Panel B of Table 6 shows the regression of eq. (41) estimated with data generated from the model. Under complete markets, β_{F2} is much smaller than one, meaning that the κ^*_{2t} component

[13] Recall, we truncate the series expansion at order three.

[14] Since the independent variable is the decomposition of the interest rate differential, running this regression is legitimate for both complete and incomplete markets.

contributes toward a forward premium bias, but because its variation contributes relatively little to the variation in the interest rate differential, the overall forward premium bias remains small. The κ_{1t}^* component, due to its dominant role in governing the interest rate differential, dominates the Fama coefficient with β_{F1} near one. Since $\kappa_{1t}^* = E_t\left(\Delta \ln(S_{1,2,t+1})\right)$, generating a forward premium anomaly (slope coefficient less than zero) under complete markets would seem to require introducing additional shocks, such as taste shocks.

Under incomplete markets, the κ_{2t}^* component of the interest rate differential is more important, but because both the κ_{1t}^* and κ_{2t}^* components exhibit approximately equal variation and both have slopes near one, there is almost no forward premium bias.

6.2 Heterogeneous Countries

The symmetric country model does not produce systematic risk in the sense of a non-zero meaned currency risk premium. In this section, we study how cross-country differences in monetary policy and productivity growth can affect systematic currency risk premia and the forward premium bias. We begin with heterogeneous monetary policy, then heterogeneous productivity growth, followed by a combination of heterogeneous monetary policy and productivity growth.

6.2.1 Heterogeneous Monetary Policy

Table 7 reports effects from monetary policy heterogeneity but symmetric productivity. We begin with country 1 following the benchmark Taylor rule ($\xi_1 = 1.5$ and $\zeta_1 = 0.5$) and letting country 2 deviate by targeting inflation ($\xi_2 = 4$ and $\zeta_2 = 0.1$), by setting very procyclical interest rates ($\xi_2 = 1.5$ and $\zeta_2 = 2.0$), and through a lack of monetary discipline (accommodating inflation and being nearly unresponsive to the output gap $\xi_2 = 1.1$ and $\xi_2 = 0.1$).

We denote the mean value of a variable x_t by $\mu(x_t)$. Under complete markets, $\mu(\kappa_{2t}^{(2)})$ is greater than $\mu(\kappa_{2t}^{(1)})$ when country 2 is an inflation targeter, lacks monetary discipline, and accommodates inflation and these policies generate a corresponding positive currency risk premium. A procyclical interest rate policy in country 2 produces the reverse pattern, as is also predicted by the complete markets mechanism. None of these cases produce a sizable forward premium bias, however.

Backus et al. (2013) also study monetary policy induced international currency risk. They work with a two-country complete markets endowment economy model where exogenous symmetric consumption growth follows a stochastic volatility (and long-run risk) process. Their interest rates are governed by Taylor-type feedback rules (but without interest rate smoothing), which generates endogenous inflation. Two central predictions from their paper are (i) the currency risk premium rp_t, becomes increasingly negative as monetary policy in country 2 becomes relatively more procyclical (increasing $\zeta_2 - \zeta_1$) and (ii) the currency risk premium becomes increasingly negative when monetary policy in country 2 becomes relatively more accommodative to inflation (increasing $\xi_1 - \xi_2$).

Table 7: Heterogeneous Monetary Policy under LCP

	Complete Markets				Incomplete Markets			
		Country 2				Country 2		
	Inflation Targeter	Procyclical Interest Rate	Undis-ciplined	Accommo-dates Inflation	Inflation Targeter	Procyclical Interest Rate	Undis-ciplined	Accommo-dates Inflation
ξ_1	1.5	1.5	1.5	2.5	1.5	1.5	1.5	2.5
ζ_1	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
ξ_2	4.0	1.5	1.1	1.3	4.0	1.5	1.1	1.3
ζ_2	0.1	2.0	0.1	0.5	0.1	2.0	0.1	0.5
$\mu(rp_t)$	0.077	-0.230	0.277	0.049	0.077	-0.324	0.354	0.063
$\mu(er_t)$	0.077	-0.207	0.223	0.042	0.077	-0.294	0.324	0.090
β_F	1.014	1.079	1.008	1.007	1.014	0.993	0.942	0.999
$\mu(\kappa_{2t}^{(1)})$	18.944	19.160	19.110	19.011	18.944	30.656	29.016	29.669
$\mu(\kappa_{2t}^{(2)})$	19.104	18.699	19.663	19.108	19.104	29.089	30.795	30.118

Notes: $\mu(\bullet)$ is the mean of the variable stated. $rp_t = E_t(i_{1,t} - i_{2,t} - \Delta\ln(S_{1,2,t+1}))$ is the currency risk premium, $er_t = i_{1,t-1} - i_{2,t-1} - \Delta\ln(S_{1,2,t})$ is the ex post currency excess return, β_F is the Fama coefficient, $\kappa_{2t}^{(1)}$ is the second conditional cumulant of country 1's nominal SDF, and $\kappa_{2t}^{(2)}$ is the second conditional cumulant of country 2's nominal SDF. Except for the monetary policy parameters, parameterization follows from Section 4. Averages over 10 replications of 5,000 periods.

The results in our production model are consistent with Backus et al. (2013)'s prediction (i) when comparing the procyclical interest rate results from Table 7 against the results in Table 4 with a risk aversion coefficient of 40 as the benchmark. In the specification labeled 'Accomodates Inflation', our model results are contrary to their prediction (ii). Here, country 2 is relatively more accommodative to inflation ($\xi_2 = 1.3$, $\zeta_2 = 0.5$, $\xi_1 = 2.5$, and $\zeta_1 = 0.5$), but the currency risk premium increases from 0.004 in Table 4 to 0.in 049 Table 7 under complete markets and from 0.006 in Table 4 to 0.063 in Table 7 under incomplete markets.[15]

In Table 7 under incomplete markets, when $\mu(\kappa_{2t}^{(2)})$ is greater than $\mu(\kappa_{2t}^{(1)})$, the currency risk premium is positive and vice versa when $\mu(\kappa_{2t}^{(2)})$ is less than $\mu(\kappa_{2t}^{(1)})$, as is the case under complete markets. Although the signs and magnitudes of the currency risk premia are similar under complete and incomplete markets, the mechanisms are different.

6.2.2 Heterogeneous Productivity Growth

This section maintains symmetric monetary policy ($\xi = 1.5$ and $\zeta = 0.5$) to isolate the effects of cross-country heterogeneity in productivity growth. To set reasonable parameters for productivity growth across countries, we estimate the productivity growth with long-run risk and stochastic

[15] In Backus et al. (2013)'s endowment model, setting the cross-country correlations of shocks in the consumption process to 0.99, $(\xi_1, \zeta_1, \xi_2, \zeta_2) = (4.423, 0.2, 1.264, 0.866)$ and risk aversion to 90 produces $\beta_F = -1.019$. These settings do not produce a forward premium anomaly in our production model. Setting the cross-country shock correlations to 0.99 and the risk aversion coefficient to 40 gives $\beta_F = 1.036$.

volatility process (eqs. (35)–(37)) for Japan, Australia, and Canada. The posterior means and the upper and lower 5% bands from a Bayesian estimation are shown in Table 8.

Table 8: Posterior Means–Productivity Growth Processes for Australia, Canada, and Japan

	Japan			Australia			Canada		
	Posterior Mean	5%	95%	Posterior Mean	5%	95%	Posterior Mean	5%	95%
μ_g	0.004	0.002	0.006	0.001	-0.001	0.002	0.001	-0.001	0.002
θ	1.288	0.843	1.838	1.634	1.108	2.117	0.618	0.197	1.020
ρ_x	0.744	0.666	0.824	0.731	0.652	0.817	0.742	0.664	0.823
ρ_σ	0.839	0.761	0.920	0.842	0.765	0.921	0.841	0.762	0.919
η	-1.859	-4.375	0.512	-1.753	-4.212	0.690	-1.159	-3.708	1.340
μ_σ	-5.864	-6.391	-5.458	-6.376	-6.872	-5.910	-6.021	-6.334	-5.683

Notes: The productivity growth processes for Australia, Canada, and Japan are governed by eqs. (35)–(37). Posterior means and the upper and lower 5% bands from a Bayesian estimation are reported.

Table 9 shows simulated TFP growth generated by the estimated process (labeled unadjusted, as was the case for the United States) overstates the volatility in the data. The overstatement for Japan is minor and a bit more substantial for Australia and Canada. For all three countries, we apply an adjustment factor as we did for the United States. The panel labeled 'Adjusted' shows the standard deviations of the simulated productivity growth ($\sigma(\Delta a_t)$) and its components.

Table 9: Volatility of Productivity Growth and Components for Australia, Canada, and Japan

	Data	Simulated					
		Unadjusted			Adjusted		
	$\sigma(\Delta a_t)$	$\sigma(\Delta a_t)$	$\sigma(x_t)$	$\sigma(\sigma_t)$	$\sigma(\Delta a_t)$	$\sigma(x_t)$	$\sigma(\sigma_t)$
Australia	3.582	4.082	1.149	0.238	3.581	0.983	0.211
Canada	2.275	3.372	2.107	0.761	2.271	1.427	0.525
Japan	4.392	4.869	1.906	0.345	4.386	1.673	0.314

Notes: $\sigma(\bullet)$ is the volatility or standard deviation of the variable stated as percent per annum. a_t is log productivity, x_t is the long-run risk component, and σ_t is the stochastic volatility component.

Figure 7 plots productivity growth and log levels from the data along with a realized simulation of the adjusted process. As can be seen, the model does a reasonable job of capturing major features of the productivity data.

Figure 7: Productivity Data and Realized Simulation

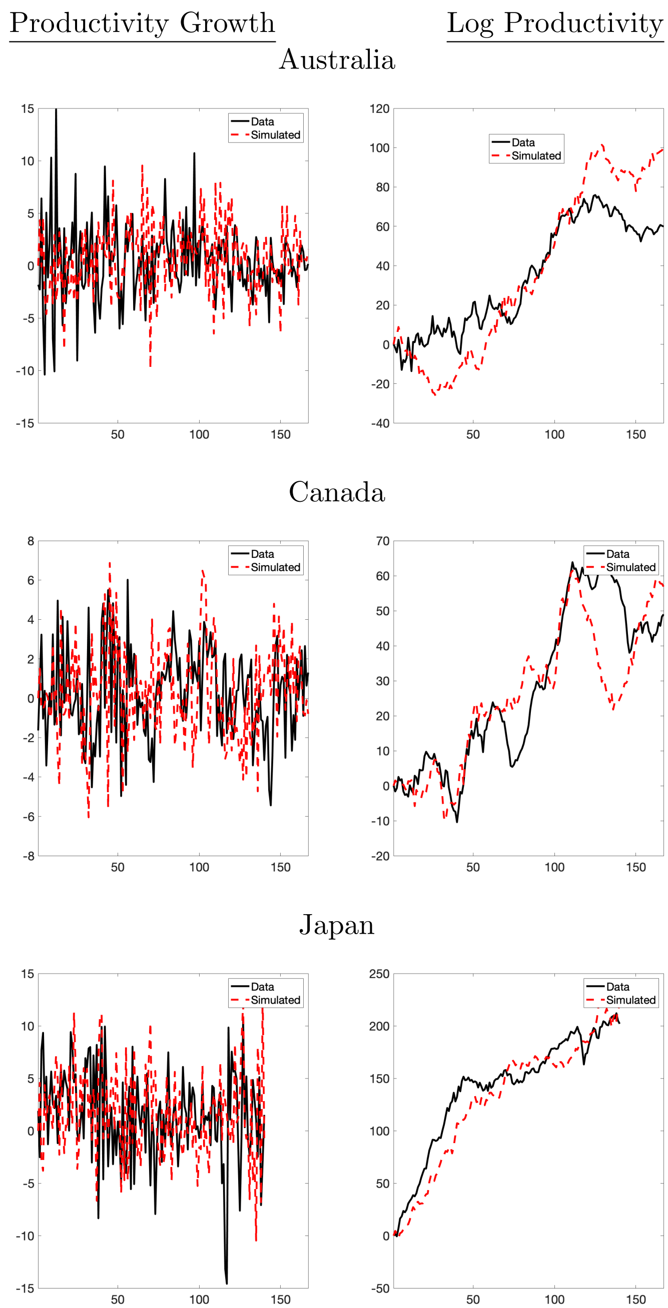


Table 10: Heterogeneous Productivity Growth under LCP (Australia, Canada, United States, and Japan)

	United States (1)–Japan (2)			Australia (1)–Japan (2)			Canada (1)–Japan (2)		
	Data	Complete Markets	Incomplete Markets	Data	Complete Markets	Incomplete Markets	Data	Complete Markets	Incomplete Markets
$\mu(rp_t)$	–	0.621	1.071	–	0.472	0.745	–	0.530	1.071
$\mu(er_t)$	0.490	0.619	1.046	4.856	0.470	0.771	2.887	0.573	1.054
β_F	-2.820	1.008	0.956	-0.465	0.998	0.952	-2.863	1.002	1.002
$\mu(\kappa_{2t}^{(1)})$	–	39.937	31.054	–	46.873	48.883	–	38.461	34.949
$\mu(\kappa_{2t}^{(2)})$	–	41.178	94.782	–	47.817	97.158	–	39.522	94.955
$\sigma(\Delta \ln S_{1,2,t})$	20.990	6.869	5.317	24.825	6.976	5.376	23.525	6.863	5.387
$\sigma(\Delta \ln Q_{1,2,t})$	21.204	6.582	4.631	24.978	6.666	4.676	24.257	6.548	4.657
$\sigma(\Delta \ln y_{1,t})$	3.713	4.393	3.050	4.128	4.217	3.555	3.378	4.330	2.983
$\sigma(\Delta \ln y_{2,t})$	3.986	3.959	4.188	3.986	3.865	4.205	3.986	4.044	4.193
$\sigma(\Delta \ln c_{1,t})$	3.198	3.185	3.625	3.118	3.489	4.090	2.709	3.167	3.619
$\sigma(\Delta \ln c_{2,t})$	3.841	4.736	4.564	3.841	4.482	4.607	3.841	4.849	4.572
$\sigma(\pi_{1,t})$	2.623	3.612	1.234	4.322	2.672	1.258	3.609	3.751	1.345
$\sigma(\pi_{2,t})$	4.745	5.882	1.528	4.745	4.694	1.546	4.745	5.985	1.541
$\Delta \ln y_{1,t}$–AR(1)	0.362	-0.165	-0.042	-0.058	-0.058	0.016	0.386	-0.182	-0.057
$\Delta \ln y_{2,t}$–AR(1)	0.129	0.283	0.095	0.129	0.293	0.093	0.129	0.282	0.101
$\Delta \ln c_{1,t}$–AR(1)	0.083	-0.087	-0.093	0.145	-0.052	-0.048	0.078	-0.103	-0.117
$\Delta \ln c_{2,t}$–AR(1)	-0.163	-0.093	0.007	-0.163	-0.079	0.005	-0.163	-0.103	0.012
$\pi_{1,t}$–AR(1)	0.798	0.878	0.590	0.595	0.838	0.555	0.652	0.879	0.603
$\pi_{2,t}$–AR(1)	0.553	0.901	0.509	0.553	0.882	0.508	0.553	0.899	0.511

Notes: $\mu(\bullet)$ is the mean of the variable stated, $\sigma(\bullet)$ is the volatility or standard deviation of the variable stated, and AR(1) is the first-order autocorrelation coefficient of the variable stated. $rp_t = E_t(i_{1,t} - i_{2,t} - \Delta \ln(S_{1,2,t+1}))$ is the currency risk premium, $er_t = i_{1,t-1} - i_{2,t-1} - \Delta \ln(S_{1,2,t})$ is the ex post currency excess return, β_F is the Fama coefficient, $\kappa_{2t}^{(1)}$ is the second conditional cumulant of country 1's nominal SDF, $\kappa_{2t}^{(2)}$ is the second conditional cumulant of country 2's nominal SDF, $S_{1,2,t}$ is the nominal exchange rate, $Q_{1,2,t}$ is the real exchange rate, $y_{1,t}$ is output in country 1, $y_{2,t}$ is output in country 2, $c_{1,t}$ is consumption in country 1, $c_{2,t}$ is consumption in country 2, $\pi_{1,t}$ is inflation in country 1, and $\pi_{2,t}$ is inflation in country 2. Country-specific productivity growth process parameters are employed. Except for the productivity growth process parameters, parameterization follows from Section 4. Averages over 10 replications of 5,000 periods.

Table 10 shows the simulation results using using adjusted country-specific productivity processes for the United States, Australia, Canada, and Japan. In each case, Japan is country 2. The table also include moments from the data. Though we are not conducting a quantitative moment matching exercise, the estimated TFP growth processes are used as guidance for a thought experiment on the effect of reasonable productivity asymmetries across countries and the comparison to the data moments serves as a reality check on the model.

Except for exchange rate volatility, the implied moments shown below the line are not unreasonably far from the data. In the data, consumption growth for Japan is always more volatile than for the partner country. This is the case in the model under both complete and incomplete markets. In the data, Japan's output growth is more volatile than the United States and Canada, while Australia's output growth is more volatile than Japan's. Relative output growth volatility matches the data for the United States–Japan and Canada–Japan under incomplete markets and for Australia–Japan under complete markets.

In regard to uncertainty risk, due to historically low interest rates, the Japanese yen is typically thought of as the carry trade funding currency.[16] As can be seen in Table 10, $\mu(\kappa_{2t})$ is always higher in Japan. The model generates a positive mean currency risk premium for our countries paired with Japan under complete and incomplete markets. There is also a modest forward premium bias for the United States and Australia under incomplete markets.

6.2.3 Heterogeneous Monetary Policy and Productivity Growth

Here, we combine country 2 departures from the benchmark Taylor-rule with cross-country heterogeneity in productivity growth using the United States (country 1) and Japan (country 2). Results are shown in Table 11.

The implied mean currency risk premium is larger under incomplete markets than under complete markets, has the correct sign and exceeds the mean ex post currency excess return in the data. Assessments of uncertainty are also consistently much higher in Japan under incomplete markets than under complete markets. The complete markets story where the foreign-home uncertainty differential drives the currency risk premium appears also to be at work under incomplete markets when the difference $\mu(\kappa_{2t}^{(2)}) - \mu(\kappa_{2t}^{(1)})$, is large as in Table 11. Compared to entries in Table 10, the mean currency risk premium increases and there is more of a forward premium bias when Japan is an inflation targeter and when Japan's monetary policy is undisciplined.

[16]The carry trade is a trading strategy where you short the low interest rate currency and go long the high interest rate currency.

Table 11: Heterogeneous Monetary Policy and Productivity Growth under LCP (United States and Japan)

		Japan Inflation Targeter		Japan Undisciplined		Japan Undisciplined*	
	Data	Complete Markets	Incomplete Markets	Complete Markets	Incomplete Markets	Complete Markets	Incomplete Markets
ξ_2	–	4.0	4.0	1.1	1.1	1.1	1.1
ζ_2	–	0.1	0.1	0.1	0.1	0.1	0.1
$\mu(rp_t)$	–	0.993	1.494	1.272	2.100	1.608	2.364
$\mu(er_t)$	0.490	1.050	1.486	1.191	2.114	1.602	2.361
β_F	-2.820	0.996	0.954	0.910	0.863	0.508	0.630
$\mu(\kappa_{2t}^{(1)})$	–	39.020	31.423	41.747	31.231	74.284	45.631
$\mu(\kappa_{2t}^{(2)})$	–	41.007	96.067	44.290	97.037	77.506	112.610

Notes: $\mu(\bullet)$ is the mean of the variable stated. $rp_t = E_t(i_{1,t} - i_{2,t} - \Delta\ln(S_{1,2,t+1}))$ is the currency risk premium, $er_t = i_{1,t-1} - i_{2,t-1} - \Delta\ln(S_{1,2,t})$ is the ex post currency excess return, β_F is the Fama coefficient, $\kappa_{2t}^{(1)}$ is the second conditional cumulant of country 1's nominal SDF, and $\kappa_{2t}^{(2)}$ is the second conditional cumulant of country 2's nominal SDF. Monetary policy parameters for the United States (country 1) are $\xi_1 = 1.5$ and $\zeta_1 = 0.5$. Country-specific productivity growth process parameters are employed. Except for the monetary policy and productivity growth process parameters, parameterization follows from Section 4. *: Cross-country correlations of exogenous shocks are set to 0.99 as in Backus et al. (2013). Averages over 10 replications of 5,000 periods.

The specifications in Table 11 do not generate a negative Fama coefficient. Under incomplete markets, and undisciplined monetary policy in Japan, we only obtain $\beta_F = 0.863$. The endowment economy model of Backus et al. (2013) is able to generate a forward premium anomaly through differences in monetary policy. They use a more general recursive utility function with intertemporal elasticity of substitution of 1.5 and risk aversion of 90, but importantly, they assume that the cross-country correlation of shocks to exogenous consumption growth and stochastic volatility dynamics are 0.99. To see how near perfectly correlated shock innovations work in our production model, the last column of the table sets each of the innovation correlations to be 0.99 combined with undisciplined monetary policy in Japan. Here, the Fama coefficient declines to $\beta_F = 0.51$ under complete markets and 0.63 under incomplete markets. There is a bit more forward premium bias but no forward premium anomaly. Generating a forward premium anomaly in general equilibrium without additional shocks remains a challenge.[17]

[17] Chen et al. (2021) make a similar point in the context of a general equilibrium model with financial frictions.

Table 12: Heterogeneous Monetary Policy and Productivity Growth under LCP (Australia, Canada, and United States)

	United States (1)–Australia (2)				United States (1)–Canada (2)				Canada (1)–Australia (2)			
	Data	Bench-mark	United States Undis-ciplined	Australia Procyclical Interest Rate	Data	Bench-mark	United States Undis-ciplined	Canada Procyclical Interest Rate	Data	Bench-mark	Canada Undis-ciplined	Australia Procyclical Interest Rate
ξ_1	–	1.5	1.1	1.5	–	1.5	1.1	1.5	–	1.5	1.1	1.5
ζ_1	–	0.5	0.1	0.5	–	0.5	0.1	0.5	–	0.5	0.1	0.5
ξ_2	–	1.5	1.5	1.5	–	1.5	1.5	1.5	–	1.5	1.5	1.5
ζ_2	–	0.5	0.5	2.0	–	0.5	0.5	2.0	–	0.5	0.5	2.0
Complete Markets												
$\mu(rp_t)$	–	0.218	-0.103	-0.136		0.015	-0.314	-0.276	–	0.194	-0.203	-0.148
$\mu(er_t)$	-4.366	0.233	-0.122	-0.181	-2.398	-0.024	-0.270	-0.290	-1.968	0.206	-0.227	-0.174
β_F	-0.553	0.999	0.989	1.028	-0.074	0.999	0.994	1.039	0.840	0.989	1.007	1.137
$\mu(\kappa_{2t}^{(1)})$	–	25.255	25.604	24.681	–	20.380	20.922	20.481	–	26.070	26.838	25.936
$\mu(\kappa_{2t}^{(2)})$	–	25.690	25.399	24.407	–	20.409	20.295	19.928	–	26.459	26.431	25.639
$\mu(\kappa_{1t}^{(1)})$	–	-14.896	-8.881	-14.472	–	-12.317	-7.181	-12.270	–	-15.189	-8.671	-15.054
$\mu(\kappa_{1t}^{(2)})$	–	-14.207	-14.101	-13.642	–	-12.272	-12.144	-12.627	–	-14.518	-14.747	-14.520
$\mu(\kappa_{1t}^{*}+\kappa_{2t}^{*}/2)$	–	0.942	-5.324	0.808	–	0.045	-5.260	-0.631	–	0.880	-6.281	0.585
$\rho(rp_t, bot_{1,t})$	–	0.028	-0.034	0.015	–	0.056	0.055	0.066	–	0.011	-0.040	0.004
Incomplete Markets												
$\mu(rp_t)$	–	0.352	-0.010	-0.113	–	0.010	-0.352	-0.389	–	0.319	-0.171	-0.130
$\mu(er_t)$	-4.366	0.339	-0.030	-0.111	-2.398	0.035	-0.339	-0.380	-1.968	0.300	-0.147	-0.091
β_F	-0.553	0.985	0.980	1.005	-0.074	0.984	0.984	1.000	0.840	0.984	0.945	0.985
$\mu(\kappa_{2t}^{(1)})$	–	30.874	31.212	30.467	–	30.221	30.545	29.693	–	34.658	35.163	33.900
$\mu(\kappa_{2t}^{(2)})$	–	47.592	48.040	46.164	–	33.107	32.662	32.183	–	47.978	48.044	46.649
$\mu(\kappa_{1t}^{(1)})$	–	-16.886	-9.106	-16.629	–	-16.529	-8.795	-16.251	–	-18.744	-9.384	-18.271
$\mu(\kappa_{1t}^{(2)})$	–	-24.283	-24.418	-24.929	–	-18.005	-17.834	-18.752	–	-24.421	-24.470	-25.123
$\mu(\kappa_{1t}^{*}+\kappa_{2t}^{*}/2)$	–	1.042	-6.904	-0.352	–	0.031	-8.006	-1.180	–	0.922	-8.657	-0.443
$\rho(rp_t, bot_{1,t})$	–	-0.239	-0.291	-0.269	–	-0.347	-0.255	-0.156	–	-0.287	-0.295	-0.294

Notes: $\mu(\bullet)$ is the mean of the variable stated and $\rho(\bullet)$ is the correlation of the variables stated. $rp_t = E_t(i_{1,t} - i_{2,t} - \Delta\ln(S_{1,2,t+1}))$ is the currency risk premium, $er_t = i_{1,t-1} - i_{2,t-1} - \Delta\ln(S_{1,2,t})$ is the ex post currency excess return, β_F is the Fama coefficient, $\kappa_{2t}^{(1)}$ is the second conditional cumulant of country 1's nominal SDF, $\kappa_{2t}^{(2)}$ is the second conditional cumulant of country 2's nominal SDF, $\kappa_{1t}^{(1)}$ is the first conditional cumulant of country 1's nominal SDF, $\kappa_{1t}^{(2)}$ is the first conditional cumulant of country 2's nominal SDF, $\kappa_{1t}^{*} + \kappa_{2t}^{*}/2 = (\kappa_{1t}^{(2)} - \kappa_{1t}^{(1)}) + \frac{\kappa_{2t}^{(2)} - \kappa_{2t}^{(1)}}{2}$, and $bot_{1,t}$ is saving in country 1 ($\frac{y_{1,t} - c_{1,t}}{y_{1,t}}$). Country-specific productivity growth process parameters are employed. Except for the monetary policy and productivity growth process parameters, parameterization follows from Section 4. Averages over 10 replications of 5,000 periods.

In Table 12, we exclude Japan from the country pairs. We set the United States (country 1) against Australia (country 2), the United States (country 1) against Canada (country 2) and Canada (country 1) against Australia (country 2). To be consistent with the data, the currency risk premium in these cases should be negative. In the columns labeled 'Benchmark' monetary policy is symmetric and the currency risk premium is never negative. In each of the other columns, we find that monetary policy heterogeneity can turn the mean currency risk premium negative.

Under complete markets, these monetary policies make $\mu(rp_t)$ negative driven primarily because $\mu(\kappa_{2t}^{(2)}) - \mu(\kappa_{2t}^{(1)})$ is negative. Under incomplete markets, this mechanism does not fully explain the currency risk premium. Although monetary policy can turn the mean currency risk premium to be negative as in the data, $\mu(\kappa_{2t}^{(2)})$ is greater than $\mu(\kappa_{2t}^{(1)})$.[18] While the conditional mean of the log nominal SDF (κ_{1t}) is irrelevant for the currency risk premium under complete markets, it matters under incomplete markets. Notice that under incomplete markets in Table 12, $\mu(\kappa_{1t}^{(2)})$ is less than $\mu(\kappa_{1t}^{(1)})$. Country 2 has a lower valuation of future cash flows than country 1. Ignoring $\kappa_{3t}/3!$, which is trivial, the sum of the first two conditional cumulant differentials ($\kappa_{1t}^* + \kappa_{2t}^*/2!$) is less than zero, which implies that the interest rate in country 2 is higher than the interest rate in country 1, and thus, since the interest rate differential component of the currency risk premium dominates the exchange rate change component, the mean currency risk premium is negative. In other words, overall, country 2 wants to save less than country 1 so bond prices are lower and interest rates are higher in country 2 relative to country 1. Higher saving in country 1 relative to country 2 should be associated with a low currency risk premium rp_t, and this is borne out with negative correlations between rp_t and country 1's balance of trade $\rho(\text{rp}_t, \text{bot}_{1,t})$. Note that under complete markets, this correlation is small and close to zero.

7 Conclusion

This paper seeks to deepen our understanding of the currency risk premium and forward premium bias under a variety of economic environments encompassed by a two-country dynamic stochastic general equilibrium New Keynesian model.

We find stochastic volatility shocks to productivity growth are the primary drivers of variation in uncertainty-risk factors and the currency risk premium, whereas monetary policy shocks are the primary drivers of exchange rate variation. The currency in which export prices are has little effect on the aggregate amount of risk in the economy, and therefore is not central for understanding the systematic currency risk premium or forward premium bias.

[18] In these cases, $\mu(\kappa_{2t}^{(2)}) > \mu(\kappa_{2t}^{(1)})$ primarily because country 2's utility ($V_{2,t+1}$) is more volatile than country 1's utility ($V_{1,t+1}$), but the difference between $\mu(\kappa_{2t}^{(2)})$ and $\mu(\kappa_{2t}^{(1)})$ is relatively small under incomplete markets compared to Tables 10 and 11. We also note that the complete markets mechanism appears to hold in Tables 10 and 11 because the precautionary saving motive strongly dominates the intertemporal substitution and consumption smoothing motive.

The currency risk premium typically reflects the foreign-domestic differential of a series expansion of the log nominal stochastic discount factor's higher-ordered conditional cumulants. We referred this series as an uncertainty-risk factor. It is preference based and endogenous to cross-country monetary policy and productivity heterogeneity. Under complete markets, the relationship between the currency risk premium and uncertainty-risk factors is exact. Higher uncertainty risk in the foreign country increases foreign precautionary saving, lowers the foreign interest rate and increases the currency risk premium to induce domestic agents to borrow the foreign currency to satisfy the precautionary saving imbalance. This mechanism does not always apply under incomplete markets. In these cases, the risk premium also reflects cross-country differences in the stochastic discount factor's first-order conditional cumulant—a non-uncertainty-risk factor.

Under complete markets, the forward premium bias is determined by the covariance between the one-period-ahead foreign-home differential in log stochastic discount factors and the differential in both the non-uncertainty-risk and uncertainty-risk factors. Our setup generates a very modest forward premium bias and no forward premium anomaly, whether under complete or incomplete markets. This underscores an important difference between endowment and production models. A forward premium anomaly is unlikely to be seen in a production model without introducing taste shocks or a very different type of incomplete markets environment, say one with noise traders.

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