Precautionary Saving of Chinese and US Households*

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Abstract

We employ a model of precautionary saving to study why household saving rates are so high in China and so low in the US. The use of recursive preferences gives a convenient decomposition of saving into precautionary and non precautionary components. This decomposition indicates that over 80 percent of China's saving rate and nearly all of the US saving arises from the precautionary motive. The difference in the income growth rate between China and the US is vastly more important for explaining saving rate differences than differences in income risk. We estimate the preference parameters and find that Chinese and US households are more similar in their attitude toward risk than in their intertemporal substitutability of consumption.

Keywords: Precautionary Saving, Recursive Preferences, China

JEL: E2, J1

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1 Introduction

From 1989 to 2009, the household saving rate in China averaged slightly over 20 percent of disposable income. This saving rate is high by international standards and contrasts sharply with the 4 percent average in the US over roughly the same time period. Both policy makers and academics have developed an interest in understanding the divergent saving rates in the world's two largest economies. In China, the high saving rate helps to drive investment-led growth, contributes to its external imbalance, and presents a challenge to its goal of 'rebalancing' growth towards consumption. The low saving rate in the US, on the other hand, presents its own set of challenges for adequate provision of old-age support and a deepening of international indebtedness. What accounts for such a large discrepancy in saving rates between the two countries, and can a single model provide an explanation? In this paper, we study the extent to which a model of precautionary saving can answer these questions.

In the policy domain, Bernanke (2005) identifies the precautionary motive as the underlying source of high Asian saving rates and for generating external imbalances that depressed the world interest rate. This so-called 'savings glut' also may have contributed to the sub-prime crisis. When opportunities for unsecured borrowing and risk-sharing are limited, the story goes, households use asset accumulation to serve as a precautionary buffer against zero or very low income states of nature. We show that both income growth and income risk at the household level are much higher in China than in the US. Since the observed differences in the properties of household income for the two countries are stark and conspicuous, it seems reasonable to ask whether these differences in income characteristics can generate the divergent saving rates observed in the data.

Our structural model of saving decisions endows households with recursive Epstein-Zin-Weil preferences (Epstein and Zin 1989, Weil 1989). These preferences generalize power utility by treating the intertemporal elasticity of substitution (IES) and the coefficient of relative risk aversion (RRA) as separate parameters. Following the precautionary saving literature (originally developed for and applied to the US), we assume that income risk is idiosyncratic and households are subject to a no-borrowing constraint. We find that riskier income does induce higher saving rates in the model; however, the differential income risk between China and the US explains a relatively small portion of the Chinese-US saving rate gap. A much more important factor is the difference in the household income growth rate across countries. Precautionary savers, as discussed by Carroll (1992, 1997), have a target asset to income (or consumption) ratio. If this target ratio is relatively insensitive to the rate of income growth, the saving rate in the high-growth country must be higher in order for assets, in the numerator, to grow at the same rate as income, in the denominator. Our analysis indicates that the US saving rate could be increased substantially if household income growth could be raised - admittedly, not an easy task. On the flip side, the model predicts that China's household saving rate will decrease as its growth begins to taper. These findings also help explain why saving rates are often high in high growth countries, as observed in Japan during the 1960s-1970s, Korea during the 1980s-1990s, and currently in China. The positive empirical relation between growth and saving has posed a puzzle in the sense that consumption smoothing arguments lead us to expect higher future income growth to depress current saving (Carroll et al. (2000)).

The data used to estimate the exogenous income dynamics for Chinese households comes from the *China Health and Nutrition Survey*. The analogous estimation for US households employs the *Panel Study for Income Dynamics*. Given the dynamics of income, we find that the observed aggregate Chinese and US household saving rates can be replicated using several configurations of the IES (σ) and RRA (γ) parameters. To further discipline the model, we estimate the preference parameters by the method of simulated moments using cross-sectional consumption data from the *Urban Household Survey* for China and the *Consumer Expenditure Survey* for the US. According to the estimates, attitudes toward risk are similar between Chinese and US households, but their intertemporal substitution elasticities are less alike. We estimate (γ, σ) to be (4.0, 1.4) for the US and (3.78, 2.16) for China. At these parameter values, the model predicts a saving rate of 26.1 percent for China and 5.3 percent for the US.

A secondary task of the paper is to quantify the size of the precautionary component of the saving rate. The recursive utility structure provides a convenient way to do this. The non-precautionary part of saving equals the implied saving rate when people are risk-neutral. The gap between the actual and risk-neutral saving rate is then due to the precautionary motive. By this accounting, the risk-neutral saving rate in the US is only 0.6 percent. Nearly all of the US saving rate can be attributed to the precautionary motive. Risk-neutral saving of Chinese households is about 4 percent, which implies a precautionary saving rate of 22 percent of income.

The high Chinese household saving rate has generated an active area of research. As saving is a multi-faceted phenomenon, research has also formed a multi-faceted approach. Due to the one-child policy's sizable impact on demographics in China, several studies have focused on the role of life-cycle effects and demographic variation on the saving rate. Modigliani and Cao (2004), Horioka and Wan (2007), and Horioka (2010) undertake empirical analyses that test for the significance of these channels; whereas, Banerjee et al. (2010), Curtis et al. (2011), Song et al. (2013), and Coeurdacier et al. (2012) quantitatively model how demographic changes affect China's saving rate via life-cycle channels. Also, in connection with the one-child policy, Wei and Zhang (2011) argue that high saving can be partly explained by the sex-ratio imbalance that has emerged as an unintended consequence of China's population control measures.

Empirical studies that address the precautionary motive include Kraay (2000) and Meng (2003) who incorporate precautionary saving aspects into their empirical analyses and Chamon and Prasad (2010) who estimate effects of the shifting state-to-private burden of educational and medical expenditures on the urban saving rate. More closely related to our paper is Chamon et al. (2013), who explicitly examine the precautionary saving motive along with life-cycle considerations. They show that the observed increase in the volatility of transitory income shocks (from downsizing of the State-Owned sector) has led to over a 5 percentage point increase in the saving rate.

Other researchers have examined the relationship between income growth and saving rates. Song and Yang (2010) focus on saving and income by age in the cross section.[1] Carroll et al. (2000) show saving rates to be increasing in income growth by giving households preferences that exhibit habit persistence, so when income growth accelerates, households that are accustomed to a low level of consumption find

[1]See Yang et al. (2010) for more on the evolution of wages within China.

it optimal to increase saving. Wen (2010) considers a general equilibrium setting, where an increase in the rate of technology growth raises the interest rate. While the higher interest rate induces a higher saving rate, Wen shows that the effect is magnified when agents save also for precautionary reasons due to limited risk-sharing opportunities. In contrast, we show that increased growth induces higher saving rates for a given interest rate. The mechanism is that precautionary savers have an optimal wealth to income ratio and when income grows fast, people need to save more aggressively to maintain their wealth to income ratio.

While work that focuses directly on precautionary saving in China is thin, it has been and continues to be a richly studied topic for US households. Gourinchas and Parker (2002) combine life-cycle and precautionary motives in a quantitative model while Parker and Preston (2005) use cross-sectional data to estimate an empirical decomposition of the saving rate into precautionary and non precautionary components. The methodology of our paper aligns more closely with the precautionary saving analyses of Zeldes (1989), Deaton (1991), and Carroll (1992, 1997). We note that these studies work with constant relative-risk aversion utility. Therefore, an additional contribution of our analysis comes from using the more general Epstein-Zin-Weil preferences.

The remainder of the paper is organized as follows. The next section discusses the aggregate household saving rates for China and the US, which we take as target values for the model simulation exercises. Section 3 presents the model for infinitely-lived Epstein-Zin-Weil households who face risky income streams. Section 4 studies properties of the model through simulations. In Section 5, we estimate the preference parameters by simulated method of moments in order to discipline the model. Then, we assess the ability of the model to explain the data when preference parameters are set to the point estimates. We also use the model to conduct impulse response analysis following aggregate permanent and transitory income shocks combined with a decline in asset values. Section 6 concludes.

2 Household Saving Rates in China and the US

Before sweeping economic reforms began in 1978, saving was relatively low in China. From 1959 to 1978, American households saved a higher fraction of income than Chinese households. As seen in Figure 1, this state of affairs changed dramatically after 1980. Since that time, Chinese saving rates have climbed enormously while US saving rates have trended down.[2] China's saving rate increased from 12 percent to 16 percent between 1980 and 1986, dropped to 11 percent by 1989, then increased more or less steadily towards 31 percent in 2011. In the US, on the other hand, the saving rate averaged 9.1 percent from 1959 to 1984, then trended downward for 23 years reaching a low of 2.4 percent in 2007, before rising somewhat during the great recession.

Broadly speaking, China underwent two sets of economic reforms during this time period. The first-round of post-central planning reforms began in 1978 and progressed through the 1980s. The initial reforms, directed at agriculture, led to increases in farm productivity and a surplus in farm labor. The

[2] Personal saving rate as a percentage of disposable income. Sources: US Department of Commerce, Bureau of Economic Analysis and various issues of the *China Statistical Yearbook*.

Table 1: Household Saving Rates over the Time-Span of Interest

	CHN	USA
1989	11.4	
1990	15.0	
1991	13.5	
1992	16.7	7.3
1993	17.3	5.8
1994	17.6	5.2
1995	17.2	5.2
1996	18.7	4.9
1997	20.6	4.6
1998	22.9	5.3
1999	24.3	3.1
2000	22.5	2.9
2001	24.0	2.7
2002	23.1	3.5
2003	24.0	3.5
2004	24.3	3.6
2005	23.5	1.6
2006	24.7	2.6
2007	26.0	2.4
2008	27.3	
2009	27.3	
Average	20.5	4.01

labor released from the land fueled an explosion of private entrepreneurship formed by Township and Village Enterprises, self-employment businesses, and private-run firms often located in rural areas and leading to a rapid rise in rural income (Huang (2008), Chapter 2).

The Tianamen Square protests in 1989 marked a turning point after which many policies were reversed. The focus shifted away from the rural economy towards the urban areas. This second phase of reform centered on regulatory and administrative restructuring of key market sectors. Of special relevance to our study was the significant downsizing of the State-Owned sector, resulting in the loss of generous benefits for health, retirement, education, and housing. The transition away from state-provision of services and income insurance has been referred to as the dismantling of the "iron rice bowl", which created new incentives for precautionary saving by households (Chamon and Prasad (2010)). This phase of the reform process is still ongoing.

The Chinese household survey panel data that we use to estimate the income dynamics are available from 1989 to 2009. Hence, we focus on the saving rate over this span of time, the period covered by the second set of Chinese economic reforms. To facilitate a comparison over (approximately) the same time period, we select waves of panel data from 1992 to 2007 for US households. During these periods, as a percentage of income, the average aggregate Chinese household saving rate was 20.5 percent whereas the average US rate was a much lower 4.0 percent. Our model is not equipped to capture the growth in the saving rate over time.[3] The question we ask of the model is whether it can explain these average saving rates.

3 A Model of Household Saving

This section presents the model. We begin with a description of the household's exogenous income process. Subsection 3.2 presents household preferences, and subsection 3.3 explores the household saving decisions analytically within the model.

3.1 The Income Process

We use the permanent-transitory income component model of Zeldes (1989) and Carroll (1992, 1997). Households draw different realizations of exogenous labor income from the same initial distribution. The expected income growth rate is common across individuals within a country. Households are infinitely lived and experience idiosyncratic realizations of permanent and transitory income shocks in each period t.[4] Markets are implicitly incomplete; agents cannot purchase contingent claims or other insurance instruments to diversify away from the labor income risk, nor can they borrow.

Let $Y_{i,t}$ be "household income" for individual $i \in [1, N]$. Income is comprised of a permanent part $P_{i,t}$ and a transitory part $e^u_{i,t}$ and evolves according to

$$Y_{i,t} = P_{i,t} e^{u_{i,t}}. \tag{1}$$

Log permanent income $\ln(P_{i,t})$ follows a random walk with drift. In levels, it evolves according to

$$P_{i,t} = e^g P_{i,t-1} e^{n_{i,t}}, \tag{2}$$

where g is the common growth rate and $e^{n_{i,t}}$ is the log normally distributed innovation to permanent income where $n_{i,t} \overset{iid}{\sim} N(\mu_n, \sigma_n^2)$ with $\mu_n = -\sigma_n^2/2$.

The transitory component is drawn from a mixture of a lognormal variate and a near zero event which translates into zero income for that year. It evolves according to

$$u_{i,t} = \begin{cases} N\left(\mu_u, \sigma_u^2\right) & \text{with probability } (1-p) \\ -\infty & \text{with probability } p \end{cases},$$

where p is the probability of drawing zero income and $\mu_u = -\sigma_u^2/2 - \ln(1-p)$.[5] This mixture of distributions is often employed in analyses of precautionary saving because the income data appears to

[3] For analyses that explain the evolution of the saving rate, see Curtis et al. (2011), Coeurdacier et al. (2012), and Song and Yang (2010).

[4] See Aiyagari (1994) for a related model with only transitory income.

[5] We follow Carroll (1997) by setting the mean of $n_{i,t}$ to $-\sigma_n^2/2$ so that $E\left(e^{n_{i,t}}\right) = e^{\mu_n + \frac{\sigma_n^2}{2}} = 1$ and by adjusting the mean of the transitory shock so that $E\left(e^{u_{i,t}}\right) = 1$.

be distributed log normally except for a concentration of observations at the lower tail of the income distribution.[6] The permanent and transitory shocks are assumed to be orthogonal to each other, $Cov(u_{i,t}n_{i,t}) = 0$. We estimate the four parameters (g, σ_n, σ_u, p) governing the income process from household-level data separately for the US and China.

Household Survey Data. The Chinese data comes from the *China Health and Nutrition Survey* (CHNS), which contains income information for a panel of households in the years 1989, 1991, 1993, 1997, 2000, 2004, 2006, and 2009. The survey relies on a multistage random cluster process to track about 4,400 households, varying in terms of geography and socioeconomic status.[7] We set "labor income" equal to total household non-capital income, including income earned by any family member and any transfer payments. This measure of income most closely resembles the concept of income in the model developed below. We restrict observations to households in which the same individual was the head of the household for each year (for which data exists) and for which the head was older than 24 and younger than 60, with complete data on education and occupation.

The US data comes from the *Panel Study of Income Dynamics* (PSID). We impose the same data restrictions used for the Chinese data. To make the time-spans covered for US households roughly comparable to the Chinese data, we use data from the 1992, 1994, 1998, 2001, 2005 and 2007 waves of the PSID.

Estimates of the Income Process Parameters. Estimation of the income process parameters (g, σ_n, σ_u, p) follows Carroll (1992).[8] The strategy is to remove aggregate time trends, predictable life cycle or occupation dependent fluctuations, and household fixed effects (hence the need for panel data) from the income data, then use the remaining variation to estimate the parameters (σ_n, σ_u, p) characterizing the shocks to the income process. The growth rate (g) is calculated as the average real growth of income across households over the entire sample period (see the appendix for more detail). Table 2 reports parameter estimates for the income process and compares our estimates to those reported by Carroll (1992) for an earlier time period (1968-1985) in the US and by Chamon et al. (2013) for China.[9]

Our estimated average income growth g for the Chinese households is 7.3 percent per year. China's income growth is high and evidently quite risky. The estimated probability of suffering a near zero income event p is 2.24 percent and the estimated standard deviation for shocks to the transitory and permanent components of income are $\sigma_u = 0.580$ and $\sigma_n = 0.127$. Our estimates of the income shock process are similar to those reported in Chamon et al. (2013) based on the same data, but a different estimation methodology.

In sharp contrast, households in the US PSID sample experienced very low real income growth but

[6] Carroll (1992) contains a more complete explanation for this income process. Both the US and Chinese data we employ below appear to be distributed log-normally except at the lower end of the distribution.

[7] Detailed information on the survey can be found at www.cpc.unc.edu/projects/china. The survey contains information at the individual and household level. We aggregate to the household level for our analysis.

[8] Following Carroll (1992), a near-zero income event is defined as annual non-capital income of less than 10 percent of the specific household's average annual income.

[9] Chamon et al. do not incorporate the near-zero income events.

Table 2: Estimated Income Process

		Our Calculations		Chamon et al. (2010)	Carroll (1992)
		China	US	China	US
		1989-2009	1992-2007	2006	1968-1985
g	Income Growth (%)	7.30	0.60	n.r.	2.00
p	Prob(e^u) = 0 (%)	2.24	0.10	n.a.	0.65
σ_u	Transitory	0.580	0.410	0.604	0.160
σ_n	Permanent	0.127	0.121	0.121	0.126

n.r.: not reported. n.a.: not applicable.

were exposed to much less risk. Over our sample, the estimated growth rate g is just 0.60 percent per year from 1992 to 2007. The estimated probability of experiencing zero or near zero income is 0.10 percent, which is only 4 percent as large as the probability in China.

Carroll (1992) also used data from the PSID to characterize the income uncertainty of US households for the years 1968-1985 (also see Meghir and Pistaferri (2004)). Thus, we can see how the income process has changed over time in the US. Compared to the 1968 - 1985 numbers reported in Carroll (1992), our results indicate that the probability of near zero income has decreased. However, our estimated transitory income shock volatility σ_u is more than twice as large as Carroll's estimate (0.41 versus 0.16). Our estimate of the standard deviation of the shock to permanent income ($\sigma_n = 0.12$) is about the same as from the earlier period. According to our estimates, about one-third of households in a given year experience a positive or negative transitory shock of at least 40 percent of their income, while a (different) third experience at least a 12 percent shock to their permanent income.

The estimated income process for Chinese households sharply differs from that for US households. Chinese households have enjoyed an income growth rate over 10 times higher than for the US (7.3 versus 0.6 percent annually). However, Chinese households face a much higher probability of a zero income shock (2.2 versus 0.1 percent), as well as larger shocks to transitory and (to a lesser extent) permanent income.

3.2 Preferences and Budget Constraints

In this section, we present a model of household saving decisions where household preferences are given by the Epstein-Zin (1989)-Weil (1989) model of recursive, non-expected utility. Use of these preferences allow us to decompose saving into precautionary and non-precautionary motives, compare US and Chinese saving rates, and evaluate the relative quantitative importance of each component of the income process in generating the saving rate.

Let $C_{i,t}$ be consumption of household $i = 1, \ldots, N$ at time t. We write the utility of the infinitely-lived household as

$$V_{i,t}=\left\{C_{i,t}^{1-\frac{1}{\sigma}}+e^{-\delta}\left[E_t\left(V_{i,t+1}^{1-\gamma}\right)\right]^{\frac{1-\frac{1}{\sigma}}{1-\gamma}}\right\}^{\frac{\sigma}{\sigma-1}}, \tag{3}$$

where $\delta > 0$ is the subjective discount rate, V is recursively defined utility and E_t is the conditional expectations operator, $\sigma > 0$ is the intertemporal elasticity of substitution (IES) and $\gamma \geq 0$ is the coefficient of relative risk aversion (RRA).

Household resources can be consumed ($C_{i,t}$) or invested in an asset ($A_{i,t}$) with gross return e^r. Households cannot borrow and face the sequence of budget constraints

$$A_{i,t+1}=\left(A_{i,t}+Y_{i,t}-C_{i,t}\right)e^r \tag{4}$$

$$A_{i,t}\geq 0. \tag{5}$$

The household's problem is to maximize (3) subject to (4) where the exogenous "labor income" Y, is generated according to (1).

3.3 Analytical Characteristics of the Model

Closed-form solutions to the model are not available, but we can deduce some of its properties from an analysis of the household's Euler equation. Throughout, we assume the (log) normality of and conditional homoskedasticity of the stochastic variables. Derivations of results discussed in this section are contained in the appendix.[10]

To ease notation, we drop the i subscript and let

$$Z_{t+1}\equiv\frac{V_{t+1}}{\left[E_t\left(V_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}. \tag{6}$$

By analogy to the development in Parker and Preston (2005), we refer to Z_{t+1} as a preference shifter. With this notation, we can write the household's Euler equation as,

$$1=e^{r-\delta}E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}}Z_{t+1}^{\frac{1-\gamma\sigma}{\sigma}}\right]. \tag{7}$$

Assuming that consumption and utility are log-normally distributed (which is equivalent to taking a second-order approximation of (7) around the deterministic steady state) gives the log-linearized version of the Euler equation,

$$E\left[\Delta\ln C_{t+1}-(1-\gamma\sigma)\ln Z_{t+1}\right]=\sigma\left(r-\delta\right)+\frac{1}{2\sigma}\text{Var}\left[\Delta\ln C_{t+1}-(1-\gamma\sigma)\ln Z_{t+1}\right], \tag{8}$$

which reduces to the familiar log-linearized Euler equation under constant relative risk aversion (CRRA) utility by setting $\gamma=1/\sigma$,

$$E\left(\Delta\ln C_{t+1}\right)=\frac{r-\delta}{\gamma}+\frac{\gamma}{2}\text{Var}\left(\Delta\ln C_{t+1}\right). \tag{9}$$

[10] The analytical results with the log-normality assumption are similar to those using the second order approximation. Again, see the appendix for detail.

An effect that raises the expected future consumption growth rate is inferred to raise the saving rate, since high consumption growth means low current consumption and hence high saving. Under CRRA utility, saving behavior is summarized by a mean-variance relationship for consumption growth. A high consumption variance household has higher demand for precautionary saving. This depresses current consumption and raises expected consumption growth. The first term in (9) is typically identified with the effect of the IES on consumption growth, because the coefficient on $(r-\delta)$ is the IES $1/\gamma$. This is made clear in (8), where the IES σ shows up explicitly. The second term in (9) is the effect of precautionary saving on mean consumption growth. An increase in the volatility of consumption growth raises expected consumption growth.

Comparing (9) to (8), we see that under recursive preferences the mean-variance relationship between consumption growth generalizes to $\Delta \ln C_{t+1} - (1-\gamma\sigma)\ln Z_{t+1}$, which includes the preference shifter term and introduces additional scope for dynamics.

As we do not have explicit expressions for the saving rate, we infer the effect of parameter values on the saving rate through their effect on the expected consumption growth rate. To assess how variations in the parameter values affect the saving rate, express (8) as

$$E\left(\Delta \ln C_{t+1}\right) = \iota + \phi + \psi. \tag{10}$$

where

$$\iota \equiv \sigma\left(r-\delta\right), \tag{11}$$

$$\phi \equiv \left(1-\gamma\sigma\right) E\left(\ln Z_{t+1}\right), \tag{12}$$

$$\psi \equiv \frac{1}{2\sigma}\mathrm{Var}\left[\Delta \ln C_{t+1} - \left(1-\gamma\sigma\right)\ln Z_{t+1}\right], \tag{13}$$

We will refer to ι as the intertemporal substitution effect, ϕ as the 'preference shifter,' and ψ as the precautionary effect on expected consumption growth. The preference shifter represents the preference for the timing of resolution of uncertainty. The term $E\left(\ln Z_{t+1}\right)$ can be interpreted as the cost of carrying uncertainty to the future and $\gamma\sigma$ as the risk adjusted elasticity of substitution for uncertainty resolution. If ϕ is positive (negative), individuals prefer later (earlier) resolution of uncertainty and raise (lower) consumption growth by consuming less (more) today. Under the log-normality assumption the cost of carrying uncertainty to the future is $E\left(\ln Z_{t+1}\right) = (\gamma-1)\mathrm{Var}(\ln Z_{t+1})/2$. Substituting into Equation (12) gives

$$\phi = \frac{\left(1-\gamma\sigma\right)\left(\gamma-1\right)\mathrm{Var}\left(\ln Z_{t+1}\right)}{2}. \tag{14}$$

Rate of time preference. The only concrete prediction that the model gives is that the saving rate is decreasing in the rate of time preference, δ. Less patient individuals (high δ) place relatively more importance on the present over the future and consume relatively more in the present. The rate of time preference does not directly impact the precautionary component of saving.

Intertemporal elasticity of substitution. The relationship between the entire saving rate and the IES is non monotonic. If people are impatient $(\delta > r)$, then increasing σ lowers expected consumption growth (and hence saving) directly by depressing the intertemporal substitution effect, $\partial\iota/\partial\sigma < 0$. When shifting consumption across time periods is easy for impatient people, they will shift consumption towards the present.

The effect of increasing the IES on the preference shifter is

$$\frac{\partial\phi}{\partial\sigma} = -\frac{\gamma(\gamma-1)\mathrm{Var}\left(\ln Z_{t+1}\right)}{2} + \frac{\left(1-\gamma\sigma\right)\left(\gamma-1\right)}{2}\frac{\partial\mathrm{Var}\left(\ln Z_{t+1}\right)}{\partial\sigma}. \tag{15}$$

The first term in (15) is a direct effect and is negative when risk aversion is not too low, $\gamma > 1$. In this case, raising σ results in a stronger preference for the early resolution of uncertainty and lowers the preference shifter $\partial\phi/\partial\sigma < 0$. Thus, the consumption growth (and hence saving) fall with σ. The second term works through the variance of utility and is an indirect effect. Making it easier for people to move consumption across time periods with higher σ results in higher volatility of consumption and utility (hence Z_{t+1}). So, the indirect effect is negative when the risk aversion and intertemporal substitution are high ($\gamma > 1$ and $\gamma\sigma > 1$).

The effect of increasing the IES on the precautionary component is,

$$\begin{aligned}\frac{\partial\psi}{\partial\sigma} &= -\frac{1}{2\sigma^2}\mathrm{Var}\left[\Delta\ln C_{t+1} - (1-\gamma\sigma)\ln Z_{t+1}\right] \\ &\quad + \frac{\gamma}{\sigma}\left[(\gamma\sigma-1)\mathrm{Var}\left(\ln Z_{t+1}\right) + \mathrm{Cov}\left(\Delta\ln C_{t+1}, \ln Z_{t+1}\right)\right] + \frac{1}{2\sigma}\left[\frac{\partial\mathrm{Var}\left(\Delta\ln C_{t+1}\right)}{\partial\sigma}\right. \\ &\quad \left. + \left(\gamma\sigma-1\right)^2\frac{\partial\mathrm{Var}\left(\ln Z_{t+1}\right)}{\partial\sigma} + 2\left(\gamma\sigma-1\right)\frac{\partial\mathrm{Cov}\left(\Delta\ln C_{t+1}, \ln Z_{t+1}\right)}{\partial\sigma}\right],\end{aligned} \tag{16}$$

The first two terms in (16) are the direct effect. The first term is clearly negative. Given volatility, $\mathrm{Var}[\Delta\ln C_{t+1} - (1-\gamma\sigma)\ln Z_{t+1}]$, raising σ directly lowers the precautionary saving motive as people can easily substitute consumption across time. The second term represents the change in the contribution of variation in the preference shifter $\ln Z_{t+1}$ on the volatility. When $\gamma\sigma > 1$, the variation of the preference shifter contributes positively on the overall volatility. In this case the precautionary saving motive rises with σ.

Alternatively, we can decompose the effect into two components as shown in the second term in (16). The first component is the direct contribution of variations of the preference shifter on the overall volatility, which is increasing in σ when the substitutability is not too low, $\gamma\sigma > 1$. The second component is the indirect contribution working through the comovements with the consumption growth. The stochastic part of $\ln Z_{t+1}$ is the (log) utility forecast error $\ln V_{t+1} - E_t\left(\ln V_{t+1}\right)$.[11] A surprise improvement in utility is positively correlated with consumption growth, making the covariance term positive. For the overall direct effect, when $\gamma\sigma > 1$, the two terms have opposite signs. Thus, the direct effect of increasing σ on the precautionary component is ambiguous. Combining the first and second terms gives

$$\frac{1}{2\sigma^2}\left[\left(\gamma\sigma\right)^2\mathrm{Var}\left(\ln Z_{t+1}\right) - \mathrm{Var}\left(\ln Z_{t+1} - \Delta\ln C_{t+1}\right)\right], \tag{17}$$

which is increasing in γ and σ. Thus, for the overall direct effect, increasing σ lowers the precautionary saving motive when risk aversion is low and raises ψ when risk aversion is high. The last term in (16)

[11] See the appendix for the derivation.

is the indirect effect working through the changes in the variations of consumption growth and utility. As the variability of consumption and utility rise with the substitutability, the indirect effect is positive when the substitutability is not too low, $\gamma\sigma > 1$.

Although the overall relationship between the intertemporal elasticity of substitution and the saving rate cannot be unambiguously signed, we conjecture that increasing σ lowers the saving when risk aversion is low and raises the saving rate when risk aversion is high. Combining all effects, we have

$$\begin{aligned} \frac{\partial E\left(\Delta \ln C_{t+1}\right)}{\partial \sigma} &= (r-\delta) + \frac{\gamma}{2}\mathrm{Var}\left(\ln Z_{t+1}\right) - \frac{1}{2\sigma^2}\mathrm{Var}\left(\ln Z_{t+1} - \Delta \ln C_{t+1}\right) \\ &+ \frac{1}{2\sigma}\left\{\frac{\partial \mathrm{Var}\left(\Delta \ln C_{t+1}\right)}{\partial \sigma} + (\gamma\sigma - 1)(\sigma - 1)\frac{\partial \mathrm{Var}\left(\ln Z_{t+1}\right)}{\partial \sigma}\right. \\ &\left. + 2(\gamma\sigma - 1)\frac{\partial \mathrm{Cov}\left(\Delta \ln C_{t+1}, \ln Z_{t+1}\right)}{\partial \sigma}\right\}. \end{aligned} \tag{18}$$

The first three terms are the direct effects and the last term is the indirect effect. The overall indirect effect, the last term, is positive when $\gamma\sigma > 1$ and $\sigma > 1$. The overall direct effect is increasing in γ and σ given volatility. For the direct effect, increasing σ lowers (raises) the saving rate when γ is relatively low (high).

For a moderate value of γ, the saving rate profile has a U shape with respect to σ. The desire to accumulate a buffer-stock of assets to hedge against adverse income shocks intensifies with greater risk aversion. Raising σ makes moving consumption around across time easier and leads to higher saving if γ is high enough (for there to be buffer stock asset accumulation). On the other hand, if risk-aversion is low, people do not build up a buffer stock. There is less desire to sacrifice current consumption, and when it is easy for people to move consumption across time periods, they will, due to their impatience, move it to the present.

Risk Aversion. The overall effect of increasing risk-aversion on the saving rate is also ambiguous. In terms of our decomposition, risk aversion has no effect on the intertemporal substitution effect, ι. Increasing risk aversion has the following effect on the preference shifter,

$$\begin{aligned} \frac{\partial \phi}{\partial \gamma} &= \frac{(1-\gamma\sigma)\,\mathrm{Var}\left(\ln Z_{t+1}\right)}{2} - \frac{\sigma(\gamma-1)\,\mathrm{Var}\left(\ln Z_{t+1}\right)}{2} \\ &+ \frac{(1-\gamma\sigma)(\gamma-1)}{2}\frac{\partial \mathrm{Var}\left(\ln Z_{t+1}\right)}{\partial \gamma} \end{aligned} \tag{19}$$

The first term is the effect of change in the uncertainty cost, $E\left(\ln Z_{t+1}\right)$ which is increasing in risk aversion. If the substitutability is large enough, $\gamma\sigma > 1$, an increase in the uncertainty cost lowers ϕ and hence saving. The second term is the direct effect of change in the risk adjusted elasticity of substitution. If $\gamma > 1$, the uncertainty cost is positive. Then, raising risk adjusted substitutability increases the desire for early resolution of uncertainty. This channel lowers ϕ and hence saving. Combining these two effects, increasing risk aversion has a negative impact (lowering the saving rate) when risk aversion is relatively high $\gamma > \frac{\sigma+1}{2\sigma}$. When risk aversion is low (high), increasing γ raises (lowers) the preference shifter and contributes towards higher (lower) saving. Thus, for the direct effects, the preference shifter profile has a hump shape with respect to the risk-aversion coefficient. The third term is the indirect effect. Our conjecture is that consumption and utility volatility declines with higher risk aversion, which leads the last term to exert a positive impact when $\gamma\sigma > 1$ and $\gamma > 1$.

The effect of increasing risk aversion on the precautionary component is,

$$\begin{aligned}\frac{\partial\psi}{\partial\gamma} &= [(\gamma\sigma-1)\mathrm{Var}\left(\ln Z_{t+1}\right)+\mathrm{Cov}\left(\Delta C_{t+1},\ln Z_{t+1}\right)]+\frac{1}{2\sigma}\left[\frac{\partial\mathrm{Var}\left(\Delta\ln C_{t+1}\right)}{\partial\gamma}\right. \\ &\quad \left. +\left(\gamma\sigma-1\right)^2\frac{\partial\mathrm{Var}\left(\ln Z_{t+1}\right)}{\partial\gamma}+2\left(\gamma\sigma-1\right)\frac{\partial\mathrm{Cov}\left(\Delta\ln C_{t+1},\ln Z_{t+1}\right)}{\partial\gamma}\right].\end{aligned} \tag{20}$$

The first term in (20) is also the second term in equation (16). Hence, the sign of this term is ambiguous at low levels of risk aversion but clearly positive if $\gamma\sigma > 1$. The predicted profile of the saving rate with respect to the direct effect of risk aversion is either the saving rate rises with γ or that it displays a U shape. The second term is negative when $\gamma\sigma > 1$ since the variability of consumption and utility is expected to fall with γ.

Combining all effects, we have

$$\begin{aligned}\frac{\partial E\left(\Delta\ln C_{t+1}\right)}{\partial\gamma} &= \left(\frac{\sigma-1}{2}\right)\mathrm{Var}\left(\ln Z_{t+1}\right)+\mathrm{Cov}\left(\Delta C_{t+1},\ln Z_{t+1}\right) \\ &\quad +\frac{1}{2\sigma}\left\{\frac{\partial\mathrm{Var}\left(\Delta\ln C_{t+1}\right)}{\partial\gamma}+\left(\gamma\sigma-1\right)\left(\sigma-1\right)\frac{\partial\mathrm{Var}\left(\ln Z_{t+1}\right)}{\partial\gamma}\right. \\ &\quad \left. +2\left(\gamma\sigma-1\right)\frac{\partial\mathrm{Cov}\left(\Delta\ln C_{t+1},\ln Z_{t+1}\right)}{\partial\gamma}\right\}.\end{aligned} \tag{21}$$

The overall indirect effect, the last term, is negative when $\gamma\sigma > 1$ and $\sigma > 1$, but is ambiguous for other values. The first two terms are the direct effects. The overall direct effect is positive for $\sigma > 1$.

When $\sigma > 1$, increasing γ raises (lowers) the saving rate when γ is relatively low (high) so that the indirect effect is relatively small or positive (high or negative). Thus the saving rate profile should exhibit a 'hump shape' with respect to γ, and the peak occurs earlier with lower σ. Note, if the volatility of consumption (hence utility) is relatively high so that the positive direct effect always dominates, the saving rate will monotonically rise with γ.

CRRA utility. The separation of intertemporal substitution and risk aversion gives additional richness to saving behavior beyond what is present under CRRA utility where γ regulates both risk aversion and intertemporal substitution. As a comparison, under CRRA utility, differentiating (9) with respect to γ gives

$$\frac{\partial E\left(\Delta\ln C_{t+1}\right)}{\partial\gamma}=\frac{\delta-r}{\gamma^2}+\frac{1}{2}\mathrm{Var}\left(\Delta\ln C_{t+1}\right)+\frac{\gamma}{2}\frac{\partial\mathrm{Var}\left(\Delta\ln C_{t+1}\right)}{\partial\gamma}. \tag{22}$$

The direct effect, given in the first two terms are positive with $\delta > r$. The indirect effect, however has a negative effect. Higher risk aversion (or lower substitutability $\sigma = 1/\gamma$) leads to lower consumption volatility. Under CRRA utility, the saving rate profile should be increasing in γ as long as the indirect effect is not too high.

Income Shocks σ_n, σ_u, and p. The effect of the income shocks on saving are straightforward. An increase in the volatility of the income shocks raises the volatility of consumption and utility.

The precautionary saving component ψ increases with volatility. The increase in volatility raises the saving rate. Volatility also affects the preference shifter. The preference shifter effect is dominated by the precautionary saving effect in the simulations below because volatility is strongly related to precautionary saving.

Income Growth g. Consumption volatility should be increasing with the income growth rate, g. We infer this by looking at the steady state version of (8) where average consumption grows at the same rate as income $\ln E\left(e^{\Delta \ln C_{t+1}}\right) = \ln E\left(e^{\Delta \ln Y_{t+1}}\right) = g + \mu_n$. As the growth rate increases on the left hand side of (8), the only thing that can increase on the right hand side are variances of utility or consumption growth. Furthermore, in the steady state, assets, cash on hand, and consumption should all grow at the same rate, $g + \mu_n$.

Let $M = A + Y$ be 'cash-on-hand,' and s be the saving to total income ratio, where total income is labor income plus interest on assets. Then the direct relationship between the saving rate and the growth rate implied by the budget constraint is[12]

$$\frac{s}{1-s} = \left(\frac{M}{C} - 1\right)\left(1 - e^{-(g+\mu_n)}\right). \tag{23}$$

Holding M/C constant, an increase in the growth rate has a positive effect on the saving rate s. This formula is a bit cumbersome, however. We can get the same intuition by looking at saving as a fraction of labor income, S/Y. For a given growth rate g, in the steady state, wealth will be proportional to income $W = \omega Y$. Hence, in the steady state,

$$\frac{S}{Y} = \frac{\Delta W}{Y} = \frac{\Delta W}{\Delta Y}\frac{\Delta Y}{Y} = e^{g+\mu_n}\frac{W}{Y}. \tag{24}$$

An increase in the growth rate,

$$\frac{\partial(S/Y)}{\partial g} = e^{g+\mu_n}\left[\frac{W}{Y} + \frac{\partial(W/Y)}{\partial g}\right], \tag{25}$$

has a positive direct effect (W/Y) and an ambiguous indirect effect $\partial(W/Y)/\partial g$. For a given target or desired wealth-to-income ratio, saving must increase with growth because a higher g causes the denominator Y to grow faster, and households need to save more aggressively to get the numerator W to grow at the new higher rate.

The target wealth-to-income ratio (equivalently M/C) need not be invariant to g, however, which gives rise to the indirect effect. Higher future income from higher income growth makes households less vulnerable to income risk. Households may reduce the target wealth-to-income ratio (and M/C), which can depress the saving rate. Assessing the relative strength of these two effects must be done numerically. Clearly, the saving rate is zero when growth is zero and positive for some positive growth rates. Also, the direct effect is diminishing in g as shown in (23). Hence, the saving rate either increases with g or, if the indirect effect dominates when income growth is high, exhibits a hump-shaped pattern with respect to g.

[12] Here, we ignore the stochastic nature of the model for illustration.

3.4 Stationarity and Convergence

The exogenous income process has a random walk component, so the model must be transformed to induce stationarity. We do this by normalizing variables by the permanent income. Let lower case letters denote the normalized variables $c_t = C_t/P_t$, $a_t = A_t/P_t$, and $v_t = V_t/P_t$. Normalizing the budget constraint in this way yields

$$a_{t+1} = (a_t + y_t - c_t)\, e^{(r-g-n_t)}. \tag{26}$$

Similarly, the stationary form of utility is

$$v_t = \left\{ c_t^{\frac{\sigma-1}{\sigma}} + e^{-\delta + \left(\frac{\sigma-1}{\sigma}\right)g} \left[E_t \left(v_{t+1}^{1-\gamma} e^{(1-\gamma)(g+n_{t+1})} \right) \right]^{\frac{\sigma-1}{\sigma(1-\gamma)}} \right\}^{\frac{\sigma}{\sigma-1}}, \tag{27}$$

and the normalized form of the Euler equation is

$$1 = E_t \left\{ e^{r-\delta} \left(\frac{c_{t+1}}{c_t} e^{g+n_{t+1}} \right)^{-\frac{1}{\sigma}} \left[\frac{v_{t+1}}{\left(E_t \left(v_{t+1}^{1-\gamma} e^{(1-\gamma)(g+n_{t+1})} \right) \right)^{\frac{1}{1-\gamma}}} \right]^{\frac{1-\gamma\sigma}{\sigma}} \right\}. \tag{28}$$

Convergence requires that two conditions be jointly satisfied. First, as in Deaton (1991) and Carroll (1997), convergence of the model, from the Euler equation (28), requires

$$\sigma (r - \delta) + \frac{[\gamma (\sigma + 1) - 1]\, \sigma_n^2}{2} \leq g - \frac{\sigma_n^2}{2}. \tag{29}$$

Clearly, impatience $\delta > r$ helps to achieve stationarity. The left hand side of (29) is increasing in γ. When $\delta > r + \gamma\sigma_n^2/2$, the left hand side is decreasing in σ and the first term helps to achieve stationarity. Note that this condition is reduced to Carroll's and Deaton's condition for CRRA utility upon setting $\gamma\sigma = 1$. The second condition comes from the maximized utility function (27), which is given by

$$\sigma (r - \delta) + \frac{[\gamma (\sigma + 1) - 1]\, \sigma_n^2}{2} + \sigma \left(g - r - \gamma\sigma_n^2 \right) < g - \frac{\sigma_n^2}{2}. \tag{30}$$

When income growth is relatively high, $g > r + \gamma\sigma_n^2$, the stationarity condition is governed by the utility function (30). In this case, once the first stationarity condition is satisfied, higher risk aversion helps to achieve stationarity as the left hand side is decreasing in γ. Impatience δ also helps to achieve stationarity. When $g > \delta + \gamma\sigma_n^2/2$, higher intertemporal substitutability raises the left hand side of (30). In this case, the first term may prevent convergence for a high enough σ.

4 Quantitative Implications

This section reports the simulated saving rates generated by embedding the income process estimates from Section 3.1 into the model presented in Section 3.2. Policy functions of the stationary model are obtained by value function iteration. The implied levels of the variables (income, assets, consumption and saving) are then obtained by "un-normalizing" the variables–that is multiplying them by permanent income. Details are contained in the appendix.

We first study the model's properties using parameter values with ranges typically assumed or estimated in the literature. In doing so, we are able to find admissible parameter values under which the model households save at rates similar to those observed in the data.

4.1 Preference Parameter Values

The parameters for the income process (g, σ_n, σ_u, p) come from the estimates based on the US and Chinese household survey data as reported in Table 2. To simulate the model, we also need values for the three preference parameters (σ, γ, δ) and the interest rate r. There exists a large literature aimed at estimating relative risk aversion and intertemporal elasticity of substitution but without much agreement and mostly using CRRA utility. The choices for σ and γ draw on estimates reported in the literature.

We consider values of the risk aversion coefficient γ between 0 and 8 to be admissible. Studies using survey data generally find γ to be in this range ($0 < \gamma < 10$ in Dohmen et al. (2005), $7.18 < \gamma < 8.59$ in Eisenhauer and Ventura (2010), and $5 < \gamma < 10$ in Vissing-Jøgensen and Attansio (2003)). Studies that use asset pricing data and constant relative risk aversion utility (which cannot separate risk aversion from intertemporal substitution) typically obtain vastly larger values, which we ignore.

Studies that estimate the intertemporal elasticity of substitution typically report values between 0.2 and 1. A collection of IES estimates reported in the literature is shown in Table 3. A restrictive feature of most of the existing studies, however, is the assumption of CRRA utility. Amongst these, however, the Beaudry and Van Wincoop (1996) and Gruber (2006) studies obtain relatively high estimates of the IES. The two recent studies by Chen et al. (2007) and Bansal and Shaliastovich (2013) employ recursive preferences and obtain estimates greater than 1. On the basis of these empirical studies, we consider three values for the $IES = (0.8, 1.2, 1.8)$.

We take the real interest rate for China to be 1.6 percent per annum. This figure is the average real interest rate on bank deposits from 2003 to 2012 provided by the World Bank's *World Development Indicators*. For the US, we set the real interest rate at 1.19 percent which we obtain from the average real three-month treasury bill rate from 1992 to 2007. In anticipation of our own parameter estimates, we set the rate of time preference $\delta = 0.022$ for China and $\delta = 0.018$ for the US, situating the households to be relatively impatient.[13]

4.2 Simulated Saving Rates

For each experiment, we simulate income and saving decisions for $N = 20,000$ households. Household saving equals the aggregate of total income (summed over the 20,000 households) minus aggregate consumption, where total income equals the sum of labor income and interest income on assets. Households begin with zero wealth and build up their target wealth-to-income ratios over time. The saving rate is initially high as households accumulate towards their target asset-to-income ratio. For China, saving

[13] High impatience has often been assumed in models of precautionary saving. We will actually generate estimates for these parameters below, but for now we impose the values for δ.

Table 3: Estimates of the Intertemporal Elasticity of Substitution from the Literature

Authors	Special Features	Range of Estimates
Biederman and Goenner (2007)	30 Year Investment Horizon	$(0.2, 0.8)$
Felices (2005)	British Household Panel Survey	$(0.05, 0.17)$
Hall (1988)	Aggregate US Consumption	0.2
Noda and Sugiyama (2010)	Japanese Data	$(0.2, 0.5)$
Ogaki and Reinhart (1998b)	Long-run Data	$(0.27, 0.77)$
Ogaki and Reinhart (1998a)	Durable Goods	$(0.32, 0.45)$
Patterson and Pesaran (1992)	UK and US Data	0.3
Skinner (1985)		$(0.2, 0.5)$
Vissing-Jørgensen (2002)	Holders of Stocks, Bonds,	stocks: $(0.3, 0.4)$
	and No Assets	bonds: $(0.8, 1)$
		no assets: 0.2
Beaudry and Van Wincoop (1994)	State-Level Consumption	1
Gruber (2005)		2
Chen et al. (2007)	Recursive Preferences	$(1.11, 2.22)$
Bansal-Shaliastovich (2013)	Recursive Preferences	1.81

and asset ratios stabilize relatively quickly after about 20 periods. For the US, stabilization of the ratios requires 50 to 60 periods. The ratios we report are stabilized values and reflect the steady-state saving rates and asset ratios. Compared to China, households in the US have an income process with a lower chance of receiving zero income, a slightly smaller variance in the permanent shock, a smaller variance in the transitory income shock, and a much lower income growth rate.

Table 4 reports the model generated household saving rates and asset to consumption ratios. The implied asset-consumption ratios are much higher for US households than for the Chinese. When risk aversion (γ) is low, increasing the IES (σ) lowers the saving rate for both China and the US. For China this occurs when $\gamma \leq 2$ and for the US when $\gamma = 0$. When risk aversion is high and the precautionary motive is strong, increasing σ leads to higher saving. The general relationship between the saving rate and the IES is consistent with our discussion in the previous section.

The saving rate–risk-aversion profile is such that for a given value of σ, the saving rate is either increasing in γ or hump shaped. This pattern is present also in the asset-consumption ratio. For China, this profile is monotonic over the range of γ considered. For the US, there is a dramatic increase in saving when γ increases from 0 to 2, but then the saving rate is relatively insensitive to further increases in γ. For the US, the saving rate (and asset-consumption ratio) displays a mild hump-shape with respect to γ at the higher RRA values. As the US has smaller volatility in income shocks, the consumption and utility volatility is relatively lower than that in China. Thus, consistent with our discussion in the previous section, the saving rate is expected to exhibit a hump shape with respect to γ.

Table 4: Implied Saving Rates and Target Asset Ratios

China						
	Saving Rate			Assets to consumption		
	σ			σ		
γ	0.8	1.2	1.8	0.8	1.2	1.8
0	0.046	0.041	0.036	0.634	0.563	0.484
2	0.055	0.053	0.050	0.781	0.735	0.694
3	0.067	0.062	0.065	0.962	0.875	0.929
4	0.076	0.082	0.102	1.129	1.252	1.652
5	0.096	0.110	0.182	1.503	1.784	3.324
6	0.123	0.170	0.333	2.034	3.025	7.840
7	0.166	0.234	0.367	2.934	4.554	8.949
8	0.207	0.315	0.381	3.863	7.020	9.350

USA						
	Saving Rate			Assets to consumption		
	σ			σ		
γ	0.8	1.2	1.8	0.8	1.2	1.8
0	0.007	0.006	0.005	1.153	1.041	0.869
2	0.043	0.043	0.045	7.801	7.853	8.192
3	0.048	0.050	0.054	8.633	9.051	9.683
4	0.049	0.051	0.055	8.842	9.305	10.053
5	0.048	0.053	0.056	8.799	9.631	10.365
6	0.047	0.051	0.057	8.619	9.466	10.403
7	0.046	0.049	0.056	8.454	8.954	10.383
8	0.044	0.049	0.056	7.909	8.933	10.358

The model can match the observed high aggregate household saving rate of 20.5 percent for China and 4.0 percent for the US using different pairs of $\sigma \in [0.8, 1.8]$ and $\gamma \in [0, 8]$. For China, $\sigma = 0.8$ and $\gamma = 8$, $\sigma = 1.2$ and $6 < \gamma < 7$, and $\sigma = 1.8$ with $5 < \gamma < 6$ all produce saving rates in the neighborhood of the data. For the US, $\sigma = 0.8$, $0 < \gamma < 2$ and $2 < \gamma < 3$ for $\sigma = 1.2$ or 1.8 give a saving rate near 4 percent.

The model also implies that nearly all of the US household saving rate is driven by the precautionary motive. When risk aversion is shut off, the implied saving rate is basically nil, in contrast to the corresponding $3.6 - 4.6$ percent rate for Chinese households. With the smaller variance in transitory shocks, there is less need for US households to save for consumption smoothing purposes. The relatively low rate of income growth is also a factor. Recall that we are looking at saving rates at the steady state. US households have built up large stocks of assets, but because of the low growth rate, they do not have to save much to maintain their desired wealth ratios.

The remainder of this section considers variations in the key parameters governing the model.

Constant Relative Risk Aversion Utility. First, we contrast the results under recursive preferences with those under CRRA utility. Table 5 shows the results where we restrict $\gamma = 1/\sigma$ and vary γ between 0.8 and 8. A risk aversion coefficient between 2 and 3 allows the model to match the US saving ratio, but this model does not come close to explaining the Chinese saving ratio for any value γ below 8.

Table 5: Implied Saving Rates under Constant Relative Risk Aversion Utility

γ	China	USA
0.8	0.045	0.010
2	0.061	0.043
3	0.078	0.050
4	0.092	0.053
5	0.105	0.052
6	0.118	0.067
7	0.130	0.062
8	0.138	0.067

Variations in Growth. While the riskiness of transitory income between China and the US differs substantially, perhaps the most dramatic contrast is in the expected growth rate of income. US households expect almost no growth in labor income, whereas Chinese households expect over 7 percent growth per year.

Next, to show how income growth affects saving, we vary g from 0.5 percent to 7 percent while holding all other parameters of the income process at their estimated values. We consider $\sigma = 1.2$ and various values of γ. Results for $\sigma = 0.8$ and 1.8 are similar and not reported to save space. Table 6 reports the household saving rates and asset ratios for China. The saving rate displays a hump shaped pattern with respect to g for $\gamma = 4$ or 6, but saving is generally increasing in g for $\gamma = 0$, 2, 8. As expected, the asset-consumption ratio declines with the growth rate.

Analogous results for the US are shown in table 7. Similar to China, the saving rate displays a hump for $\gamma = 2$, 4, 6, but is non-decreasing in g for $\gamma = 0$, 8. It is possible to get Americans to save a higher fraction of income with more rapid income growth. At an annual growth rate of 4 percent and $\gamma = 4$, the US household saving ratio is predicted to be 17.5 percent.

If households want to maintain the same target wealth-to-income ratio, the saving rate needs to rise with higher income growth. However, higher income growth makes households less vulnerable to income risk, leading them to reduce their target wealth-to-income ratio (and hence their saving rate declines). The first effect (maintaining the target wealth-to-income ratio) dominates for low income growth, but the second effect dominates when the income growth rate is high. The critical point where the two effects cancel out each other is increasing in risk aversion as the income risk is relatively more important. The generally positive relationship between growth and saving is noteworthy as it provides a potential explanation for why high-growth economies tend to have high saving rates.

Also note that risk-neutral ($\gamma = 0$) saving rate in China exceeds that of the US for any of the growth rates considered. Chinese households desire to save more (buffer stock saving) on account of the higher volatility of their income.

Variations in near-zero income event probability. Table 8 shows the results of varying the probability of receiving zero income p for Chinese households under alternative values of γ and $\sigma = 1.2$.[14] Increasing p increases the saving rate; however, the effect is small. Increasing p from 0.8 percent to 2.8 percent only increases the saving rate by about one percentage point for an RRA less than 8 and even less than that for RRA equal to 8. These results are consistent with those reported in Carroll (1997).[15] Since zero income shocks are rare and agents literally have forever to recover, the effect on saving is small.

Overall, the experiments in this section confirm the hypothesis given in the analytical section above. The effect of the precautionary motive to save dominates the preference shifter effect in equation (8).

5 Simulated Moments Estimates of Preference Parameters

The analysis of the previous section investigated the general quantitative properties of the model by treating (δ, σ, γ) as free parameters. In this section, we estimate the preference parameters by simulated

[14]We also have experimented with different values for the variances of the shocks to permanent and transitory income. The results are straightforward; larger variances induce higher precautionary saving. Since this channel is clear and quantitatively not as important as income growth for determining saving rates, we do not focus on it.

[15]In Carroll (1997), raising p from 0.1 percent to 1 percent increases the saving rate by 0.6 percentage points from 0.4 percent to 1 percent.

Table 6: Variations in China's Income Growth, Implied Saving Rates and Asset-Consumption Ratios

	γ				
g	0	2	4	6	8
A. Saving Rate					
1.005	0.007	0.043	0.054	0.053	0.050
1.01	0.013	0.057	0.077	0.083	0.082
1.02	0.021	0.041	0.124	0.136	0.141
1.03	0.024	0.044	0.152	0.184	0.190
1.04	0.029	0.047	0.106	0.218	0.234
1.05	0.034	0.049	0.095	0.244	0.274
1.06	0.037	0.049	0.087	0.207	0.303
1.07	0.040	0.051	0.082	0.173	0.312
B. Asset-Consumption Ratio					
1.005	1.442	6.234	7.916	8.229	8.243
1.01	1.309	5.280	7.389	8.085	8.153
1.02	1.071	2.299	7.071	7.824	8.136
1.03	0.806	1.619	6.444	7.800	8.055
1.04	0.721	1.276	3.313	7.526	8.045
1.05	0.676	1.022	2.271	7.215	8.187
1.06	0.617	0.829	1.678	4.821	8.085
1.07	0.570	0.738	1.315	3.253	7.295

Calculations assume $\sigma = 1.2$.

Table 7: Variations in US Income Growth, Implied Saving Rates and Asset-Consumption Ratios

			γ		
g	0	2	4	6	8
		A. Saving Rate			
1.005	0.006	0.038	0.047	0.045	0.042
1.01	0.009	0.059	0.078	0.081	0.081
1.02	0.014	0.037	0.133	0.143	0.143
1.03	0.017	0.034	0.162	0.192	0.193
1.04	0.018	0.032	0.175	0.227	0.244
1.05	0.020	0.031	0.078	0.252	0.281
1.06	0.023	0.032	0.065	0.260	0.304
1.07	0.023	0.031	0.053	0.207	0.334
		B. Asset-Consumption Ratio			
1.005	1.084	7.970	9.716	9.434	8.865
1.01	0.924	6.997	9.003	9.223	9.148
1.02	0.721	2.286	8.415	8.905	8.804
1.03	0.536	1.212	7.243	8.509	8.498
1.04	0.439	0.829	6.418	8.043	8.741
1.05	0.381	0.631	1.819	7.660	8.592
1.06	0.368	0.531	1.221	7.032	8.271
1.07	0.322	0.435	0.811	4.206	8.172

Calculations assume $\sigma = 1.2$.

Table 8: Variations in p and Chinese Saving Ratios

	Percent Chance of Receiving Zero Income, p					
γ	0.8	1.2	1.6	2.0	2.4	2.8
0	0.033	0.034	0.036	0.038	0.039	0.039
2	0.045	0.046	0.046	0.049	0.050	0.050
4	0.061	0.065	0.068	0.068	0.072	0.073
6	0.125	0.126	0.127	0.129	0.131	0.169
8	0.228	0.227	0.228	0.227	0.232	0.231

Calculations assume $\sigma = 1.2$.

method of moments and investigate the extent to which the estimated model can explain observed saving behavior of Chinese and US households.

Our estimation employs only consumption data. Other research estimating the recursive utility function, e.g. Chen et al. (2007) and Bansal and Shaliastovich (2013), combine consumption and asset returns data. As is well known, risk aversion must be very high to be consistent with asset returns data. Chen et al.'s estimates of the IES range between 1.11 and 2.22 while their estimates of the risk aversion coefficient are above 60. Bansal and Shaliastovich's estimates of (γ, σ) equal $(20.9, 1.81)$. Our estimation approach is consistent with our model, which only has implications for consumption moments. Therefore, we do not necessarily generate super high estimates of γ.

For the US, the consumption data comes from the *Consumer Expenditure Survey* (CEX) from 1992 to 2007. Since real consumption did not grow over this period, we pool the data into a single cross section. The four moments used in estimation are the mean, variance, skewness, and kurtosis of the cross-sectional distribution for household consumption. The US data set consists of 21,138 observations. For China, the data comes from a single cross section consisting of 15,039 observations from the 2007 *Urban-Rural Household Survey.*[16]

We simulate are $N_s = 50,000$ individuals over many periods, t. The simulated moments are calculated over observations from $t = 60$ to 76 for the US to correspond to the 16 years of data in our CEX sample. For China, the simulated moments are computed at $t = 20$, since we have only one cross section in the data.[17]

Table 9 shows the estimation results. The time-preference δ and risk aversion γ estimates for China and the US are quite similar. Households are impatient in the sense that the rate of time preference exceeds the interest rate, $\delta > r$. The risk-aversion coefficient is not estimated to be exceedingly high (as is the case when using asset returns). The point estimates for the IES are substantially greater than 1 for both countries.

Our estimates of σ make contact with the literature on long-run risk (Bansal and Yaron (2004)). Models of long-run risk employ recursive preferences and can explain several asset pricing anomalies including the equity premium puzzle, the low risk-free rate, and their volatilities, provided two key ingredients are present. First, some long run risk must exist to vary the expected growth rate of consumption over time. Second, the intertemporal elasticity of substitution must exceed 1. Chen et al. (2007) also estimate this elasticity to be greater than 1. However, their estimates of the risk-aversion coefficient are much larger and range between 17 and 60, depending on whether they use aggregate consumption or consumption of stock holders. Our estimates support the long-run risk framework.

Saving Rates and Decomposition under Estimated Parameters. Next, we ask to what extent observed saving rates in China and the US can be explained by the model at the estimated parameter values, and what part of that saving rate is driven by the precautionary motive. The results are shown in Table 10.

[16]Yi Huang kindly provided these data.

[17]Recall that convergence to steady-state ratios takes 60 periods for the US and 20 periods for China.

Table 9: Simulated Moment Estimates of Preference Parameters

	δ (se)	σ (se)	γ (se)	Crit
China	0.022	2.160	3.780	1.498
	(0.000)	(0.046)	(0.056)	
USA	0.018	1.443	4.047	3.570
	(0.010)	(1.818)	(5.064)	

Table 10: Total and Risk-Neutral Saving Rate Implied by Estimated Parameters

	Parameter Settings			Saving
Country	γ	σ	δ	Rate
China	$\hat{\gamma}$	$\hat{\sigma}$	$\hat{\delta}$	0.261
	0	$\hat{\sigma}$	0.039	0.030
	0	1.8	$\hat{\delta}$	0.036
US	$\hat{\gamma}$	$\hat{\sigma}$	$\hat{\delta}$	0.053
	0	$\hat{\sigma}$	$\hat{\delta}$	0.006

For China, the saving rate implied by the estimated parameters is 26.1 percent. The model over predicts the average saving rate during the time-span of the CHNS data but nearly matches the saving in 2007, the year of the consumption data used to estimate the preference parameters.

The model does not converge when risk-aversion is shut down holding σ and δ at the estimated values because the utility blows up. Although we are unable to obtain the exact risk-neutral saving rate, we approximate that measurement from two directions. Our first approach is to raise the rate of time preference until convergence is achieved. Doing this gives a risk-neutral saving rate of 3.0 percent. Our second approximation is to lower the IES to 1.8, which gives risk-neutral saving of 3.6 percent. These experiments allow us to reasonably conclude that China's risk-neutral saving does not exceed 4.0 percent and that the precautionary component of the saving rate is about 22 percent of income in the model.

For the US, the 5.3 percent saving rate somewhat over-predicts the 4.0 percent average from 1992 to 2007. The implied risk-neutral saving rate is a minuscule 0.6 percent, suggesting that virtually all of the household saving rate in the US is driven by the precautionary motive.

Impulse Responses. Here, we study the model's impulse responses to aggregate negative shocks to income and asset values.

Due to the partial equilibrium nature of the model, negative income shocks affect household asset values only to the extent to which they draw down their assets for consumption. In a severe economic downturn or a financial crisis, however, asset values decline independently of drawdowns to finance consumption. To generate a more realistic environment facing households in times of aggregate distress, we also have asset values fall exogenously by 20 percent at the time of the negative income shock.[18] These exercises show that the model reacts in a reasonable manner, and the impulse responses also highlight the different reactions between US and Chinese households.

Parameter values are set at their point estimates. The model runs normally for $t_1 - 1$ periods with people receiving their idiosyncratic income shocks. In period t_1, everyone receives the worst possible draw of the transitory ($u_{i,t}$) or permanent ($n_{i,t}$) shock, and asset values fall by 20 percent. After period t_1, the model runs normally once again.

Figure 2 shows the response to a negative aggregate transitory income shock. Both countries take a hit to the income level (panel a) and asset values in t_1, after which growth in China resumes and the US simply recovers. Consumption growth (panel b) in China is depressed for only on period. In the US, the level of consumption takes about 8 periods to recover from the shock. Assets (panel c) lose a substantial portion of their value and are also partially eaten in the period of the shock. In China, asset accumulation resumes the period after the shock, with pre-shock wealth fully recovered after 3 periods. In the US, the rebuilding of asset stocks takes considerably longer, around 13 years. The saving to income ratio (panel d) for both countries sharply declines when the shock hits. In the subsequent periods, saving remains elevated because agents save toward their target asset to income ratio. The consumption to income ratio (not shown) is the inverse image of the asset to income ratio, so for the US it remains low following the shock. The consumption-asset ratio (not shown) declines in the period of the shock then shoots up as consumption recovers. Over time, this ratio converges back to the (inverse of the) target asset-consumption ratio.

Figure 3 shows the impulse responses to a negative permanent shock to income. With almost no growth, US income (panel a) is permanently lower following the shock; whereas, China's growth resumes and fully recovers after 4 years. Similarly, consumption (panel b) in China takes 4 periods to recover, but consumption does not fully recover in the US within the time frame examined. The decline in asset values (panel c) in the US is persistent; whereas, asset growth in China recovers after only 3 periods. The decline in the saving-income ratio (panel d) is persistent because the shock to income is permanent, reducing the need to rebuild assets in order to achieve the target wealth to income ratio.

Using the same model, the US and China have very different responses to the negative shocks. The underlying reason is that incomes in China grow faster than in the US. We think this finding helps explain the sluggish recovery of consumption in the US following the great recession. It may be too early to tell whether the US experienced a permanent or transitory shock to income; regardless, our model predicts that either type of shock depresses consumption for many years. China, on the other hand, recovers quickly within our model, as it did in the real world.

[18] We also have carried out impulse response simulations with just aggregate income shocks.

6 Conclusion

In this paper, we use household survey data to show that Chinese households have experienced more volatility in their income process along with a much higher rate of income growth relative to households in the US. We then embed the country-specific estimated income processes into a model of saving decisions by infinite lived households with Epstein-Zin-Weil recursive preferences. The model can generate the high current level of Chinese saving and the low level of US saving.

The model abstracts from a few relevant factors, such as life-cycle effects and environmental changes facing households. China has undergone large changes in demographics due to the one-child policy. Recent years also have seen a dismantling of old-age income security. Furthermore, medical and educational expenditures, which were previously provided by the state, have been shifted to households. Nevertheless, our relatively simple set-up generates powerful insights into why household saving is so high in China and so low in the US.

In particular, the recursive preference structure allows a convenient decomposition of the saving rate into precautionary and non-precautionary components. According to this decomposition, the precautionary motive drives most of the saving rate in China and nearly all the saving in the US. Since the social safety net in China is less comprehensive than in the US, Chinese households face substantially higher transitory income risk. Somewhat surprisingly, however, the higher income growth rate in China, and not the elevated income risk, accounts for most of the China-US saving rate gap. In the model, saving is increasing over a range of income growth rates as households save aggressively to maintain a desired asset-to-income ratio. This result sheds some light on the somewhat puzzling empirical fact that high growth countries (like China) often have high saving rates.

Figure 1: Historical Saving Rates

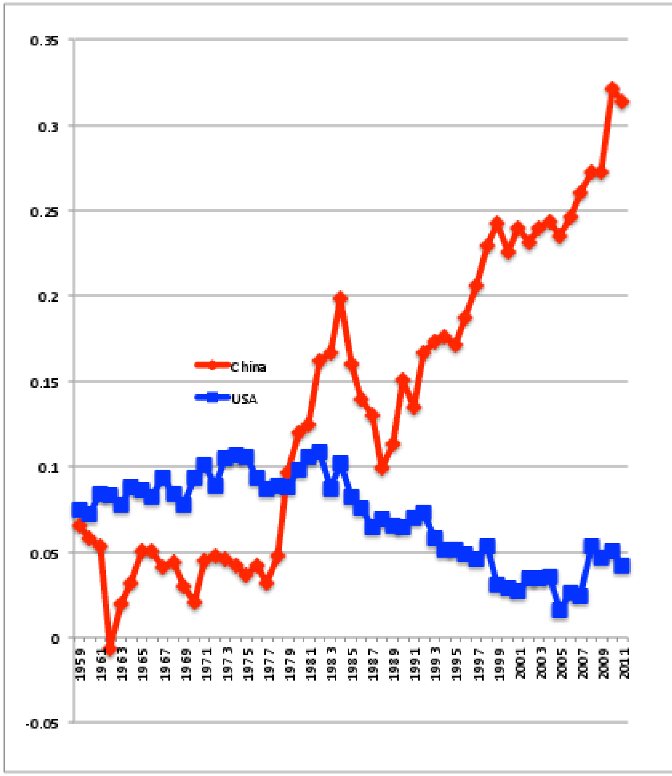
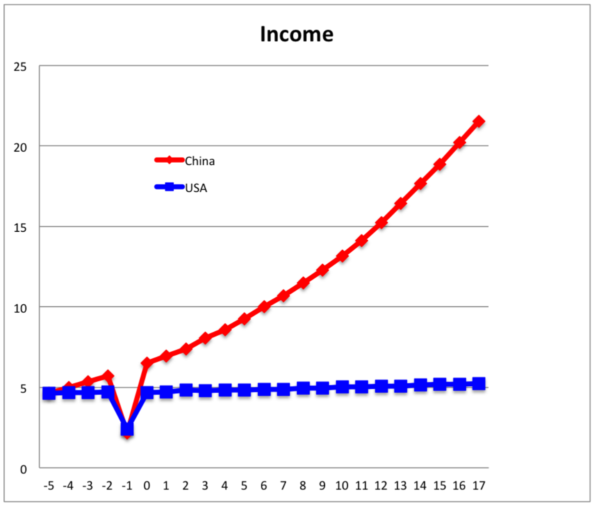
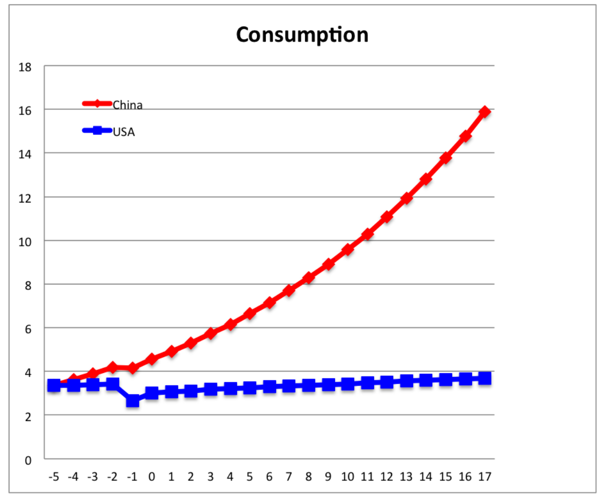


Figure 2: Impulse Responses to Transitory Income Shock

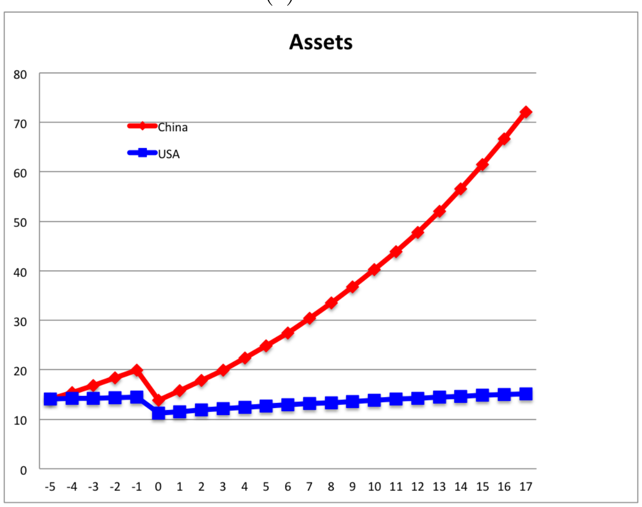
(a) Income



(b) Consumption



(c) Assets



(d) Saving-Income Ratio

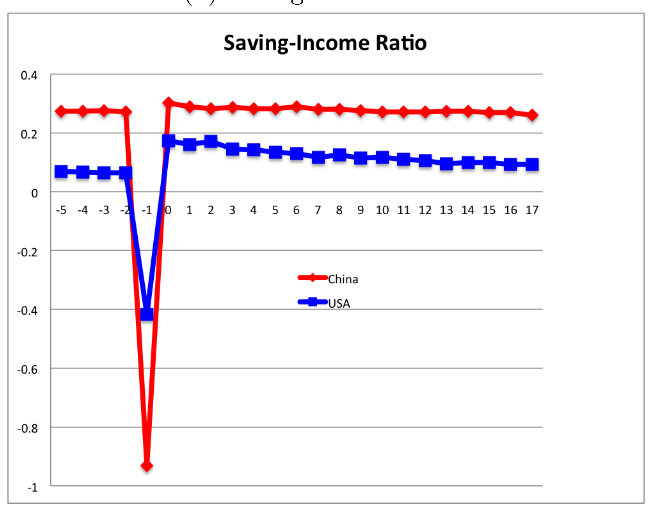
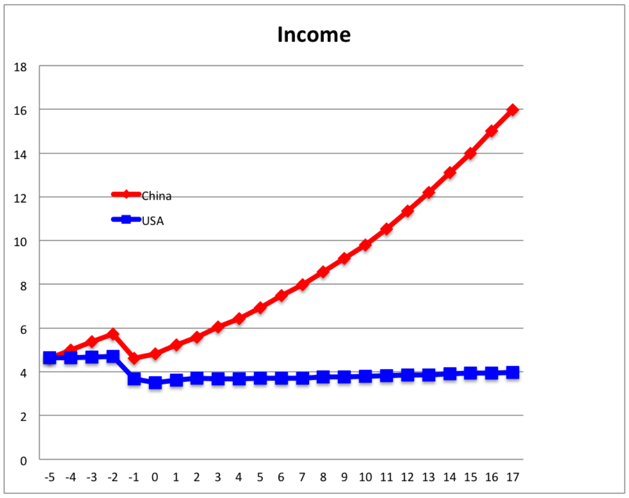
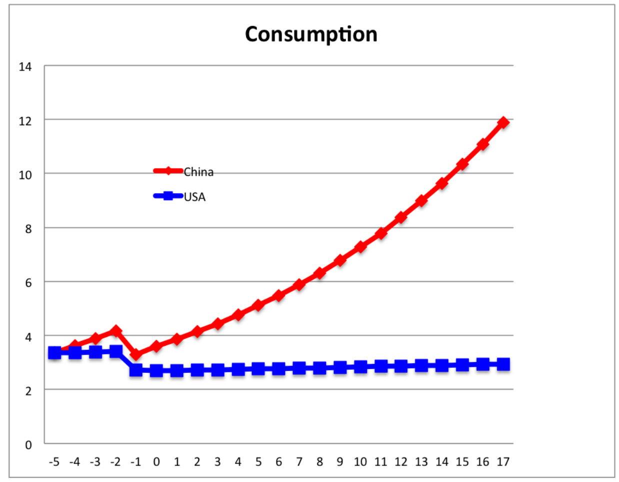


Figure 3: Impulse Responses to Permanent Income Shock

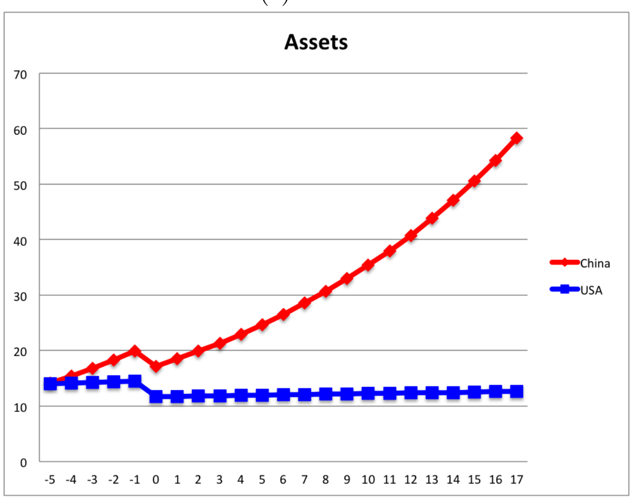
(a) Income



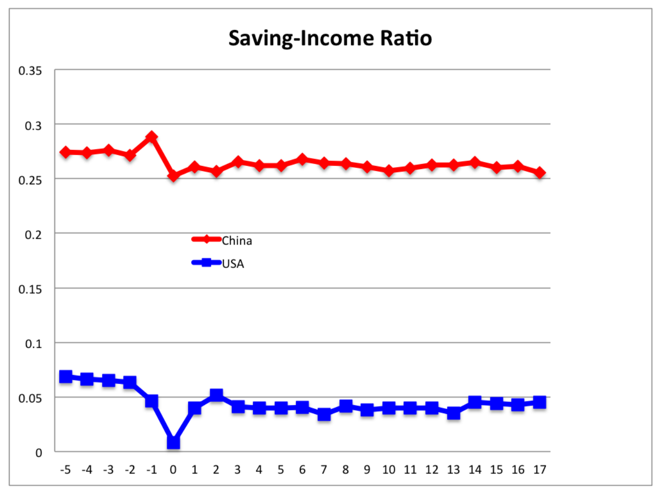
(b) Consumption



(c) Assets



(d) Saving-Income Ratio



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Appendix

A. Estimation of the Labor Income Process

Carroll (1992) contains a more complete explanation and justification for the estimation procedure. Here we sketch the steps involved.

The procedure for estimating the probability of zero (transitory) income is as follows:

1. For each year, divide actual household income by the cross-sectional mean of income. Call the result detrended household income. Normalization by the mean is intended to remove cycle and trend components.

2. Regress detrended income on age, occupation, education, the interactions of these terms, age squared, and gender. Use this regression to predict life-cycle (age-specific) movements in income for each household.

3. Divide detrended income by predicted income. Call this $YL_{i,t}$.

4. Take the average income over all observations for household i. Call this average permanent income.

5. Take $YL_{i,t}$ and divide by average permanent income. This creates up to 8 observations per household for a total of 4,550 observations on urban households. The entire procedure was repeated separately for 12,163 rural households, since their income stream could be different. Categorize a zero-income event as occurring when YL divided by average permanent income is less than 0.1. A substantial portion of the observations are concentrated near zero income. Following Carroll (1992), negative observations are counted as zero. A total of 69, about 1.5%, of the observations of urban households occur at or below 0.10 (i.e. 90% below trend income). The percentage for rural households is 2.5%. A weighted average across urban and rural households gives $p = 0.0224$.

Table 11: Frequency of Zero Non-Capital Income Events, China

Head of Household	Observations	Near-Zero Events	% Near-Zero Events
Urban Chinese	4,550	69	1.52
Rural Chinese	12,163	307	2.52

The entire process was repeated separately for the US PSID data, resulting in $p = 0.0010$.

To determine the relative magnitudes of the transitory and permanent shocks (σ_n, σ_u), we further restrict the sample to heads of households whose marital status never changed, who never ran a business as their primary occupation, and who never experienced a near-zero income event. Note, determining who owns a business in the Chinese data is less straightforward than in the US data. These restrictions should all reduce variability. The variance of the shocks are then estimated by regressing the sample variance of $\ln YL_{it-m} - \ln YL_{it}$ on m and a constant for all values of m that can be calculated.

B. Derivations of Main Equations in the Text

For notational convenience, we drop the i subscript.

Derivation of equation (8), the Euler equation. Begin with the utility function (7). If the household is given extra consumption today (dC_t), it lowers tomorrow's assets by $dA_{t+1} = -e^r dC_t$. Exploiting the envelope theorem, if the household is on the optimal path, this infinitesimal reallocation results in no change in welfare. That is,

$$\frac{\partial V_t}{\partial C_t} dC_t = E_t \left(\frac{\partial V_t}{\partial V_{t+1}} \frac{\partial V_{t+1}}{\partial A_{t+1}} \right) \left(e^r dC_t\right) \tag{31}$$

where

$$\frac{\partial V_t}{\partial C_t} = \left(1 - e^{-\delta}\right) V_t^{\frac{1}{\sigma}} C_t^{-\frac{1}{\sigma}} \tag{32}$$

$$\frac{\partial V_t}{\partial V_{t+1}} = V_t^{\frac{1}{\sigma}} e^{-\delta} \left[E_t \left(V_{t+1}^{1-\gamma} \right) \right]^{\frac{\gamma\sigma-1}{\sigma(1-\gamma)}} V_{t+1}^{-\gamma} \tag{33}$$

Since the intertemporal marginal rate of substitution is

$$\mathrm{IMRS}_{t+1} = \frac{E_t \left(\frac{\partial V_t}{\partial V_{t+1}} \frac{\partial V_{t+1}}{\partial A_{t+1}} \right)}{\frac{\partial V_t}{\partial C_t}}, \tag{34}$$

using (31) in (34) gives

$$E \left(\mathrm{IRMRS}_{t+1} e^r\right) = 1$$

Now the budget constraint gives

$$\frac{\partial A_{t+1}}{\partial A_t} = \left(1 - \frac{\partial C_t}{\partial A_t} \right) e^r \tag{35}$$

Furthermore,

$$\begin{aligned}
\frac{\partial V_t}{\partial A_t} &= \frac{\partial V_t}{\partial C_t} \frac{\partial C_t}{\partial A_t} + E_t \left[\frac{\partial V_t}{\partial V_{t+1}} \frac{\partial V_{t+1}}{\partial A_{t+1}} \underbrace{\left(\frac{\partial A_{t+1}}{\partial A_t} \right)}_{(35)} \right] \\
&= \frac{\partial V_t}{\partial C_t} \frac{\partial C_t}{\partial A_t} + E_t \left[\frac{\partial V_t}{\partial V_{t+1}} \frac{\partial V_{t+1}}{\partial A_{t+1}} \left(1 - \frac{\partial C_t}{\partial A_t} \right) e^r \right] \\
&= \left\{ \frac{\partial V_t}{\partial C_t} - E_t \left[\frac{\partial V_t}{\partial V_{t+1}} \frac{\partial V_{t+1}}{\partial A_{t+1}} e^r \right] \right\} \frac{\partial C_t}{\partial A_t} + E_t \left[\frac{\partial V_t}{\partial V_{t+1}} \frac{\partial V_{t+1}}{\partial A_{t+1}} e^r \right] \\
&= \frac{\partial V_t}{\partial C_t}
\end{aligned}$$

where the last equality comes from the envelope condition. Notice that the term in braces (by equation (31)) is zero. This means,

$$\frac{\partial V_t}{\partial A_t} = \frac{\partial V_t}{\partial C_t}. \tag{36}$$

Now substitute (36), (32), and (33) into (34) to get

$$\begin{aligned}
\mathrm{IMRS}_{t+1} &= E_t\left(\frac{\frac{\partial V_t}{\partial V_{t+1}}\frac{\partial V_{t+1}}{\partial C_{t+1}}}{\frac{\partial V_t}{\partial C_t}}\right) \\
&= E_t\left\{\frac{V_t^{\frac{1}{\sigma}}e^{-\delta}\left[E_t\left(V_{t+1}^{1-\gamma}\right)\right]^{\frac{\gamma\sigma-1}{\sigma(1-\gamma)}}V_{t+1}^{-\gamma}V_{t+1}^{\frac{1}{\sigma}}C_{t+1}^{-\frac{1}{\sigma}}}{V_t^{\frac{1}{\sigma}}C_t^{-\frac{1}{\sigma}}}\right\} \\
&= \left\{e^{-\delta}\left[\frac{V_{t+1}}{\left(E_t\left(V_{t+1}^{1-\gamma}\right)\right)^{\frac{1}{1-\gamma}}}\right]^{\frac{1-\gamma\sigma}{\sigma}}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}}\right\}.
\end{aligned}$$

Using this IMRS to price the asset available to households, gives the Euler equation in the text.

Derivation of log-linearized Euler equation (8) by second-order approximation. That (8) follows from (7) under log-normality of consumption growth and utility is obvious. Here, we show that a second-order approximation around a deterministic steady state gives the same result. To see this, we use the deterministic steady state as the evaluation point:

$$1 = \exp\left\{r-\delta-\left(\frac{1}{\sigma}\right)g_c+\left(\frac{(1-\gamma\sigma)}{\sigma}\right)\ln\overline{Z}\right\},$$

where $g_c = \overline{\Delta\ln C}$, steady state consumption growth. Note, we do not take normalization of variables here. Hence, the steady state value C grows over time, $g_c > 0$. The second order approximation gives

$$\begin{aligned}
0 = E_t&\left\{-\left(\frac{1}{\sigma}\right)\left[\Delta\ln\left(C_{t+1}\right)-g_c\right]+\left(\frac{(1-\gamma\sigma)}{\sigma}\right)\ln\left(Z_{t+1}/\bar{Z}\right)\right\} \\
&+\frac{1}{2\sigma^2}E_t\left\{\left[\Delta\ln\left(C_{t+1}\right)-g_c-(1-\gamma\sigma)\ln\left(Z_{t+1}/\bar{Z}\right)\right]^2\right\}.
\end{aligned}$$

Using the deterministic steady state condition, $r-\delta-\left(\frac{1}{\sigma}\right)g_c+\left(\frac{1-\gamma\sigma}{\sigma}\right)\ln\overline{Z}=0$, we have

$$\begin{aligned}
E_t\Delta\ln C_{t+1} &= \sigma\left(r-\delta\right)+\left(1-\gamma\sigma\right)E_t\ln Z_{t+1} \\
&\quad+\frac{1}{2\sigma}E_t\left\{\left[\Delta\ln C_{t+1}-(1-\gamma\sigma)\ln\left(Z_{t+1}\right)-\left[g_c-(1-\gamma\sigma)\ln\left(\overline{Z}\right)\right]\right]^2\right\}.
\end{aligned}$$

Taking unconditional expectations on both sides gives the result.

Property of $\ln Z_{t+1}$. Before proceeding, we state and prove a pair of useful results.

Result 1 *Under log-normality of consumption and utility,*

$$\ln Z_{t+1} = \epsilon_{v,t+1}+\left(\frac{\gamma-1}{2}\right)\mathrm{Var}\left(\ln v_{t+1}+n_{t+1}\right), \tag{37}$$

$$E\left(\ln Z_{t+1}\right) = \left(\frac{\gamma-1}{2}\right)\mathrm{Var}\left(\ln v_{t+1}+n_{t+1}\right), \tag{38}$$

$$\mathrm{Var}\left(\ln Z_{t+1}\right) = \mathrm{Var}\left(\ln v_{t+1}+n_{t+1}\right), \tag{39}$$

where $\epsilon_{v,t+1} = \ln v_{t+1}-E_t\left(\ln v_{t+1}\right)+n_{t+1}-\mu_n$ *and* $v_{t+1} = V_{t+1}/P_{t+1}$.

To obtain (38), let $\ln v_{t+1} = \ln(V_{t+1}/P_{t+1})$ be conditionally (on date t information) normally distributed with conditional mean $\mu_{v,t} = E_t(\ln v_{t+1})$ and variance $\omega_{v,t} = \mathrm{Var}_t(\ln v_{t+1})$ =Var$(\ln v_{t+1})$ with the assumption of conditional homoskedasticity.[19] Then,

$$\begin{aligned}
\ln Z_{t+1} &= \ln V_{t+1} - \frac{1}{1-\gamma}\ln\left[E_t\left(V_{t+1}^{1-\gamma}\right)\right] \qquad (40)\\
&= \ln v_{t+1} + n_{t+1} - \frac{1}{1-\gamma}\ln\left[E_t\left(v_{t+1}e^{n_{t+1}}\right)^{1-\gamma}\right]\\
&= \ln v_{t+1} - E_t(\ln v_{t+1}) + n_{t+1} - \mu_n + \left(\frac{\gamma-1}{2}\right)\mathrm{Var}\left(\ln v_{t+1} + n_{t+1}\right),
\end{aligned}$$

where the last equation uses the lognormality and conditional homoskedasticity. The last equation is (37). Taking expectations on both sides gives (38), and taking variances gives (39).

Property of $E(\ln Z_{t+1})$. With the log-normality assumption, we have

$$E(\ln Z_{t+1}) = \left(\frac{\gamma-1}{2}\right)\mathrm{Var}(\ln Z_{t+1}).$$

Thus, $E(\ln Z_{t+1})$ is positive (negative) for $\gamma > 1$ (< 1), and is increasing in γ.

Without log-normality we have the same qualitative properties. From Jensen's inequality,

$$\begin{aligned}
\exp\{E[(1-\gamma)\ln v_{t+1}]\} &\leq E\left(e^{(1-\gamma)\ln v_{t+1}}\right),\\
(1-\gamma)E(\ln v_{t+1}) &\leq \ln E\left(e^{(1-\gamma)\ln v_{t+1}}\right).
\end{aligned}$$

This gives

$$(1-\gamma)\left[E(\ln v_{t+1}) - \frac{1}{1-\gamma}\ln E\left(v_{t+1}^{1-\gamma}\right)\right] = (1-\gamma)E_t(\ln Z_{t+1}) \leq 0.$$

Thus, $sgn(E\ln Z_{t+1}) = sgn(\gamma - 1)$. We also have $\frac{\partial E(\ln Z_{t+1})}{\partial\gamma} > 0$, since $\frac{1}{1-\gamma}\ln\left[E_t\left(v_{t+1}^{1-\gamma}\right)\right]$ is decreasing in γ from the property of the generalized mean.

Derivation of (15). Substituting (38) into (12) and differentiating with respect to σ gives (15).

Derivation of (16). The derivation of equation (16) uses the following result.

Result 2 *Let $y(x)$ be a random variable that depends on a parameter x. Then*

$$\frac{\partial \mathrm{Var}[y(x)]}{\partial x} = 2E\left[y(x)\frac{\partial y(x)}{\partial x}\right] - 2E[y(x)]\,E\left(\frac{\partial y(x)}{\partial(x)}\right). \qquad (41)$$

Direct differentiation of $\mathrm{Var}[y(x)] = E\left[y(x)^2\right] - [E(y(x))]^2$ *with respect to x gives the result (41).*

[19]Note that if $\mathrm{Var}_t(\ln v_{t+1})$ =Var$(\ln v_{t+1})$ with the assumption of conditional homoskedasticity for $\ln v_{t+1}$, $\mathrm{Var}_t(\ln v_{t+1} + n_{t+1})$ =Var$(\ln v_{t+1} + n_{t+1})$.

Using the rule (41) to differentiate ψ with respect to σ gives,

$$
\begin{aligned}
\frac{\partial \psi}{\partial \sigma} =& -\frac{1}{2\sigma^2}\mathrm{Var}\left(\Delta \ln C_{t+1} - (1-\gamma\sigma)\ln Z_{t+1}\right) \\
&+ \frac{\gamma}{\sigma}\left\{\mathrm{Cov}\left(\Delta \ln C_{t+1}, \ln Z_{t+1}\right) - (1-\gamma\sigma)\mathrm{Var}\left(\ln Z_{t+1}\right)\right\} + \frac{1}{2\sigma}\left[\frac{\partial \mathrm{Var}\left(\Delta \ln C_{t+1}\right)}{\partial \sigma}\right. \\
&\left. + (\gamma\sigma - 1)^2 \frac{\partial \mathrm{Var}\left(\ln Z_{t+1}\right)}{\partial \sigma} + 2(\gamma\sigma - 1)\frac{\partial \mathrm{Cov}\left(\Delta \ln C_{t+1}, \ln Z_{t+1}\right)}{\partial \sigma}\right].
\end{aligned}
$$

For the direct effects of the first two terms, we have

$$
\begin{aligned}
\left.\frac{\partial \psi}{\partial \sigma}\right|_{\text{direct}} =& \frac{1}{2\sigma^2}\left[2\gamma\sigma\mathrm{Cov}\left(\Delta \ln C_{t+1}, \ln v_{t+1}\right) - 2\gamma\sigma(1-\gamma\sigma)\mathrm{Var}\left(\ln v_{t+1}\right)\right. \\
&\left. - \mathrm{Var}\left(\Delta \ln C_{t+1}\right) - (1-\gamma\sigma)^2\mathrm{Var}\left(\ln v_{t+1}\right) + 2(1-\gamma\sigma)\mathrm{Cov}\left(\Delta \ln C_{t+1}, \ln v_{t+1}\right)\right] \\
=& \frac{1}{2\sigma^2}\left[2\mathrm{Cov}\left(\Delta \ln C_{t+1}, \ln v_{t+1}\right) - \mathrm{Var}\left(\Delta \ln C_{t+1}\right) + \left(\gamma^2\sigma^2 - 1\right)\mathrm{Var}\left(\ln v_{t+1}\right)\right] \\
=& \frac{1}{2\sigma^2}\left[(\gamma\sigma)^2\mathrm{Var}\left(\ln v_{t+1}\right) - \mathrm{Var}\left(\Delta \ln C_{t+1} - \ln v_{t+1}\right)\right].
\end{aligned}
$$

Derivation of (19). Substitute (38) in (12), and differentiating with respect to γ gives the result.

Derivation of (20). Substitute (38) in (13), and using the rule (41) to differentiate ψ with respect to γ gives

$$
\begin{aligned}
\frac{\partial \psi}{\partial \gamma} =& \frac{1}{2\sigma}\left[2\sigma(\gamma\sigma-1)\mathrm{Var}\left(\ln Z_{t+1}\right) + 2\sigma\mathrm{Cov}\left(\Delta C_{t+1}, \ln Z_{t+1}\right)\right] \\
&+ \frac{1}{2\sigma}\left[\frac{\partial \mathrm{Var}\left(\Delta \ln C_{t+1}\right)}{\partial \gamma} + (\gamma\sigma-1)^2\frac{\partial \mathrm{Var}\left(\ln Z_{t+1}\right)}{\partial \gamma}\right. \\
&\left. + 2(\gamma\sigma-1)\frac{\partial \mathrm{Cov}\left(\Delta \ln C_{t+1}, \ln Z_{t+1}\right)}{\partial \gamma}\right] \\
=& \left[(\gamma\sigma-1)\mathrm{Var}\left(\ln Z_{t+1}\right) + \mathrm{Cov}\left(\Delta C_{t+1}, \ln Z_{t+1}\right)\right] + \frac{1}{2\sigma}\left[\frac{\partial \mathrm{Var}\left(\Delta \ln C_{t+1}\right)}{\partial \gamma}\right. \\
&\left. + (\gamma\sigma-1)^2\frac{\partial \mathrm{Var}\left(\ln Z_{t+1}\right)}{\partial \gamma} + 2(\gamma\sigma-1)\frac{\partial \mathrm{Cov}\left(\Delta \ln C_{t+1}, \ln Z_{t+1}\right)}{\partial \gamma}\right].
\end{aligned}
$$

Derivation of (23). Begin with the budget constraint

$$A_{t+1} = e^r\left(A_t + Y_t - C_t\right),$$

and divide both sides by C_t to get

$$
\begin{aligned}
\left(\frac{A_{t+1}}{C_{t+1}}\right)e^{g_{c,t+1}} &= e^r\left(\frac{A_t + Y_t}{C_t} - 1\right) \\
\left(\frac{A_{t+1}}{C_{t+1}}\right)e^{g_{c,t+1}-r} &= \frac{M_t}{C_t} - 1,
\end{aligned}
\tag{42}
$$

where $g_{c,t+1} = \ln\left(C_{t+1}/C_t\right)$ and $M_t = A_t + Y_t$, cash-at-hand. The saving rate is defined as

$$S_t = 1 - \frac{C_t}{\left(\frac{e^r-1}{e^r}\right)A_t + Y_t}.$$

Then, we have

$$\frac{1}{1-S_t}=\frac{A_t+Y_t}{C_t}-e^{-r}\left(\frac{A_t}{C_t}\right).$$

Applying (42), we obtain

$$\begin{aligned}\frac{S_t}{1-S_t}&=\left(\frac{A_{t+1}}{C_{t+1}}\right)e^{g_{c,t+1}-r}-e^{-r}\left(\frac{A_t}{C_t}\right)\\&=\left(\frac{A_{t+1}}{C_{t+1}}\right)e^{g_{c,t+1}-r}\left[1-\left(\frac{A_t/C_t}{A_{t+1}/C_{t+1}}\right)e^{-g_{c,t+1}}\right]\\&=\left(\frac{M_t}{C_t}-1\right)\left[1-\left(\frac{A_t/C_t}{A_{t+1}/C_{t+1}}\right)e^{-g_{c,t+1}}\right].\end{aligned}$$

Lastly, taking the steady state and the condition that income growth equals the consumption growth $g_c=g+\mu_n$ gives (23).

Stationary Transformation. Normalize the income process by P_t to get

$$\frac{Y_t}{P_t}=e^{u_t}.$$

Normalizing the budget constraint gives

$$\frac{A_{t+1}}{P_t}\frac{P_{t+1}}{P_t}=e^r\left(\frac{A_t}{P_t}+\frac{Y_t}{P_t}-\frac{C_t}{P_{t-1}}\right).$$

Let lower case with tilde denote variables normalized by P_t :

$$a_{t+1}e^{g+n_{t+1}}=e^r\left(a_t+e^{u_t}-c_t\right). \tag{43}$$

Now, normalizing the utility function (3) gives

$$\begin{aligned}\left(\frac{V_t}{P_t}\right)^{\frac{\sigma-1}{\sigma}}&=\left(\frac{C_t}{P_t}\right)^{\frac{\sigma-1}{\sigma}}+e^{-\delta}\left\{E_t\left[\left(\frac{V_{t+1}}{P_{t+1}}\frac{P_{t+1}}{P_t}\right)^{1-\gamma}\right]\right\}^{\frac{\sigma-1}{(1-\gamma)\sigma}} \qquad (44)\\ v_t^{\frac{\sigma-1}{\sigma}}&=c_t^{\frac{\sigma-1}{\sigma}}+e^{-\delta}\left\{E_t\left[\left(v_{t+1}e^{g+n_{t+1}}\right)^{1-\gamma}\right]\right\}^{\frac{\sigma-1}{(1-\gamma)\sigma}}\\&=c_t^{\frac{\sigma-1}{\sigma}}+e^{-\delta+\left(\frac{\sigma-1}{\sigma}\right)g}\left\{E_t\left[\left(v_{t+1}e^{n_{t+1}}\right)^{1-\gamma}\right]\right\}^{\frac{\sigma-1}{(1-\gamma)\sigma}}.\end{aligned}$$

Normalizing the Euler equation (7) gives

$$\begin{aligned}1&=e^{r-\delta}E_t\left\{\left(\frac{C_{t+1}/P_{t+1}}{C_t/P_t}\left(\frac{P_{t+1}}{P_t}\right)\right)^{-\frac{1}{\sigma}}\left[\frac{V_{t+1}/P_{t+1}}{E\left[\left(\frac{V_{t+1}}{P_{t+1}}\frac{P_{t+1}}{P_t}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\left(\frac{P_{t+1}}{P_t}\right)\right]^{\frac{1-\gamma\sigma}{\sigma}}\right\} \qquad (45)\\&=e^{r-\delta}E_t\left\{\left(\frac{c_{t+1}}{c_t}\right)^{-\frac{1}{\sigma}}e^{-\frac{1}{\sigma}(g+n_{t+1})}\left[\frac{v_{t+1}e^{g+n_{t+1}}}{E\left[\left(v_{t+1}e^{g+n_{t+1}}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1-\gamma\sigma}{\sigma}}\right\}\\&=e^{r-\delta-\frac{g}{\sigma}}E_t\left\{\left(\frac{c_{t+1}}{c_t}\right)^{-\frac{1}{\sigma}}e^{-\gamma n_{t+1}}\left[\frac{v_{t+1}}{E\left[\left(v_{t+1}e^{n_{t+1}}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1-\gamma\sigma}{\sigma}}\right\}.\end{aligned}$$

Convergence Criterion (30) and (29). The stationarity conditions concern non-exploding a_t and v_t. The borrowing constraint always prevents degeneration of a_t and v_t. If a household builds up enough assets a with stationarity, the household can completely stabilize the consumption by setting $c_t = \bar{c}$, which equals the expected income $E(Y_t/P_t)$ plus the expected annuity $(e^{r-g}-1)E(a_t)$. Therefore, the stationarity conditions are (i) the normalized utility in (44) does not explode, and (ii) $E(a_t)$ does not explode, which is equivalent to the condition that the right hand side of (45) is less than with $c_t = \bar{c}$. From (44), with $c_t = \bar{c}$, we have constant utility $v_t = \bar{v}$. Applying this to the Euler equation

$$\left(\frac{c}{v}\right)^{\frac{\sigma-1}{\sigma}} = 1 - \exp\left[-\delta + \left(\frac{\sigma-1}{\sigma}\right)g - \frac{\gamma(\sigma-1)\sigma_n^2}{2\sigma}\right].$$

Since $\bar{c} > 0$ and $\bar{v} > 0$, the stationarity condition from the utility function is given by

$$-\sigma\delta + (\sigma-1)g - \frac{\gamma(\sigma-1)\sigma_n^2}{2} < 0.$$

From (45) with $c_t = \bar{c}$ and $v_t = \bar{v}$ together with the inequality, we have

$$\sigma(r-\delta) - g + \frac{\gamma(1+\sigma)\sigma_n^2}{2} \leq 0.$$

C. Simulated Moments Estimation

Letting $c_{i,t}$ be the logarithm of real consumption expenditures for household i in year t, the four moments we use are

$$h(c_{i,t}) = \begin{pmatrix} h_1(c_{i,t}) \\ h_2(c_{i,t}) \\ h_3(c_{i,t}) \\ h_4(c_{i,t}) \end{pmatrix} = \begin{pmatrix} c_{i,t} \\ \left(\frac{c_{i,t}-\bar{c}}{\bar{c}}\right)^2 \\ \left(\frac{c_{i,t}-\bar{c}}{\bar{s}}\right)^3 \\ \left(\frac{c_{i,t}-\bar{c}}{s}\right)^4 \end{pmatrix},$$

where $\bar{c} = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} c_{i,t}$ is the grand sample mean and $s = \sqrt{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(c_{i,t}-\bar{c})^2}$ is the sample standard deviation. The higher-ordered moments are scaled so that they are all similar in magnitude.

Let $\beta = (\delta, \sigma, \gamma)$ be the parameter vector. For the simulated observations, let

$$h(\beta) = \begin{pmatrix} h_1(\beta) \\ h_2(\beta) \\ h_3(\beta) \\ h_4(\beta) \end{pmatrix} = \begin{pmatrix} c_{i,t}(\beta) \\ \left(\frac{c_{i,t}(\beta)-\bar{c}}{\bar{c}}\right)^2 \\ \left(\frac{c_{i,t}(\beta)-\bar{c}(\beta)}{s}\right)^3 \\ \left(\frac{c_{i,t}(\beta)-\bar{c}(\beta)}{s}\right)^4 \end{pmatrix}.$$

We simulate $N_s = 50000$ individuals over many periods, t. The simulated moments are calculated over observations from $t = 60$ to 76 for the US to correspond to the 16 years of data in our CEX sample. For China, the simulated moments are computed at $t = 20$ since we have only one cross section.

Note, $\bar{c}$ in $h_2(\beta)$ is the mean computed from the sample, and s in $h_3(\beta)$ and $h_4(\beta)$ is the standard deviation computed from the sample. The idea is to scale the sample and simulated moments with the same scaling factors so that we can treat them like constants.

Now let

$$H(c) = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} h(c_{i,t})$$

be the vector of sample moments and

$$H(c(\beta)) = \frac{1}{N_s T_s}\sum_{i=1}^{N_s}\sum_{t=1}^{T_s} h(c_{i,t}(\beta))$$

be the vector of simulated moments and define $u(c_{i,t}) = h(c_{i,t}) - H(c)$. Then, $\Omega_{NT} = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} u(c_{i,t})\, u(c_{i,t})'$ is the sample covariance matrix. The simulated method of moments estimator selects the vector β that minimizes

$$g(\beta, c)' W_{NT}^{-1} g(\beta, c),$$

where $g(\beta, c) = H(c) - H(c(\beta))$ and $W_{NT} = \left(1 + \frac{NT}{N_s T_s}\right)\Omega_{NT}$. Asymptotically,

$$\sqrt{NT}\left(\hat{\beta} - \beta_0\right) \xrightarrow{D} N(0, V_\beta)$$
$$V_\beta = \left(1 + \frac{1}{n}\right)(B'\Omega B)^{-1},$$

where $n = \lim\left(\frac{NT}{N_s T_s}\right)$ and $B = E\left[\frac{\partial g(\beta,c)}{\partial \beta'}\right]$. Inference is drawn using the sample counterparts.