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THE INTERNATIONAL TRANSMISSION OF REAL BUSINESS CYCLES*

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1. INTRODUCTION

This paper is concerned with the international transmission of real business cycles. We present a stochastic model of two barter economies, each inhabited by an infinitely-lived representative firm and consumer/worker. The countries are linked together by free trade in goods and securities. Expectations are rational, full and complete information is costlessly available, and financial capital is perfectly mobile internationally. In our two-country model, the same industry in each country may be exposed to different, country-specific shocks. This leads to an allocation of world investment that depends on the international distribution of these shocks, an effect not present in a closed-economy problem.[2] The international problem is further complicated by the fact that the two representative consumer/workers cannot be treated as a single representative world consumer/producer due to the international immobility of labor.

We find that the construction of the equilibrium is useful and delivers insights beyond that obtained solely from analysis of the planner's problem.[3] The gains from trading claims to risky incomes emerge in our analysis and is explicitly recognized by modeling an international market for securities. This allows us to explicitly study the role of the capital market in the international allocation of risk and in the international transmission mechanism.[4]

The main thesis of our paper can be summarized. Shocks to productivity which raise output abroad are positively transmitted to the home country. A distinction between income and output transmission is useful in our setup. Positive income transmissions occur because agents share their country-specific income risks in

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[2] For example, Long and Plosser (1983).

[3] Dellas (1986) studies only the planning problem in an analysis of the international transmission of business cycles. His is a two-good, two-country version of the Long and Plosser (1983) model in which both goods are consumed and each country specializes in production. Because both commodities are required inputs to production, a positive transmission is in part an artifact of the technology. Furthermore, the trade account is always in balance in Dellas' model thus precluding the possibility for **net** capital flows and the analysis of current account issues.

[4] The asset markets we model are of the type studied by Lucas (1978) and Brock (1982). Our paper is related to Helpman and Razin (1978), Stulz (1983), and Grinols (1984) who analyze international capital markets and production under uncertainty; however, they do not explicitly address business cycle implications. Also, a related paper by Stockman and Svensson (1985) came to our attention after this paper was completed.

the capital market. Thus, a positive shock abroad raises income at home and abroad. Part of this income gain is consumed and part is invested as agents attempt to spread the gain over time. This leads to higher future output at home and abroad; that is, the transmission of output is positive. Moreover, these transmissions are Pareto-efficient and are a natural consequence of engaging in trade. Although the volatility of national output and consumption may be lower in autarchy, agents in both countries experience welfare improvements upon opening their economies to trade.

Until recently, macroeconomic analyses of interdependent economies have been based on the generalizations of the Fleming (1962)-Mundell (1964) model.[5] In these models, rigidities arise either from imperfect information or fixed nominal wage contracts. Therefore, foreign disturbances transmitted to the home country are costly in terms of welfare and may be offset through appropriate indexation schemes or exchange-rate management.[6] In our analysis, economic agents choose consumption and investment rules that generate transmissions in preference to no-transmissions (autarchic) rules.

There are, however, some empirical difficulties with the existing literature on international transmissions. First, depending upon on exchange-rate regime, the Fleming-Mundell model predicts that disturbances can be negatively transmitted.[7] Since national output movements have generally been observed to be positively correlated during both the fixed and flexible exchange-rate periods,[8] the reverse transmission hypothesis is unconvincing, and undue importance seems to have been attached to alternative monetary arrangements. Second, these models do not address the risk-allocating function of the capital markets and cannot explain the international cross-hauling of securities. Third, asset demands are not well defined. Capital mobility in the Fleming-Mundell framework means interest-rate arbitrageurs ensure that rates of return are equalized internationally. The current account, then, is a residual and adjusts only to maintain internal balance. As recently emphasized, a current account surplus should be viewed as the result of planned, savings-investment behavior.[9]

Although we ignore exchange-rate considerations, the dynamics generated in our analysis lead to positively correlated outputs across countries. Asset demands arise from the consumption-savings decision and, with short-term securities, gross

[5] See, for example, Flood (1979), Sachs (1979), Cox (1980), Turnovsky (1977, 1981), and a survey by Mussa (1979).

[6] For example, Flood and Marion (1982) and Aizenman and Frenkel (1985).

[7] That is, with fixed exchange rates, an expansionary fiscal policy abroad raises output abroad but lowers it at home; whereas, under flexible exchange rates, both foreign and domestic output rise. Exactly the reverse is implied for a foreign monetary expansion. Much of the work generalizing the Fleming-Mundell model has tried to reverse the negative transmission results with limited success. See the references cited in Footnote 4.

[8] We have estimated vector auto regressions for various groupings of OECD GDP's and have found that innovation in one country's GDP is followed by a rise in other countries' GDP's. This relationship was present during both fixed and flexible exchange periods. These results are available from the authors upon request.

[9] For example, Sachs (1981), Dornbusch (1985), Aizenman (1983), and Frenkel and Razin (1984).

capital flows are large relative to net capital flows in order to effect efficient international risk bearing.

The remainder of the paper is organized as follows. We present our general two-country, one-good model in Section 2 and discuss an explicit parameterization in Section 3. In Section 4, we analyze the implied dynamics of national outputs, gross national products, and the current account. In Section 5, we discuss the limitations and possible generalizations of the analysis.

2. THE GENERAL MODEL

Consider a two-country model consisting of a "home" and a "foreign" country. We adopt a representative agent setup to describe the aggregate behavior of firms and consumer/workers in each country. Each country is inhabited by a large, fixed number of infinitely-lived consumer/workers (hereafter abbreviated as "consumers") with identical wealth and tastes. Wealth may differ internationally but tastes are assumed to be the same.

Firms at home and abroad produce the same good through a constant returns-to-scale technology which combines units of the commodity (capital) with labor services. Production takes one period and is subject to country-specific technology shocks. Factor costs are equity-financed and incurred *ex ante*, implying that stockholders bear all residual risks. The two countries are linked by free trade in commodities and securities.

Production Technology. Home output, revealed at the beginning of the next period, y', is produced by combining labor services, n, and capital, k, in the current period. The technology for producing the commodity is described by $F(n, k)$ which exhibits constant returns to scale and the standard curvatures. Capital is a commodity input which fully depreciates in the production process.[10] Home output is subject to a multiplicative, country-specific shock, λ, which is common to all firms. Firms in the foreign country have access to the same production technology $F(\cdot, \cdot)$; however, output is subject to a different technology shock, λ^*. Next period's outputs at home and abroad are

$$y' = \lambda' F(n, k); \qquad y^{*\prime} = \lambda^{*\prime} F(n^*, k^*) \tag{1}$$

where primes are used to denote values obtained next period and stars are used to denote foreign variables.

We assume that (λ, λ^*) is an independently and identically distributed, two-element process whose joint density attains positive probability mass only for positive and finite values of λ and λ^*.

Production Decisions. We describe in detail only the problem solved by the domestic firm, as the foreign firm's problem is entirely symmetric.

[10] An alternative but equivalent interpretation is that the undepreciated portion of capital can be converted to the consumption good and, hence, is subsumed in the output term, y.

A sufficient set of variables to describe the state of the world economy consists of the inherited capital stocks, the level of labor services, and the current technology shocks. Let $s' = (n, n^*, k, k^*, \lambda', \lambda^{*\prime})$ denote next period's state vector. At the beginning of each period, the state is revealed and, in equilibrium, all prices and quantities reflecting the actions of individual decision makers will be functions of s.

The conditional distribution of s' given current and past information depends only on the current state, s. We denote this conditional c.d.f. by $G(s' \mid s)$ and conditional expectations are taken with respect to this distribution.

Firms at home and abroad issue one perfectly divisable share of equity each period to finance the acquisition of the commodity input (capital) and labor services. Revenues obtained next period are paid out to stockholders as dividends. There are no retained earnings and the value of existing equities goes to zero. These shares are assumed to be traded internationally in organized securities markets.

Let q be the price of newly issued equity by the representative domestic firm and δ be the discount factor used by shareholders. We assume the existence of complete capital markets which is sufficient for both home and foreign equity holders to share the same discount factor, δ. Thus, the price of a share is simply the expected discounted value of next period's dividends (revenues)

$$q(s) = E[y(s')\delta(s') \mid s]. \tag{2}$$

The firm is assumed to know this discount factor and maximizes its net market value, $q - wn - k$, where w is the market-clearing wage and the price of the numeraire good is unity.

The first-order conditions for the firm's problem equates factor costs to their expected discounted marginal products:

$$w(s) = E[\lambda' F_n(n, k)\delta(s') \mid s]; \qquad \text{and} \tag{3}$$

$$1 = E[\lambda' F_k(n, k)\delta(s') \mid s]. \tag{4}$$

Given complete information and constant returns to scale, equations (3) and (4) are interpreted as industry equilibrium conditions. The net values of firms equal zero, since $q = wn + k$. An analogous problem is solved by the foreign firm.

Consumer/Workers. The problem solved by the consumers at home and abroad are entirely analogous so we describe in detail only the domestic consumer's problem.

The domestic consumer enters each period with real income, ζ, which is allocated to consumption, c, purchases z units of domestic equity, x units of foreign equity, and (possibly) an N-element vector, $\mathbf{m}$, of Arrow-Debreu state securities from foreign consumers.[11] Negative elements of $\mathbf{m}$ indicate sales of state securi-

[11] With multiplicative uncertainty, equities are sufficient to complete the market. We introduce the possibility of state security trading as a convenient way to characterize some aspects of the equilibrium.

ties. Let the price vector of the state securities be $\mathbf{p}$. The jth element of $\mathbf{p}$ is $p(A_j \mid s)$ which is the price of a state security promising to pay off one unit of the consumption good if $s' \in A_j$ given the current state, s, where

$$\text{Prob}\left\{s' \in \bigcup_{j=1}^{N} A_j \mid s\right\} = 1.$$

Let $\mathbf{v}(s')$ be the payoff vector from state security trading where the jth element, $v_j(s')$, is one if $s' \in A_j$ and zero otherwise.

Income in the next period is derived from dividends from home equities, $z \cdot y'$, foreign equities, $x \cdot y^{*\prime}$, state securities, $\mathbf{v}'\mathbf{m}$, and labor income, $w'n'$. Thus, the consumer faces the sequential budget constraints

$$\zeta = c + qz + q^*x + \mathbf{pm}, \qquad \text{and} \tag{5}$$

$$\zeta' = y'z + y^{*\prime}x + \mathbf{v}'\mathbf{m} + w'n' \tag{6}$$

The state vector relevant to the consumer's problem is (ζ, s).

Let the consumer's endowment of time each period be normalized to unity so that leisure is $l = 1 - n$. He wishes to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(1 - n_t, c_t), \tag{7}$$

subject to (5) and (6). Here E_t is the mathematical expectation conditioned at time t information, $0 < \beta < 1$ is a subjective discount factor, and $u(\cdot)$ is the single-period, utility function which exhibits standard curvatures.

The functional equation for the value $V(\zeta, s)$ of (11), when the domestic consumer is in state (ζ, s) and behaves optimally forever, is

$$V(\zeta, s) = \max\{u(1 - n, c) + \beta E[V(\zeta', s') \mid s]\}.$$

The first-order, necessary conditions for an optimum are:[12]

$$u_1(1 - n(s), c(s)) = w(s)u_c(1 - n(s), c(s)); \tag{8.a}$$

$$q(s)u_c(1 - n(s), c(s)) = \beta E[V_\zeta(\zeta', s')y(s') \mid s]; \tag{8.b}$$

$$q^*(s)u_c(1 - n(s), c(s)) = \beta E[V_\zeta(\zeta', s')y^*(s') \mid s]; \tag{8.c}$$

$$p(A_j \mid s)u_c(1 - n(s), c(s)) = \beta E[V_\zeta(\zeta', s')v_j(s') \mid s], \qquad j = 1, \ldots, N; \qquad \text{and} \tag{8.d}$$

$$V_\zeta(\zeta, s) = u_c(1 - n(s), c(s)). \tag{8.e}$$

These conditions have the usual interpretations: the utility costs of foregone leisure or consumption are equated with the discounted expected value of its return. An analogous problem faces the foreign consumer.

From (8.d) and (8.e) (and their foreign counterparts), it is seen that domestic

[12] We proceed informally and assume enough regularity conditions hold for a unique optimum to exist and for the value function to be differentiable.

and foreign consumers do indeed share the same discount factor

$$\delta' = \frac{\beta u_c(1 - n', c')}{u_c(1 - n, c)} = \frac{\beta u_{c*}(1 - n^{*\prime}, c^{*\prime})}{u_{c*}(1 - n^*, c^*)}. \tag{9}$$

Since a fixed quantity of equities are issued each period and the value of old equities goes to zero, (8.b)–(8.e) (or their foreign counterparts) deliver the standard asset pricing formulas found in Lucas (1978) and Brock (1982).

Finally, aggregate demands satisfy the world resource constraint,

$$y + y^* = c + c^* + k + k^*, \tag{10}$$

and equilibrium in securities markets require:

$$z + z^* = 1; \tag{11}$$

$$x + x^* = 1; \qquad \text{and} \tag{12}$$

$$\mathbf{m} + \mathbf{m}^* = \mathbf{0}, \tag{13}$$

where z^*, x^*, and $\mathbf{m}^*$ are shares of the domestic equity, foreign equity, and state securities purchased by the foreign consumer.

3. A SPECIFIC PARAMETERIZATION

In this section, the model is parameterized to yield explicit solutions. First, we assume that home and foreign utility is logarithmic and separable in consumption and leisure:

$$u(1 - n_t, c_t) = \theta_n \ln(1 - n_t) + \theta_c \ln(c_t) \tag{14}$$

$$u(1 - n_t^*, c_t^*) = \theta_n \ln(1 - n_t^*) + \theta_c \ln(c_t^*).$$

Second, we assume Cobb-Douglas production,

$$y_{t+1} = \lambda_{t+1} n_t^{\alpha} k_t^{1-\alpha}; \qquad y_{t+1}^* = \lambda_{t+1}^* n_t^{*\alpha} k_t^{*1-\alpha}, \tag{15}$$

where $0 < \alpha < 1$ is "labor's share."

Because capital markets are complete, the equilbrium will be supported by the solution to a planner's problem in which $\{c, c^*, n, n^*, k, k^*\}$ is chosen to solve

$$V(\bar{y}, s) = \max \{\phi u(1 - n, c) + (1 - \phi)u(1 - n^*, c^*) + \beta E[V(\bar{y}', s') \mid s]\}$$

subject to the aggregate resource constraint, the international immobility of labor, and specifications of preferences and technology. Here, the planner attaches a weight ϕ and $(1 - \phi)$ to the domestic and foreign agent respectively. Given the solution to this planning problem, the equilibrium can be conveniently characterized.

Let upper bars denote world totals (e.g., $\bar{c} = c + c^*$, $\bar{y} = y + y^*$, etc.). In the appendix, we show that the solution to this planning problem implies:

$$\bar{c}_t = [1 - \beta(1 - \alpha)]\bar{y}_t; \qquad c_t = \phi \bar{c}_t; \qquad c_t^* = (1 - \phi)\bar{c}_t; \tag{16}$$

$$\bar{k}_t = \beta(1-\alpha)\bar{y}_t\,; \qquad k_t = \psi\bar{k}_t\,; \qquad k_t^* = (1-\psi)\bar{k}_t\,; \tag{17}$$

$$\begin{aligned} n &= \beta\alpha\theta_c\psi/\{\theta_n[1-\beta(1-\alpha)]\phi + \beta\alpha\theta_c\psi\}\,; \\ n^* &= \beta\alpha\theta_c(1-\psi)/\{\theta_n[1-\beta(1-\alpha)](1-\phi) + \beta\alpha\theta_c(1-\psi)\} \end{aligned} \tag{18}$$

where $\psi \equiv E[(y'/\bar{y}')\,|\,s]$ is the home country's relative share of world output and is a measure of the expected relative return from investing there. It is easily verified that (16)–(18) satisfy the first-order conditions, (3), (4), and (8.a)–(8.e). The fixity of relative investment shares, ψ and $1-\psi$, results from the assumptions of i.i.d. shocks and full depreciation of capital. Labor supplies are fixed in this setup because the income and substitution effects cancel out. *World* consumption and investment are constant shares of *world* output; whereas, the distribution of consumption and investment between the countries depends upon relative wealth and capital productivity considerations.

We now proceed to characterize the equilibrium. First, equity prices are

$$q_t = \beta\psi\bar{y}_t\,; \qquad q_t^* = \beta(1-\psi)\bar{y}_t\,. \tag{19}$$

To determine the relative consumption shares, ϕ and $1-\phi$, it is useful to calculate consumer wealth and to exploit the fact that, under infinite-horizon, logarithmic utility, the consumer consumes a constant, $1-\beta$, of his current wealth (e.g., Samuelson 1969). Wealth consists of currently received dividends plus the expected present value of current and future labor income. Each period, domestic workers' labor income is $w_t n_t = E_t[\alpha y_{t+1}\delta_{t+1}]$ and foreign workers' labor income is $w_t^* n_t^* = E_t[\alpha y_{t+1}^*\delta_{t+1}]$. As can be seen from (9), (14), and (16), the single-period discount factor is $\delta_{t+1} = \beta\bar{y}_t/\bar{y}_{t+1}$ and the τ-period discount factor is $\delta_{t+1}\cdot\delta_{t+2}\cdots\delta_{t+\tau} = \beta^\tau\bar{y}_t/\bar{y}_{t+\tau}$. The expected present value of current and future labor income is

$$E_t\sum_{j=1}^{\infty}\beta^j(\bar{y}_t/\bar{y}_{t+j})\alpha y_{t+j} = [\beta\alpha\psi/(1-\beta)]\bar{y}_t$$

at home, and

$$E_t\sum_{j=1}^{\infty}\beta^j(\bar{y}_t/\bar{y}_{t+j})\alpha y_{t+j}^* = [\beta\alpha(1-\psi)/(1-\beta)]\bar{y}_t$$

abroad. Thus, given initial shareholdings (z_{t-1}, z_{t-1}^*, x_{t-1} and x_{t-1}^*), wealth at home and abroad, W_t and W_t^*, and for the world, $\bar{W}_t$, are

$$W_t = z_{t-1}y_t + x_{t-1}y_t^* + [\beta\alpha\psi/(1-\beta)]\bar{y}_t\,; \tag{20}$$

$$W_t^* = z_{t-1}^*y_t + x_{t-1}^*y_t^* + [\beta\alpha(1-\psi)/(1-\beta)]\bar{y}_t\,; \quad \text{and} \tag{21}$$

$$\bar{W}_t = \{[1-\beta(1-\alpha)]/(1-\beta)\}\bar{y}_t\,. \tag{22}$$

Notice that each country's wealth depends upon its (arbitrary) initial income from portfolio investment and its current and future labor incomes which depend upon relative investment shares, ψ and $1-\psi$.

Exploiting the fact that consumption is $1-\beta$ times wealth, it follows from (17), (20), and (21) that

$$\phi = \{\beta\alpha\psi + [(1-\beta)(z_{t-1}y_t + x_{t-1}y_t^*)]/\bar{y}_t\}/(1-\beta(1-\alpha)) \quad \text{and} \tag{23}$$

$$(1-\phi) = \{\beta\alpha(1-\psi) + [(1-\beta)(z_{t-1}^* y_t + x_{t-1}^* y_t^*)]/\bar{y}_t\}/(1-\beta(1-\alpha)). \tag{24}$$

To obtain a perfect risk-pooling equilibrium, home and foreign marginal utilities of consumption should be perfectly correlated. With logarithmic utility, this requires that home consumption (wealth) be perfectly correlated with foreign consumption (wealth).

It can be seen from (20) and (21) that this can be attained by setting $z_t = x_t$ and $z_t^* = x_t^*$ since this renders W_{t+1} and W_{t+1}^* linear in $\bar{y}_{t+1}$. Thus, when the domestic consumer holds the same number of domestic equity shares as foreign equity shares in his portfolio and the foreign consumer does likewise, their wealths will be perfectly correlated over time. In the next period, domestic and foreign wealth will be

$$W_{t+1} = [\hat{z}_t + \beta\alpha\psi/(1-\beta)]\bar{y}_{t+1} \quad \text{and} \tag{25}$$

$$W_{t+1}^* = [1 - \hat{z}_t + \beta\alpha(1-\psi)/(1-\beta]\bar{y}_{t+1} \tag{26}$$

where $\hat{z}_t \equiv z_t = x_t$ and $1 - \hat{z}_t \equiv z_t^* = x_t^*$.

Finally, it can be seen from (20), (21), (25), and (26) that $\hat{z}_t$ must be the same in every period in order to preserve relative wealth positions, ϕ and $1-\phi$, over time. The value of $\hat{z}$ which achieves this is

$$\hat{z} = \phi + [\beta\alpha/(1-\beta)](\phi - \psi). \tag{27}$$

That is, given physical investment shares (ψ and $1-\psi$) and inherited wealth shares (ϕ and $1-\phi$), agents maintain their relative wealths through time by engaging in portfolio investment according to (27).

We have allowed ϕ to differ from ψ, possibly as a result of unilateral transfers which occured in the past. A transfer from the foreign country to the home country in the current period raises ϕ so that a higher value of $\hat{z}$ will be maintained from this point on.

4. IMPLICATIONS OF THE PARAMETERIZED MODEL

The International Transmission Mechanism and the Dynamics of GDP's and GNP's. We substitute the labor supply and investment rules, (17) and (18), into the production functions (15) and take logarithms to obtain the following system of stochastic difference equations:

$$\log y_t = b + (1-\alpha)\log \bar{y}_{t-1} + \varepsilon_t, \tag{28}$$

$$\log y_t^* = b^* + (1-\alpha)\log \bar{y}_{t-1} + \varepsilon_t^*, \tag{29}$$

$$\log \bar{y}_t = d + (1-\alpha)\log \bar{y}_{t-1} + v_t, \tag{30}$$

where $b \equiv \log(n^\alpha[\beta(1-\alpha)\psi]^{1-\alpha})$, $b^* \equiv \log(n^{*\alpha}[\beta(1-a)(1-\psi)]^{1-\alpha})$, $d \equiv \log([\beta(1-\alpha)]^{1-\alpha})$, $\varepsilon_t = \log(\lambda_t)$, $\varepsilon_t^* = \log(\lambda_t^*)$, and $v_t \equiv \log(\lambda_t n^\alpha \psi^{1-\alpha} + \lambda_t^* n^{*\alpha}(1-$

$\psi)^{1-\alpha}$). Suppose the technology shocks are independent so that ε and ε^* are uncorrelated. Then it is clear that a positive shock to domestic production increases home output and, therefore, world output. This then leads to increased foreign output next period, and the rise in domestic and foreign outputs over their "natural" levels tends to persist.

The "business cycle" is transmitted abroad through the capital market. The shock to domestic output raises current-period dividends and, hence, income. It also raises the discount factor $\{\delta_t, \delta_{t+1}, \cdots\}$ used in discounting future labor income. Both effects raise wealth at home and abroad, leading to increased portfolio and physical investment worldwide. Inputs of capital in production increase next period, raising output at home and abroad above their natural levels. GDP's are, thus, positively correlated over time. The unconditional correlation between log (y) and log (y^*) is:

$$0 < \frac{(1-\alpha)^2 \text{ Var }(v)}{[(1-\alpha)^2 \text{ Var }(v) + \text{Var }(\varepsilon)]^{1/2}[(1-\alpha)^2 \text{ Var }(v) + \text{Var }(\varepsilon^*)]^{1/2}} < 1.$$

While the correlation between log (y) and log (y^*) is less than one, logarithms of incomes received at time t are perfectly correlated because ξ and ζ^* are proportional to $\bar{y}$. This is a result of the perfect-pooling equilibrium and the assumption that the utilities are logarithmic and time-separable over an infinite horizon. With perfect capital markets, these assumptions would imply perfectly correlated national incomes regardless of the actual production technologies.

We hesitate to confront these predictions on the relative correlations of national incomes and national outputs with actual data because GNP accounting does not include all fluctuations in income, in particular, captial gains. In actual economies, capital and equities are long-lived, dividends tend to be stable, while retained earnings are more volatile. Fluctuations in equity values affect relative (international) incomes. However, the result that properly measured national incomes are more correlated than outputs is due to the perfect risk pooling, and should be robust to the life span of capital and equities.

Output and Consumption Volatility and the Gains to Trade. In autarchy, the firm's problem is unchanged; however, there are no risk-pooling opportunities so that, in equilibrium, the representative consumer holds the firm's entire stock of equity each period.

Consider the home country in autarchy. Letting the superscript "a" denote autarchic values, it is easy to show that the following decision rules will be chosen:

$$c_t^a = [1 - \beta(1-\alpha)]y_t^a, \tag{31}$$

$$k_t^a = \beta(1-\alpha)y_t^a, \qquad \text{and} \tag{32}$$

$$n^a = \beta\alpha\theta_c/[\theta_n(1-\beta(1-\alpha)) + \beta\alpha\theta_c], \tag{33}$$

where $y_t^a = \lambda_t(n^a)^\alpha(k_{t-1}^a)^{1-\alpha}$.

Upon opening the economy to trade, it can be shown that the change in the variance of the logarithms of output and consumption are

$$\mathrm{Var}(\log y_t) - \mathrm{Var}(\log y_t^a) = \{(1-\alpha)^2/\alpha(2-\alpha)\}[\mathrm{Var}(v_t) - \mathrm{Var}(\varepsilon_t)] \gtrless 0, \quad \text{and}$$

$$\begin{aligned}\mathrm{Var}(\log c_t) - \mathrm{Var}(\log c_t^a) &= (1-\alpha)[\mathrm{Var}(\log y_t) - \mathrm{Var}(\log y_t^a)] \\ &= \{1/\alpha(2-\alpha)\}[\mathrm{Var}(v_t) - \mathrm{Var}(\varepsilon_t)] \gtrless 0\end{aligned}$$

where $\mathrm{Var}(\cdot)$ is the unconditional variance operator.

It is interesting to note that, depending upon the relative magnitudes of $\mathrm{Var}(v)$ and $\mathrm{Var}(\varepsilon)$, volatility may or may not be higher in trade, but this has no direct welfare implications. Volatility is higher in trade if the variance of λ is low relative to the variance of λ^*. (This implies that volatility abroad declines in trade.) If λ and λ^* have the same variance, output and consumption volatility is reduced in trade both at home and abroad.

For simplicity, let $\phi = \psi$, for in this case it can be seen from equations (18) and (33) that employment is identical in trade as in autarchy. Therefore, a welfare comparison of the two situations involves only a comparison of the values of consumption plans in the two regimes. Upon opening the economy to trade, the home consumer accrues a welfare gain

$$\begin{aligned}&E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t/c_t^a) \\ &\qquad = [1/(1-\beta)]\left\{\log \psi - E_0 \log \frac{\lambda_1 n^\alpha \psi^{1-\alpha}}{\lambda_1 n^\alpha \psi^{1-\alpha} + \lambda_1^* n^{*\alpha}(1-\psi)^{1-\alpha}}\right\},\end{aligned}$$

which is positive by virtue of Jensen's inequality and equation (A.15) in the appendix.

Opening up trade essentially pushes out the "efficient frontier." Although the volatility of consumption may be higher in trade, agents are willing to bear the additional "risk" in exchange for higher expected "return."[13] As in the standard Heckscher-Olin model, both countries unambiguously gain from trade because tastes are the same but opportunity sets differ in autarchy.[14]

Current Account Dynamics. Recall that the value of equities goes to zero at the end of each period. Consequently, the period-to-period change in the home country's net foreign asset position is $x(q_t^* - q_{t-1}^*) - z^*(q_t - q_{t-1})$. From equations (19) and (27), the current account surplus, CA_t, measured by the change in net foreign asset holdings, can be expressed as

$$\begin{aligned}(34)\qquad CA_t &= \beta(\hat{z} - \psi)\,\Delta\bar{y}_t \\ &= \{\beta[1-\beta(1-\alpha)]/(1-\beta)\}(\phi - \psi)\,\Delta\bar{y}_t.\end{aligned}$$

[13] We use the terms "efficient frontier" and "risk" rather loosely here. They are not meant to be technical definitions in the context of this model.

[14] There are environments where some trades cannot be made, say in an overlapping generations framework, that taste differences across countries can induce a deterioration of welfare upon the opening up of trade. On this point, see Buiter (1981) and Dornbusch (1985).

Once the international distribution of wealth is given, the sign of the current account surplus depends only on the sign of $\Delta\bar{y}_t$ because each country invests a fixed fraction of its wealth, and each country's wealth is linear in $\bar{y}_t$. For example, if the home country is wealthier than the foreign country ($\phi > \psi$, say) it will hold the larger share of the outstanding equities in the world. When world output goes up, so does its wealth and, hence, its *net* foreign asset holdings rise. This relationship is time invariant, however, since redistributions of wealth do not occur in equilibrium.

It was shown in (30) that $\log(\bar{y}_t)$ follows a first-order autoregressive AR(1) process. It follows that $\Delta \log(\bar{y}_t)$ follows a mixed first-order autoregressive moving average ARMA(1, 1) process. Provided that $\phi \neq \psi$, the current account fluctuations generated by our model tend to persist and be positively correlated over time.

When $\phi = \psi$, *net* capital flows are identically zero each period but gross capital flows may still be large. Since the equities are all short term, substantial cross-hauling of securities is necessary to effect efficient risk pooling. Thus, our model is consistent with the observation that gross capital flows tend to be large relative to net capital flows.

The Correlation between Savings and Investment. It has been argued (Feldstein and Horoika 1980; Feldstein 1983) that the degree of capital mobility can be inferred from the correlation of a country's savings and investment. The reasoning underlying this argument is, with perfect capital mobility, that domestic savings respond to world opportunities for investment and domestic investment is financed by a world pool of savings. An increase in savings in one country is channeled towards investment in all countries so that the correlation between savings and investment depends upon the country's share of world capital (that is, the correlation should be one half in a world of two countries of equal size and zero for a small country). On the other hand, if capital is completely immobile as in a closed economy, savings and investment are perfectly correlated. Thus, a high correlation between savings and investment is interpreted as evidence that capital is immobile.

Obstfeld (1986) and Stulz (1986) have provided counterexamples to this line of reasoning. Our model also provides a convenient counterexample because home savings and investment are perfectly correlated even though capital is perfectly mobile.

5. CONCLUDING REMARKS, LIMITATIONS, AND POSSIBLE GENERALIZATIONS

This paper has presented a model for the international transmission of business cycles. The sources of risk in the model are business cycles emanating from country-specific technology shocks. International securities markets transmit business cycles and provide an opportunity for domestic and foreign agents to pool these risks. GDP's and GNP's are found to be correlated across countries.

The model also provides a setting in which substantial cross-hauling of securities is necessary to maintain efficient risk pooling.

One restrictive aspect of the setup is that agents maintain their *relative* wealth positions over time. This is due to the risk-pooling equilibrium in conjunction with the time-separable, logarithmic utility specification. One result is that the correlation of the current account surplus and the country's own output is always positive for the wealthy country and negative for the poor country. If there exist non-marketable risks, the relative wealths and the signs of these covariances might change over time. If we allow serial dependence in the process governing the productivity shocks, the countries' relative shares of investment and the sign of the current account's correlation with output might fluctuate. These generalizations would probably reduce the correlation between each country's savings and investment, and produce negative transmissions of the business cycle.[15, 16]

Our specification leads to simplified dynamics for key economic variables. We adopted this specification for its analytical tractability; the equilibrium can be conveniently characterized and closed-form solutions can be obtained. Incorporating non-separable utility and specifying more complicated investment technologies may be feasible and would generate more complicated dynamics that might come closer to replicating the empirical record.

Finally, we have studied only a barter model which focuses on the transmission mechanism independent of exchange-rate considerations. The introduction of money would allow for the determination of nominal exchange rates, another source of risk, and another possible business cycle transmission mechanism.[17]

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APPENDIX

Here, we discuss the solution to the planner's problem in which $\{c, c^*, n, n^*, k, k^*\}$ are chosen to solve:

$$V(\bar{y}, s) = \max\,\{\phi u(1-n, c) + (1-\phi)u(1-n^*, c^*) + \beta E[V(\bar{y}', s') \mid s]\} \tag{A.1}$$

subject to $\bar{y} = y + y^* = c + c^* + k + k^*$, $0 \leq n, n^* \leq 1$, $u(1-n, c) = \theta_n \log(1-n) + \theta_c \log(c)$, $y' = \lambda' n^\alpha k^{1-\alpha}$, $y^{*\prime} = \lambda^{*\prime} n^{*\alpha} k^{*1-\alpha}$. The first-order conditions are

$$V_{\bar{y}}(\bar{y}, s) = \beta(1-\alpha)E[V_{\bar{y}}(\bar{y}', s')y(s')/k(s) \mid s] \tag{A.2}$$

[15] In this setup, capital mobility, in the sense of diversifying risk, tends to increase the correlation between savings and investment. However, changing investment opportunities (non-constant ψ's) would reduce the correlation.

[16] In a deterministic model, Cantor and Mark (1987) show that negative transmissions are generated when shocks to technology are permanent. Thus, a generalization of the process governing the technology shocks to allow permanent disturbances may generate negative transmissions in a stochastic model as well.

[17] King and Plosser (1984) have introduced inside money into a closed-economy, real business cycle model. In the context of open economy, optimizing models, Eaton (1983) and Stockman and Svensson (1985) have incorporated money through cash-in-advance constraints.

(A.3) $$V_{\bar{y}}(\bar{y}, s) = \beta(1-\alpha)E[V_{\bar{y}}(\bar{y}', s')y^*(s')/k^*(s) \mid s]$$

(A.4) $$V_{\bar{y}}(\bar{y}, s) = \phi\theta_c/c(s)$$

(A.5) $$V_{\bar{y}}(\bar{y}, s) = (1-\phi)\theta_c/c^*(s)$$

(A.6) $$\phi\theta_n/(1-n(s)) = \beta\alpha E(V_{\bar{y}}(\bar{y}', s')y(s')/n(s) \mid s]$$

(A.7) $$(1-\phi)\theta_n/(1-n^*(s)) = \beta\alpha E[V_{\bar{y}}(\bar{y}', s')y^*(s')/n^*(s) \mid s]$$

The solution to the planner's problem takes the form

(A.8) $$V(\bar{y}, s) = K + [\theta_c/(1-\beta(1-\alpha))] \log(\bar{y})$$

where K is a constant. Substituting (A.8) into (A.2)–(A.7) implies:

(A.9) $$c = \phi[1-\beta(1-\alpha)]\bar{y},$$

(A.10) $$c^* = (1-\phi)[1-\beta(1-\alpha)]\bar{y},$$

(A.11) $$k = \beta\psi(1-\alpha)\bar{y}$$

(A.12) $$k^* = \beta(1-\psi)(1-\alpha)\bar{y}$$

(A.13) $$n = \beta\alpha\theta_c\psi/\{\theta_n[1-\beta(1-\alpha)]\phi + \beta\alpha\theta_c\psi\}, \qquad \text{and}$$

(A.14) $$n^* = \beta\alpha\theta_c(1-\psi)/\{\theta_n[1-\beta(1-\alpha)](1-\phi) + \beta\alpha\theta_c(1-\psi)\}$$

where $\psi \equiv E[(y(s')/\bar{y}(s')) \mid s]$. Substituting (A.11) and (A.12) into this expression for ψ, it can be seen that ψ must solve

(A.15) $$\psi = E\left(\left[\frac{\lambda' n^\alpha \psi^{1-\alpha}}{\lambda' n^\alpha \psi^{1-\alpha} + \lambda^{*\prime} n^{*\alpha}(1-\psi)^{1-\alpha}}\right] \middle| s\right)$$

where n and n^* are given in (A.13) and (A.14). Thus (A.13), (A.14), and (A.15) form a system of 3 equations and 3 unknowns, ψ, n, and n^*. To show that a unique solution exists, we proceed in three steps. Step 1 shows for any n and n^* ($0 \le n$, $n^* \le 1$), a unique ψ exists to solve (A.12). Steps 2 and 3 show that a unique pair (n, n^*) ($0 \le n$, $n^* \le 1$) exist which jointly solves (A.13) and (A.14).

1. Using the proof in the appendix to Section 1.5 in Brock (1982), for any fixed n and n^* ($0 \le n$, $n^* \le 1$) it can be shown that there exists a unique $\psi(n, n^*)$ which solves (A.15) where,

(A.16) $$\psi_n(n, n^*) > 0; \qquad \psi_n^*(n, n^*) < 0;$$
$$\lim_{n \to 0} \psi(n\ n^*) = 0; \qquad \lim_{n* \to 0} \psi(n, n^*) = 1.$$

To economize on notation, let $a \equiv \theta_n\phi[1-\beta(1-\alpha)]$, $b \equiv \beta\alpha\theta_c$, and $h \equiv \theta_n(1-\phi)[1-\beta(1-\alpha)]$.

2. Next, we show that for any fixed $n^*(0 \le n^* \le 1)$, if

(A.17) $$\psi_n(n, n^*) < (a + b\psi)^2/ab,$$

a unique n exists which solves (A.13), and that this solution value is decreasing in n^*.

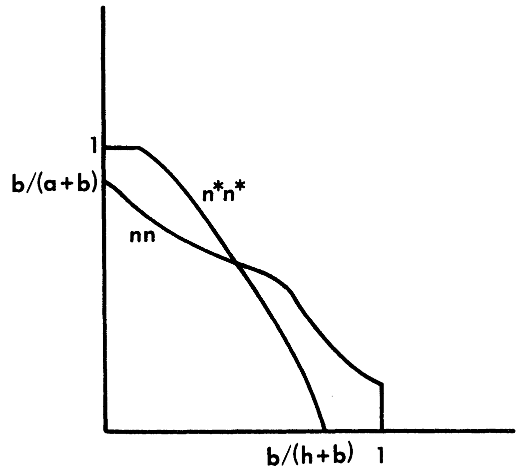

FIGURE A.1

Define on [0, 1], the function, $f(n) = b\psi(n, n^*)/(a + b\psi(n, n^*))$. Since $f'(n) = [ab/(a + b\psi)^2]\psi_n > 0$ if (A.17) holds (and we assume it does), then $f' < 1$ so that f is a contraction mapping. Now raising n^* shifts the function f down as $f_{n*}(n) = [ab/(a + b\psi)^2]\psi_{n*} < 0$. Thus it follows that the fixed point n is decreasing in n^*. This relationship is labeled nn in Figure A.1.

3. Using the same argument as in 2, it can be shown for any fixed n $(0 \leq n \leq 1)$, if

$$(A.18) \qquad -\psi_{n*}(n, n^*) = |\psi_{n*}(n\ n^*)| < [h + b(1 - \psi)]^2/hb,$$

a unique n^* exists which solves (A.14). Moreover, this solution value is decreasing in n. This relationship is labeled n^*n^* in Figure A.1.

REFERENCES

AIZENMAN, JOSHUA, "A Theory of Current Account and Exchange Rate Determinations," *European Economic Review* 23 (1983), 261–280.

BROCK, WILLIAM, "Asset Prices in a Production Economy," in *The Economics of Information and Uncertainty*, John McCall, ed. University of Chicago Press for the National Bureau of Economic Research (1982).

BUITER, WILLIAM, "Time Preference and International Lending and Borrowing in an Overlapping Generations Model," *Journal of Political Economy* 89 (1981), 769–797.

CANTOR, RICHARD AND NELSON C. MARK, "International Debt and World Business Fluctuations," *Journal of International Money and Finance* 6 (1987), 153–166.

COX, W. M. "Unanticipated Money, Output, and Prices in the Small Economy," *Journal of Monetary Economics* 6 (1980), 359–384.

DANIEL, BETTY, "The International Transmission of Economic Disturbances under Flexible Exchange Rates," *International Economic Review* 22 (1981), 401–409.

DELLAS, HARRIS, "A Real Model of the World Business Cycle," *Journal of International Money and Finance* 5 (1986), 381–394.

DORNBUSCH, RUDIGER, "Real Interest Rates, Home Goods, and Optimal External Borrowing," *Journal of Political Economy* 91 (1983), 141–153.

———, "Intergenerational and International Trade." *Journal of International Economics* 18 (1985), 123–139.

FELDSTEIN, MARTIN, "Domestic Saving and International Capital Movements in the Long Run and Short Run," *European Economic Review* 21 (1983), 129–151.

——— AND CHARLES HORIOKA, "Domestic Saving and International Capital Flows," *Economic Journal* 90 (1980), 314–329.

FLEMING, MARCUS, "Domestic Financial Policies under Fixed and Floating Exchange Rates," *I.M.F. Staff Papers* 9 (1962), 363–379.

FLOOD, ROBERT, "Capital Mobility and the Choice of Exchange-Rate Systems." *International Economic Review*, 405–416.

——— AND NANCY MARION, "The Transmission of Disturbances under Alternative Exchange-Rate Regimes." *Quarterly Journal of Economics* (1982), 43–66.

FRENKEL, JACOB AND ASSAF RAZIN, "Government Spending, Debt, and International Economic Interdependence," *Economic Journal* 95 (1985), 619–636.

GRINOLS, EARL, "Production and Risk Leveling in the Intertemporal Capital Asset Pricing Model," *Journal of Finance* 39 (1984), 1571–1595.

HELPMAN, ELHANAN AND ASSAF RAZIN, *A Theory of International Trade under Uncertainty* (New York: Academic Press, 1978).

LONG, JOHN AND CHARLES PLOSSER, "Real Business Cycles," *Journal of Political Economy* 91 (1983), 39–69.

LUCAS, ROBERT E., "Asset Prices in an Exchange Economy," *Econometrica* 46 (1978), 1429–1445.

———, "Interest Rates and Currency Prices in a Two-Country World," *Journal of Monetary Economics* 10 (1982), 334–360.

MUNDELL, ROBERT, "Capital Mobility and Stabilization Policy under Fixed and Flexible Exchange Rates," *Canadian Journal of Economics and Political Science* 29 (1963), 474–485.

MUSSA, MICHAEL, "Macroeconomic Interdependence and the Exchange-Rate Regime," *International Economic Policy*, Rudiger Dornbusch and Jacob Frenkel, eds. (Baltimore: Johns Hopkins University Press, 1979).

OBSTFELD, MAURICE, "Capital Mobility in the World Economy: Theory and Measurement," *Carnegie-Rochester Public Policy Conference* 24 (1986), 53–103.

———, "Intertemporal Price Speculation and the Optimal Current-Account Deficit," *Journal of International Money and Finance* 2 (1983), 215–224.

SACHS, JEFFERY, "Wages, Flexible Exchange Rates, and Macroeconomic Policy," *Quarterly Journal of Economics* 95 (1980), 731–747.

SAMUELSON, PAUL, "Lifetime Portfolio Selection by Dynamic Stochastic Programming," *Review of Economics and Statistics* 51 (1969), 139–146.

STOCKMAN, ALAN AND LARS SVENSSON, "Capital Flows, Investment, and Exchange Rates," NBER Working Paper 1598 (1985).

STULZ, RENE, "Capital Mobility in the World Economy: Theory and Measurement: A Comment," *Carnegie Rochester Series on Public Policy* 24 (1986), 53–113.

———, "The Determinants of Net Foreign Investment," *Journal of Finance* 38 (1983), 459–468.

TURNOVSKY, STEPHEN, "On the Insulation Properties of Flexible Exchange Rates," *Revue Economique* (1977), 719–746.

———, "Monetary Policy and Foreign Price Disturbances under Flexible Exchange Rates: A Stochastic Approach," *Journal of Money, Credit, and Banking* 13 (1981), 575–591.