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International Debt and World Business Fluctuations

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The international transmission of real productivity disturbances is examined in an optimizing model with two countries each inhabited by an infinitely-lived representative consumer/producer. We analyze the dynamics of output, consumption, investment, interest rates, and net external debt. Foreign productivity shocks can have positive or negative impacts on domestic output and international debt, depending on the intertemporal distribution of the shocks. Transient foreign shocks are shown to be positively transmitted and lead to increased domestic indebtedness, while permanent foreign shocks are negatively transmitted and lead to a decline in domestic indebtedness.

This paper is concerned with the international transmission of real output fluctuations and the implications of such disturbances for the behavior of real interest rates and the distribution of international debt. We present a deterministic, perfect-foresight model of two countries, each inhabited by an infinitely-lived representative consumer/producer. The countries are linked together by international goods and capital markets. In our model, output fluctuations are triggered by exogenous shocks to a country's production function and are propagated internationally through the international capital market. National output movements are positively or negatively related depending on the intertemporal distribution of exogenous disturbances to production. We show that a transitory increase to current-period foreign productivity raises both foreign and domestic output in the short run. Part of this transitory income gain to foreigners is invested in foreign production and part is lent to the home country thereby increasing the home country's net external debt. On the other hand, a permanent increase in foreign productivity raises foreign output but lowers domestic output in the short run. As a result of this permanent income gain, foreign borrowing increases principally to smooth consumption, thereby reducing the home country's net external debt.

The analysis of fluctuations triggered by productivity or supply shocks has figured prominently in recent research in closed-economy macroeconomics (Kydland and Prescott, 1982; Long and Plosser, 1983). Analysis of these issues requires a dynamic structure with production and investment. This paper applies

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these ideas to the analysis of the open economy and in particular to the international transmission of these shocks. Concern with the international transmission of fluctuations has enjoyed a long tradition in international economics (Morgenstern, 1959). Theoretical analyses of these issues have typically been based on variations of the Mundell–Fleming framework.[1]

Two unsatisfactory features are associated with the Mundell–Fleming approach. First, the transmission of an exogenous disturbance from one country to the other is exchange rate regime dependent.[2] Since national output movements have generally been observed to be positively correlated during both the fixed and flexible exchange rate periods (Layton, 1985; Gerlach, 1985), this analysis appears to place undue importance on international monetary arrangements.[3] Second, there is no role for asset accumulation. Capital mobility in the Mundell–Fleming framework refers to the notion that interest rate arbitrageurs ensure that rates of return are equalized internationally. There, the current account is a residual which adjusts to maintain internal balance while the effects of the intertemporal terms of trade on the current account are ignored (Miller, 1968; Connolly and Ross, 1970). As recently emphasized (Sachs, 1981; Dornbusch, 1983; Obstfeld, 1983; Aizenman, 1983; Frenkel and Razin, 1985), a current account surplus reflects an increase in a country's net foreign asset position and is properly viewed as the result of planned savings–investment behavior. The approach adopted in this paper is consistent with this view.

While the model presented here is somewhat stylized, we feel that our results are a useful first step in explaining the behaviour of net external debt and national output movements in the world economy. In the one-commodity world we consider, our model highlights the behavior of the intertemporal terms of trade and the role of intertemporal substitution. Asset markets are a fundamental link among national economies and capital mobility plays a prominent role in transmitting disturbances internationally. Our results are consistent with rational expectations (*i.e.*, perfect foresight in this deterministic model), full information and perfect capital mobility.

The remainder of the paper is organized as follows. In the next section, the formal structure of the model is presented. The solution and characterization of equilibrium follow in Section II. Section III examines some comparative dynamics of the model. Finally, Section IV contains some concluding remarks on the limitations of the present analysis and some possible extensions for future research.

I. The Model[4]

We consider two countries, each inhabited by an infinitely-lived representative agent, producing a single commodity. In this one-good world, the output is used for consumption as well as for an input to next period's production. Lending or borrowing from abroad is financed through a competitive international bond market. There are no restrictions on the trade of goods or assets.

I.A. The Home Country

Consider, first, the home country. The domestic agent commits his input of the commodity, along with his fixed supply of labor (normalized to unity), at time t. Production requires one period and the resulting output at $t+1$ is revealed by the

Cobb–Douglas technology.

$$\langle 1 \rangle \qquad y_{t+1} = \lambda_{t+1} k_t^{\theta}, \qquad 0 < \theta < 1,$$

where y_{t+1} is output in $t+1$, λ_{t+1} is an exogenous shock to productivity occurring at $t+1$, and k_t is the commodity input (capital) committed at t. The input is fully used up in the production process, so that capital depreciates fully during the period.

Let τ denote the current period. In each period t, the domestic resident faces a budget constraint of the form,

$$\langle 2 \rangle \qquad c_t = y_t + B_t - k_t - R_{t-1}^{-1} B_{t-1}, \qquad t \geqslant \tau; \qquad \text{and}$$

$$\left.\begin{aligned} R_{\tau-1}^{-1} B_{\tau-1} &= B^0 \\ k_{\tau-1} &= k^0 \end{aligned}\right\} \text{given,}$$

where B_t is the stock of single-period bonds in units of the commodity issued to foreigners at t, c_t is consumption of the commodity at t, and $R_t \equiv 1/(1+r_t)$, where r_t is the one-period interest rate on bonds at t. At time τ, accumulated debt, $R_{\tau-1}^{-1} B_{\tau-1}$, and last period's investment, $k_{\tau-1}$, are predetermined. To ensure that indebtedness does not grow without bound, we impose a transversality condition requiring that the present value of international debt, arbitrarily far in the future, be zero; *i.e.*,

$$\langle 3 \rangle \qquad \lim_{t \to \infty} \alpha_t(\tau) B_t = 0,$$

where

$$\alpha_t(\tau) \equiv R_\tau R_{\tau+1} \ldots R_{t-1}, \qquad (t \geqslant \tau + 1)$$

is the present value of a unit of commodity at t. Notice that $\alpha_\tau(\tau) \equiv 1$.

Preferences are assumed to be time-separable and logarithmic in consumption and are given by,

$$\langle 4 \rangle \qquad U(\tau) = \sum_{t=\tau}^{\infty} \beta^{(t-\tau)} \log c_t, \qquad 0 < \beta < 1,$$

where β is a subjective discount factor. Assuming that agents have perfect foresight, the sequentially optimal decisions of the domestic resident are given by the sequences $\{c_t\}$, $\{k_t\}$, and $\{B_t\}$, $(t = \tau, \tau+1, \tau+2, \ldots)$, which maximize $U(\tau)$ subject to the constraints ⟨1⟩–⟨3⟩. Notice that there are two investment alternatives available to the agent. He can lend to (or borrow from) foreigners and earn a return of $1/R_t$, or he can invest in domestic production and earn the marginal product of capital, $\theta y_{t+1}/k_t$. Since $\beta^{(t-\tau)} c_t^{-1}$ is the marginal utility of consumption at time t, the agent undertakes each investment to equate the marginal utility of a unit of the consumption good foregone for investment to be discounted marginal utility of the return from the investment. Thus, the first-order, necessary conditions for this problem are (as shown in the Appendix),

$$\langle 5 \rangle \qquad c_t^{-1} = \beta c_{t+1}^{-1} R_t^{-1}, \qquad t \geqslant \tau,$$

$$\langle 6 \rangle \qquad c_t^{-1} = \beta c_{t+1}^{-1} (\theta y_{t+1}/k_t), \qquad t \geqslant \tau.$$

Equating equations ⟨5⟩ and ⟨6⟩, it is seen that the return on the bond is equal to the marginal product of capital. This yields an investment decision rule in which the commitment of capital depends only on the current one-period interest rate and next period's output, *i.e.*,

⟨7⟩ $$k_t = \theta R_t y_{t+1}, \qquad t \geqslant \tau.$$

In the Appendix, we derive the following consumption rule in which domestic residents consume a constant fraction $(1-\beta)$ of their wealth, w_t,

⟨8⟩ $$c_t = (1-\beta)w_t, \qquad t \geqslant \tau,$$

where

$$w_t = \frac{\beta^{(t-\tau)}}{\alpha_t(\tau)} w_\tau \qquad \text{and,}$$

$$w_\tau = y_\tau + (1-\theta)\left[\sum_{j=\tau}^{\infty} \alpha_j(\tau) y_j\right] - B^0.$$

Current-period wealth, w_τ, is the sum of current-period home output and the present value of labor's income stream less initial indebtedness.[5] The commodity input, or 'capital' is not a durable asset since it is fully used up in the production process and, as such, has no income stream to contribute to wealth.

Substituting the investment and consumption rules ⟨7⟩ and ⟨8⟩ into the budget constraint ⟨2⟩, it is seen that borrowing in period t is undertaken to finance the excess of spending, $c_t + k_t$, over savings, $\beta(y_t - R_{t-1}^{-1} B_{t-1})$, and can be expressed as,

⟨9⟩ $$B_t = (1-\beta)(1-\theta)\left[\sum_{j=\tau}^{\infty} \frac{\alpha_{t+j}(\tau)}{\alpha_t(\tau)} y_{t+j}\right] + \theta R_t y_{t+1} - \beta(y_t - R_{t-1}^{-1} B_{t-1}), \qquad t \geqslant \tau.$$

I.B. The Foreign Country

Residents of the foreign country have access to the same technology but may experience different levels of productivity. The foreign production function is given by,

⟨10⟩ $$y^*_{t+1} = \lambda^*_{t+1} k_t^{*\theta},$$

where asterisks denote the variables of the foreign country. Foreigners face sequential budget constraints of the form,

⟨11⟩ $$c^*_t = y^*_t + B^*_t - k^*_t - R_{t-1}^{-1} B^*_{t-1}, \qquad t \geqslant \tau, \qquad \text{and}$$

$$\left.\begin{aligned} R_{\tau-1}^{-1} B^*_{\tau-1} &= B^{*0} \\ k^*_{\tau-1} &= k^{*0} \end{aligned}\right\} \text{given.}$$

Claims on home residents and capital inputs inherited from the past are taken as given. We also impose the transversality condition,

⟨12⟩ $$\lim_{t \to \infty} \alpha_t(\tau) B^*_t = 0.$$

Foreign preferences are assumed to be identical to those of domestic agents and are given by,

⟨13⟩ $$U^*(\tau) = \sum^{\infty} \beta^{(t-\tau)} \log c^*_t.$$

Thus the only differences between the two countries arise from the different levels of productivity they experience and their initial debt positions. The assumption that subjective rates of time preference are identical across countries rules out a number of 'transfer problem criteria' in the analysis.[6] We make this assumption to preserve analytical tractability of the model and to ensure that a steady state exists. However, it does not seem that this assumption is unreasonable, *a priori*.

The foreign agent chooses sequences $\{c_t^*\}$, $\{k_t^*\}$, and $\{B_t^*\}$, $(t=\tau, \tau+1, \tau+2, \ldots)$, which maximize $U^*(\tau)$ subject to the constraints ⟨10⟩–⟨13⟩, and obtains the following decision rules which are completely analogous to those of the domestic agent:

$$\langle 14\rangle \qquad c_t^* = (1-\beta)w_t^*, \qquad t \geqslant \tau,$$

where

$$w_t^* = \frac{\beta^{(t-\tau)}}{\alpha_t(\tau)} w_\tau^*, \qquad \text{and,}$$

$$w_\tau^* = y_\tau^* + (1-\theta)\left[\sum_{j=\tau}^{\infty} \alpha_j(\tau) y_j^*\right] - B^{*0};$$

$$\langle 15\rangle \qquad k_t^* = \theta R_t y_{t+1}, \qquad t \geqslant \tau; \qquad \text{and,}$$

$$\langle 16\rangle \qquad B_t^* = (1-\beta)(1-\theta)\left[\sum_{j=\tau}^{\infty} \frac{\alpha_{t+j}(\tau)}{\alpha_t(\tau)} y_{t+j}^*\right] + \theta R_t y_{t+1}^* - \beta(y_t^* - R_{t-1}^{-1} B_{t-1}^*), \qquad t \geqslant \tau.$$

II. International Market Clearing

The distinction among goods is that they are dated at different times so that the only prices to be determined are the intertemporal terms of trade or one-period market discount factors, $\{R_t\}$, $(t=\tau, \tau+1, \tau+2, \ldots)$. Once these are determined, general solutions for all quantities are readily obtained.

There are two markets which must clear each period—the market for financial claims and the market for goods. In the capital market, equilibrium requires that the home country's borrowing equals the foreign country's lending, *i.e.*, $B_t = -B_t^*$. The goods market is in equilibrium when world output, $\bar{y} = y + y^*$, equals world demand, $c + c^* + k + k^*$. By direct substitution, these conditions jointly imply that general equilibrium is attained when,

$$\langle 17\rangle \qquad \bar{y}_t = (1-\beta)\left[\bar{y}_t + (1-\theta)\sum_{j=\tau-1}^{\infty} \alpha_j(\tau)\bar{y}_j\right] + \theta R_t \bar{y}_{t+1}, \qquad t > \tau.$$

It is shown in the Appendix that the sequence of one-period discount factors and associated long-term discount factors which solves ⟨17⟩ is,

$$\langle 18\rangle \qquad R_t = \frac{\beta \bar{y}_t}{\bar{y}_{t+1}}, \qquad t \geqslant \tau;$$

$$\langle 19\rangle \qquad \alpha_t(\tau) = \frac{\beta^{(t-\tau)}\bar{y}_\tau}{\bar{y}_t}, \qquad t \geqslant \tau.$$

which can be verified by substitution into ⟨17⟩. Hence, the discount factors or the intertemporal terms of trade, are seen to be higher than normal when the quantity of current world output is high relative to future output.

Substituting the equilibrium discount factors into ⟨7⟩ and ⟨15⟩ yields the following solutions for investment:

⟨20⟩ $$k_t = \frac{\beta\theta\bar{y}_t}{1+\left(\frac{\lambda^*_{t+1}}{\lambda_{t+1}}\right)^{1/(1-\theta)}}, \qquad t \geqslant \tau,$$

⟨21⟩ $$k^*_t = \frac{\beta\theta\bar{y}_t}{1+\left(\frac{\lambda_{t+1}}{\lambda^*_{t+1}}\right)^{1/(1-\theta)}}, \qquad t \geqslant \tau.$$

Substituting ⟨20⟩ and ⟨21⟩ in the domestic and foreign production functions, yields the solutions for outputs:

⟨22⟩ $$y_{t+1} = \frac{\lambda_{t+1}(\beta\theta\bar{y}_t)^\theta}{\left(1+\left(\frac{\lambda^*_{t+1}}{\lambda_{t+1}}\right)^{1/(1-\theta)}\right)^\theta}, \qquad t \geqslant \tau,$$

⟨23⟩ $$y^*_{t+1} = \frac{\lambda^*_{t+1}(\beta\theta\bar{y}_t)^\theta}{\left(1+\left(\frac{\lambda_{t+1}}{\lambda^*_{t+1}}\right)^{1/(1-\theta)}\right)^\theta}, \qquad t \geqslant \tau,$$

Finally, adding ⟨22⟩ and ⟨23⟩, the solution for world output is,

⟨24⟩ $$\bar{y}_{t+1} = [\lambda_{t+1}^{1/(1-\theta)} + \lambda_{t+1}^{*1/(1-\theta)}]^{(1-\theta)}(\beta\theta\bar{y}_t)^\theta, \qquad t \geqslant \tau.$$

Equations ⟨22⟩–⟨24⟩ show that the relationship between home and foreign output depends crucially on the intertemporal distribution of the exogenous productivity shocks. The implied dynamics resulting from various shocks are examined in the next section.

III. Comparative Dynamics

In this section we specialize the model by considering the specific time paths governing the exogenous variables λ_t and λ^*_t. In particular, we partition time into the past ($t<\tau$), present ($t=\tau$) and future ($t>\tau$), and assume that,

$$\lambda_t = \begin{cases} \lambda^0 & t<\tau \\ \lambda_\tau & t=\tau \\ \lambda & t>\tau \end{cases} ; \qquad \lambda^* = \begin{cases} \lambda^{*0} & t<\tau \\ \lambda^*_\tau & t=\tau \\ \lambda^* & t>\tau \end{cases} .$$

That is, when decisions are made ($t=\tau$), *future* productivity factors are constant while current-period productivity factors can assume different values. A comparison between steady state equilibria is facilitated by assuming that *past* productivity factors were constant. This specification admits a convenient discussion of transient foreign productivity shocks, say $\lambda^*_\tau \neq \lambda^{*0} = \lambda^*$, or of permanent shocks, say $\lambda^*_\tau = \lambda^{*0} \neq \lambda^*$,[7] and similarly for domestic shocks.

We see from ⟨20⟩–⟨24⟩, that under this specification, the new steady state solutions for investment and output are,

$$\langle 25\rangle \qquad \begin{aligned} k &= (\lambda\beta\theta)^{1/(1-\theta)}, \\ k^* &= (\lambda^*\beta\theta)^{1/(1-\theta)}, \\ y &= [\lambda(\beta\theta)^{\theta}]^{1/(1-\theta)}, \\ y^* &= [\lambda^*(\beta a)^{\theta}]^{1/(1-\theta)}, \\ \bar{y} &= [\lambda^{1/(1-\theta)} + \lambda^{*1/(1-\theta)}](\beta\theta)^{\theta/(1-\theta)}. \end{aligned}$$

The initial steady state solutions are similar except that λ^0 and λ^{*0} replace λ and λ^* in ⟨25⟩. Thus, using the initial steady state solutions and ⟨1⟩ and ⟨10⟩, current-period world output is,

$$\langle 26\rangle \qquad \bar{y}_\tau = \lambda_\tau(\lambda^0\beta\theta)^{\theta/(1-\theta)} + \lambda^*_\tau(\lambda^{*0}\beta\theta)^{\theta/(1-\theta)},$$

and from ⟨24⟩, the time path for world output is given by,

$$\langle 27\rangle \qquad \bar{y}_t = \bar{y}\left(\frac{\bar{y}_\tau}{\bar{y}}\right)^{\theta^{(t-\tau)}}, \qquad t \geqslant \tau.$$

The time paths of domestic investment and output are obtained from equations ⟨20⟩, ⟨22⟩, and ⟨27⟩ and are given by,

$$\langle 28\rangle \qquad k_t = k\left(\frac{\bar{y}_\tau}{\bar{y}}\right)^{\theta^{(t-\tau)}}; \qquad t \geqslant \tau+1; \qquad \text{and,}$$

$$\langle 29\rangle \qquad y_t = y\left(\frac{\bar{y}_\tau}{\bar{y}}\right)^{\theta^{(t-\tau)}}; \qquad t \geqslant \tau+1,$$

where k and y are the steady state levels of domestic investment and output, respectively. Current-period wealth at home now simplifies to,

$$\langle 30\rangle \qquad w_\tau = \frac{\bar{y}_\tau}{1-\beta}\left[(1-\beta)\frac{y_\tau}{\bar{y}_\tau} + \beta(1-\theta)\frac{y}{\bar{y}}\right] - B^0.$$

Combining equations ⟨18⟩, ⟨19⟩, ⟨26⟩, and ⟨27⟩, the evolution of the short- and long-term discount rates is given by,

$$\langle 31\rangle \qquad R_t = \beta\left(\frac{\bar{y}_\tau}{\bar{y}}\right)^{\theta^{(t-\tau)}(1-\theta)}, \qquad t \geqslant \tau+1;$$

$$\langle 32\rangle \qquad \alpha_t(\tau) = \beta^{(t-\tau)}\left(\frac{\bar{y}_\tau}{\bar{y}}\right)^{1-\theta^{(t-\tau)}}, \qquad t \geqslant \tau+1.$$

We note that in the steady state, the short-term market discount factor, R, is equal to the subjective discount factor, β. Furthermore, for any given value of current-period wealth, w_τ, equations ⟨8⟩ and ⟨32⟩ completely describe the evolution of wealth and consumption over time.

The time paths of foreign investment, output, and current wealth are completely analogous and are given by,

$$\langle 33\rangle \qquad k^*_t = k^*\left(\frac{\bar{y}_\tau}{\bar{y}}\right)^{\theta^{(t-\tau)}}; \qquad t \geqslant \tau+1;$$

$$\langle 34\rangle \qquad y_t^* = y^*\left(\frac{\bar{y}_\tau}{\bar{y}}\right)^{\theta^{(t-\tau)}}; \qquad t \geqslant \tau+1; \qquad \text{and,}$$

$$\langle 35\rangle \qquad w_\tau^* = \frac{\bar{y}_\tau}{1-\beta}\left[(1-\beta)\frac{y_\tau^*}{\bar{y}_\tau} + \beta(1-\theta)\frac{y^*}{\bar{y}}\right] + B^0.$$

Finally, we note that domestic borrowing from abroad evolves according to,

$$\langle 36\rangle \qquad B_t = \beta R_{t-1}^{-1} B_{t-1} = \frac{\beta^{(t-\tau)}}{\alpha_t(\tau)} B_\tau, \qquad t \geqslant \tau+1;$$

where,

$$\langle 37\rangle \qquad B_\tau = \beta\bar{y}_\tau[(y/\bar{y}) - (y_\tau/\bar{y}_\tau)] + B^0.$$

From equation ⟨37⟩, it can be seen that current-period borrowing from abroad (*i.e.*, changes in the current account) depends on both the international and intertemporal distribution of world output between the home and foreign countries. For example, given current-period output at home and abroad, an increase in future home output that raises the home share of world output increases current consumption and investment at home which is financed by borrowing from abroad. However, if future output at home and abroad increase in the same proportion, current period borrowing is unaffected.

Following the initial adjustment in current-period borrowing, B_τ, future borrowing occurs to maintain debt at a constant level in terms of utility. This can be seen by noting that the intertemporal marginal rate of substitution of consumption between period $t-1$ and t, $c_{t-1}^{-1}/(\beta c_t^{-1})$, is equal to R_{t-1}^{-1}. Thus it is seen from equation ⟨36⟩ that, when interest rates are higher than normal ($R_t < \beta$; $t \geqslant \tau$), external debt is rising over time. Higher interest rates reduce the relative price of future goods in terms of present goods making it optimal to shift the debt burden to the future when it is cheaper to service in terms of utility. Steady state debt, B, is constant and is equivalent to a consol. The constant levels of debt service payments each period are financed by the excess of output over spending in perpetuity.

III.A. The Transmission of Transient Productivity Shocks

Suppose the system is initially in steady state equilibrium, and at time τ it is revealed that a one-period increase in λ_τ^* occurs, while λ remains constant. That is,

$$\lambda_t = \lambda^0 = \lambda, \forall\, t, \qquad \text{and} \qquad \lambda_\tau^* > \lambda^{*0} = \lambda^* = \lambda_t^*, \qquad t \neq \tau.$$

Since the shock is purely transitory, it has no effect on the steady state.

From equation ⟨26⟩ it can be seen that an increase in λ_τ^* raises current-period foreign and world output (y_τ^* and $\bar{y}_\tau$, respectively), while current-period home output, y_τ, is unchanged. Equations ⟨28⟩–⟨31⟩ imply that the market discount rates and current consumption and investment in both countries increase. From equation ⟨37⟩, B_τ is seen to rise, indicating an increase in domestic borrowing from abroad.

The windfall gain of foreign output is partially consumed, invested in foreign production, and invested in the international bonds. Higher investment abroad lowers the marginal product of capital there. Since equilibrium requires that the return to investment equal the return on the bond, the world interest rate declines

(R_τ increases). Lower interest rates imply increased investment at home thereby raising domestic output in future periods. Current-period domestic and foreign wealth increase because of the lower market interest rates and higher output levels that prevail during the transition to the steady state. Since current-period output at home is unchanged, the increase in domestic consumption and investment is financed entirely by the increase in current-period borrowing.

Since current-period investment increases in both countries, output at home and abroad increase above their steady state levels in future periods. The unanticipated transitory shock to foreign productivity is thus positively transmitted to domestic output with a one-period lag.

The dynamics of external debt can be inferred from equations ⟨31⟩, ⟨32⟩, ⟨36⟩, and ⟨37⟩. As previously mentioned, current-period borrowing at home, B_τ, increases. Since the short-term discount rates increase above the steady state levels during the transition, ($R_t \geqslant \beta$; $t \geqslant \tau$), it follows from equation ⟨36⟩ that, following the shock, external debt declines over time. Steady state indebtedness is seen to be, $B = (\bar{y}/\bar{y}_\tau)B_\tau$. Thus if net external debt is initially zero, the level of debt remains positive in the steady state and is rolled over indefinitely.

Both foreign and domestic consumption rise at time τ, then decline exponentially toward their steady state values. The foreign country, which benefits from the transitory productivity shock, consumes more in the new steady state and the home country consumes less. These consumption paths are made possible by domestic borrowing from abroad. The foreigners cannot completely smooth their consumption stream through investment because of decreasing returns to capital. However, the domestic residents borrow at lower interest rates to raise current consumption and service that debt in the future. This intertemporal transfer of consumption assures that the ratio of foreign to domestic marginal utility is constant over time.

III.B. *The Transmission of Permanent Productivity Shocks*

Suppose that the system is again initially in a steady state equilibrium, but at time τ it is revealed to agents that a permanent increase in foreign productivity will occur at time $\tau+1$ while domestic productivity remains unchanged. That is,

$$\lambda_t = \lambda^0 = \lambda, \forall\, t, \qquad \text{and} \qquad \lambda_s^* = \lambda^{*0} < \lambda^* = \lambda_t^*, \qquad s \leqslant \tau < t.$$

Current world output is predetermined and world investment is a constant fraction of $\bar{y}_\tau$ (see equations ⟨20⟩ and ⟨21⟩). The increase in future foreign productivity raises steady state foreign output and leaves steady state domestic output unaffected. However, the immediate effect is to raise the demand for k_τ^* relative to k_τ. The permanent foreign productivity shock anticipated to occur next period is thus *negatively* transmitted to the home country since current investment to next period's production increases abroad but declines at home. At time $\tau+1$, domestic and foreign outputs jump in opposite directions, then both increase monotonically toward their steady state values.

The expected increase in world output from $\bar{y}_\tau$ to $\bar{y}_{\tau+1}$ leads to a fall in the market discount factor which rises exponentially back to β as output grows. The lower discount rates tend to lower wealth; however, for the foreign country, this effect is more than offset by rising foreign output resulting from higher productivity and investment. For the home economy, the decline in domestic investment reinforces the discount rate effect leading to a fall in wealth. Hence, c_τ^* rises and c_τ falls. The rising discount rate and national output levels over time imply that future consumption at home and abroad rise above current (time τ) consumption levels.

Since current-period foreign output, y_τ^*, is predetermined, the increase in foreign spending ($c_\tau^* + k_\tau^*$) must be financed through international borrowing. The supply of credit is forthcoming in response to the higher interest rates and lower demand for domestic consumption. Therefore, B_τ falls (B_τ^* rises) and the dynamics of external debt can again be inferred from equations ⟨31⟩, ⟨32⟩, ⟨36⟩, and ⟨37⟩. Although debt declines in the current period, it is seen from equation ⟨31⟩ that during the transition to the new steady state, short-term discount factors are lower than normal (short-term interest rates are higher than normal). Thus external debt falls during the transition, and steady state indebtedness is less than current indebtedness ($B < B_\tau$, or $B^* > B_\tau^*$). As in the discussion of the transitory shocks, a comparison of debt between the two steady states depends on the level of initial steady state debt, $B_{\tau-1}$. However, for moderate levels of initial debt (*e.g.*, $B_{\tau-1} = 0$), home indebtedness to foreigners in the new steady state declines.

Agents attempt to smooth their consumption in response to the future rise in output. Foreigners want to consume some of their future income immediately. This is achieved by enticing, through high yields, home residents to trade off current consumption for an increase in steady state income arising from the interest on the foreign country's debt.

IV. Conclusions and Extensions

In this paper, we have analyzed international economic interdependence in a two-country, general-equilibrium model with production. We showed that a transitory foreign-productivity shock lowers world interest rates and raises domestic borrowing from abroad. The decline in interest rates persists over time and both foreign and domestic output increases because the unanticipated increase in income is largely invested. Conversely, a permanent increase in foreign productivity leads to a period of increased interest rates and a decline in borrowing from abroad. This shock is negatively transmitted to the home country as output increases abroad while it declines at home.

The mechanism which transmits and propagates country-specific supply shocks in our model is that higher output in one industry leads to greater use of that output as an input in all industries next period. Since each industry is separately owned by distinct agents, the efficient flow of capital must be financed by explicit borrowing. Further research might be directed towards an extension to the uncertainty case and the introduction of equities.[8] Such a framework can be used to study the variances and covariances of endogenous variables across countries. In addition, the number of produced commodities might be expanded in order to analyze the interaction of terms of trade fluctuations and world debt.

Our model is also useful for analyzing the impact of foreign economic growth on domestic lending and production. When capital flows toward highly productive investment opportunities abroad, domestic investment falls as lending abroad increases. The subsequent decline in domestic production is efficient and welfare-improving for the home country because the lost domestic output is more than offset by higher interest payments on the loans. However, in an economy with separate ownership of capital and labor, such capital flows may generate internal conflicts since they reduce the productivity of domestic labor.

Appendix

We consider the finite horizon analogues to the infinite horizon problems for the representative domestic and foreign agents presented in the text. We derive explicit solutions for the time paths of market discount rates and consumption at home and abroad. The analogous solutions presented in the text follow straightforwardly when the horizon approaches infinity.

Suppose the world ends at $\tau+T$. Agents realize that at that date borrowing and investment must be zero and consumption will be current output minus debt. The domestic agent's decision rules maximize the following problem, taking the sequence of market discount rates $\{R_t\}$ and future productivities $\{\lambda_t\}$ as given.

$$\max_{\{c_t, k_t, B_t\}_{t=\tau}^{\tau+T}} \sum_{t=\tau}^{\tau+T} \beta^{(t-\tau)} \log c_t$$

subject to:

$$\langle A1\rangle \qquad k_{\tau-1} \quad \text{and} \quad R_{\tau-1}^{-1} B_{\tau-1} \quad \text{given;}$$

$$\langle A2\rangle \qquad k_{\tau+T} = B_{\tau+T} = 0$$

$$\langle A3\rangle \qquad y_{t+1} = \lambda_{t+1} k_t^{\theta}, \qquad t=\tau, \tau+1, \ldots, \tau+T; \qquad \text{and,}$$

$$\langle A4\rangle \qquad c_t = y_t - k_t + B_t - R_{t-1}^{-1} B_{t-1}, \qquad t=\tau, \tau+1, \ldots, \tau+T.$$

The first-order conditions are:

$$\langle A5\rangle \qquad c_t^{-1} = \beta c_{t+1}^{-1} R_t^{-1}, \qquad t=\tau, \tau+1, \ldots, \tau+T-1; \qquad \text{and,}$$

$$\langle A6\rangle \qquad c_t^{-1} = \beta c_{t+1}^{-1}(\theta y_{t+1}/k_t), \qquad t=\tau, \tau+1, \ldots, \tau+T-1.$$

The analogous problem for the foreign agent leads to the following constraints and first-order conditions:

$$\langle A1^*\rangle \qquad k_{\tau-1}^* \quad \text{and} \quad R_{\tau-1}^{-1} B_{\tau-1}^* \quad \text{given;}$$

$$\langle A2^*\rangle \qquad k_{\tau+T}^* = B_{\tau+T}^* = 0;$$

$$\langle A3^*\rangle \qquad y_{t+1}^* = \lambda_{t+1}^* k_t^*, \qquad t=\tau, \tau+1, \ldots, \tau+T-1;$$

$$\langle A4^*\rangle \qquad c_t^* = y_t^* - k_t^* + B_t^* - R_{t-1}^{-1} B_{t-1}^*, \qquad t=\tau, \tau+1, \ldots, \tau+T-1;$$

$$\langle A5^*\rangle \qquad c_t^{*-1} = \beta c_{t+1}^{*-1} R_t^{-1}, \qquad t=\tau, \tau+1, \ldots, \tau+T-1;$$

$$\langle A6^*\rangle \qquad c_t^{*-1} = \beta c_{t+1}^{*-1}(\theta y_{t+1}^*/k_t^*), \qquad t=\tau, \tau+1, \ldots, \tau+T-1.$$

Equations ⟨A1⟩–⟨A6⟩ are sufficient to solve for $\{c_t, k_t, y_t, B_t\}$ and ⟨A1*⟩–⟨A6*⟩ for $\{c_t^*, k_t^*, y_t^*, B_t^*\}$ for $t=\tau, \tau+1, \ldots, \tau+T$. The sequence of current and future market discount rates is determined by the requirement that period-by-period demand equal supply for domestic debt. By Walras' Law, these rates must also be consistent with the condition for sequential goods market clearing.

Proposition 1. For a world economy of finite horizon T, the sequence of current and future market discount rates is given by,

$$R_t = \frac{\bar{y}_t \cdot \beta \sum_{j=0}^{\tau+T-t-1} (\beta\theta)^j}{\bar{y}_{t+1} \sum_{j=0}^{\tau+T-t} (\beta\theta)^j}, \qquad t=\tau, \tau+1, \ldots, \tau+T-1.$$

Proof. The proof is by induction. First we show that ⟨A7⟩ is correct for $R_{\tau+T-1}$. Market clearing in period $\tau+T-1$ requires that,

⟨A8⟩ $$\bar{y}_{\tau+T-1} = c_{\tau+T-1} + c^*_{\tau+T-1} + k_{\tau+T-1} + k^*_{\tau+T-1}.$$

Substituting ⟨A2⟩, ⟨A2*⟩, ⟨A4⟩, ⟨A4*⟩, ⟨A5⟩, and ⟨A5*⟩ into ⟨A8⟩ and solving for $R_{\tau+T-1}$ yields,

⟨A9⟩ $$R_{\tau+T-1} = \frac{\beta \bar{y}_{\tau+T-1}}{(1+\beta\theta)\bar{y}_{\tau+T}},$$

which is consistent with the solution ⟨A7⟩. Now assume that ⟨A7⟩ describes the solution for $\{R_t\}_{t=\tau+T-n}^{\tau+T-1}$. The market clearing condition for $\tau+T-n-1$ is,

⟨A10⟩ $$\bar{y}_{\tau+T-n-1} = c_{\tau+T-n-1} + c^*_{\tau+T-n-1} + k_{\tau+T-n-1} + k^*_{\tau+T-n-1}.$$

The conditions ⟨A5⟩ and ⟨A5*⟩ imply,

⟨A11⟩ $$c_{\tau+T-n-1} + c^*_{\tau+T-n-1} = (c_{\tau+T-1} + c^*_{\tau+T-1})\frac{\alpha_{\tau+T}(\tau)}{\beta^{n+\tau}\alpha_{\tau+T-n-1}(\tau)}.$$

Noting that $c_{\tau+T} + c^*_{\tau+T} = \bar{y}_{\tau+T}$ and substituting the conjectured solutions for $\{R_t\}_{t=\tau+T-n}^{t+T-1}$ into ⟨A11⟩ yields,

⟨A12⟩ $$c_{\tau+T-n-1} + c^*_{\tau+T-n-1} = R_{\tau+T-n-1}\bar{y}_{\tau+T-n}\beta\left(\sum_{j=0}^{n}(\beta\theta)^j\right)^{-1}$$

Substituting ⟨A6⟩, ⟨A6*⟩, and ⟨A12⟩ into ⟨A10⟩ and solving for $R_{\tau+T-n-1}$ yields,

⟨A13⟩ $$R_{\tau+T-n-1} = \frac{\bar{y}_{\tau+T-n-1}\beta\sum_{j=0}^{n}(\beta\theta)^j}{\bar{y}_{\tau+T-n}\sum_{j=0}^{n+1}(\beta\theta)^j}$$

which is consistent with the proposed solution ⟨A7⟩.

Proposition 2. For a world economy of finite horizon T, the sequence of domestic consumption rates is given by,

⟨A14⟩ $$c_t = \left(\sum_{j=0}^{\tau+T-t}\beta^j\right)^{-1}\left[y_t - R_{t-1}^{-1}B_{t-1} + (1-\theta)\sum_{j=1}^{\tau+T-t}\left(y_{t+j}\frac{\alpha_{t+j}(\tau)}{\alpha_t(\tau)}\right)\right],$$
$$t = \tau, \tau+1, \ldots, \tau+T.$$

Proof. (Sketch) The proof is again by induction. The proposed solution is obviously valid for $c_{\tau+T}$ and $c_{\tau+T-1}$. By exploiting equations ⟨A4⟩–⟨A6⟩, one can show that the assumption that ⟨A14⟩ is valid for $\{c_t\}_{t=\tau+T-n}^{\tau+T}$ implies that ⟨A14⟩ is valid for $c_{\tau+T-n-1}$.

Notes

1. The literature on international macroeconomic interdependence is extensive. See Mussa's (1979) survey article on international transmissions and the references therein.
2. For example, under fixed exchange rates, an expansionary fiscal policy abroad raises foreign output but lowers it at home; whereas, under flexible rates, both foreign and domestic output rise.
3. This empirical regularity is not interpreted as proof of the violation of the Mundell–Fleming hypothesis since, in an n-country world, disturbances originating from the nth country could result in the observed positively correlated movements in outputs in the other $n-1$ countries.
4. Frenkel and Razin (1985) analyze a similar structure but without production.
5. $(1-\theta)$ is labor's share of output.

6. For an analysis of how differing subjective discount rates across countries are important when agents are infinitely lived, see Frenkel and Razin (1985) and also Buiter (1981) in an overlapping generations framework. Essentially, ruling out different discount rates in the present analysis makes the distribution of income across the world less important.
7. The qualitative results of this section are unchanged when permanent productivity shocks are assumed to occur at time τ; however, the algebra becomes more tedious in that case.
8. In a related paper (see Cantor and Mark (1986)), we have examined the case in which (λ_t, λ_t^*) is an i.i.d. random vector, which corresponds to the transient productivity shocks considered in this paper. The transmission results obtained in this paper and the intuition regarding capital market linkages, investment, and consumption smoothing carry over to the uncertainty case. Under uncertainty, however, debt instruments alone are unable to effect efficient risk sharing and a need for additional financial instruments arise. For the case that we considered, equity claims against the firm's earnings are sufficient to span the uncertainty and admits an efficient allocation of risk.

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